Limiting angle of friction

It may be defined as the angle which the resultant reaction R makes with the normal reaction R_N.

$$\tan\phi = F/R_N$$

Angle of Repose

If the angle of inclination α of the plane to the horizontal is such that the body begins to slide down the plane, then the angle α is called angle of repose.

$$W\sin\alpha = F = \mu R_N = \mu W\cos\alpha$$

$$\tan\alpha = \mu = \tan\phi$$

$$\alpha = \phi$$

Efficiency

The efficiency of an inclined plane, when a body slides up the plane, is defined as the ratio of force required to move the body without consideration and with consideration of friction of force.

The efficiency of an inclined plane, when a body slides down the plane is defined as the ratio of forces required to move the body with and without consideration of forces.

Inclined plane

Motion up the plane

$$F = \frac{W \sin(\alpha+\phi)}{\sin[\theta-(\alpha+\phi)]}$$

2. **Motion down the plane**

$$F = \frac{W \sin(\phi-\alpha)}{\sin[\theta+(\phi-\alpha)]}$$

Screw Threads

Square Threads

If the w.t is to be lifted

$$F = \frac{W \sin(\alpha+\phi)}{\sin[\theta-(\alpha+\phi)]}$$

$$F = \frac{W \sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$$= W \frac{\tan\alpha \tan\phi}{1-\tan\alpha \tan\phi}$$

$$= W \frac{l/\pi d \cdot \mu}{1- l/\pi d \cdot \mu}$$

$$= W \frac{l+\mu\pi d}{\pi d-\mu l}$$

Let f = force applied at the end of the bar of length L

$$fL = Fr$$

$$f = \frac{Fr}{L} = \frac{Wr}{L}\tan(\alpha+\phi)$$

If the w.t is to be lowered

$$F = \frac{W\sin(\phi-\alpha)}{\sin[\theta+(\phi-\alpha)]}$$

$$= W\tan(\phi-\alpha)$$

$$f = \frac{Wr}{L}\tan(\phi-\alpha)$$

Screw efficiency

$$\eta_{screw} = \frac{\text{W.D in lifting the load/rev}}{\text{W.D by the applied force/rev}}$$

$$= \frac{W \cdot l}{F \cdot \pi d}$$

$$= \frac{W}{F}\,\frac{l}{\pi d}$$

$$= \frac{\tan\alpha}{\tan(\alpha+\phi)}$$

$$(\eta_{screw})_{max} = \frac{1-\sin\phi}{1+\sin\phi}$$

when $\alpha = 45° - \frac{\phi}{2}$

Mechanical Advantage

$$M.A = \frac{\text{W.t lifted}}{\text{force applied}}$$

$$= \frac{W}{f}$$

$$= \frac{W}{\frac{Wr}{L}\tan(\alpha+\phi)}$$

$$= \frac{L}{r}\cot(\alpha+\phi)$$

velocity ratio

$$V.R = \frac{\text{distance moved by force/rev}}{\text{distance moved by load/rev}}$$

$$= \frac{2\pi L}{l}$$

$$= \frac{L}{\frac{l}{\pi d}\,\frac{d}{2}}$$

$$= \frac{L}{r\tan\alpha}$$

Overhauling of Screw and Self Locking of Screw

$$F = W \tan(\phi - \alpha)$$

$$T = W \tan(\phi - \alpha) \times \frac{d}{2}$$

If $\phi < \alpha$, then the torque required to lower the load will be negative i.e. The load will start moving downward without the application of any torque. Such a condition is known as overhauling of screw.

If $\phi > \alpha$, the torque required to lower the load will be +ve i.e an effort is required to lowered the load. Such a screw is known as self locking of screw.

Screw Jack

Torque required to lift the body by a Screw Jack

effort applied at the circumference of the screw to lift the load

$$F = W \tan(\alpha + \phi)$$

Torque required to overcome frict^n b/w the screw and nut

$$T_1 = F \times \frac{d}{2} = W \tan(\alpha + \phi) \frac{d}{2}$$

Torque required to overcome friction at collar

$$T_2 = \mu_1 W \left(\frac{R_1 + R_2}{2}\right) = \mu_1 W R$$

μ_1 = coefficient of frict^n of collar

Total torque required to overcome friction

$$T = T_1 + T_2$$

$$T = F \times \frac{d}{2} + \mu_1 W R$$

If an effort F is applied at the end of a lever of arm length L, the total torque required to overcome friction must be equal to the torque applied at the end of the lever

$$T = F \times \frac{d}{2} + \mu_1 WR = f \times L$$

$$d = \frac{d_o + d_c}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2}$$

1. Flat pivot Bearing

W = Load transmitted over bearing surface

R = radius of bearing surface

p = intensity of pressure per unit area of bearing surface b/w rubbing surface

μ = co efficient of frict^n

1. considering uniform pressure

When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{\pi R^2}$$

consider a ring of radius r and thickness dr of the bearing area

Area of bearing surface $A = 2\pi r\, dr$

Load transmitted to ring

$$\delta W = p * A = p * 2\pi r\, dr$$

Frictional resistance to sliding on the ring acting tangentially at radius r

$$F_r = \mu \delta W = \mu p * 2\pi r\, dr$$

$$F_r = 2\pi \mu p r\, dr$$

Frictional torque on the ring

$$T_r = F_r * r = 2\pi \mu p r\, dr * r$$

$$T_r = 2\pi \mu p r^2\, dr$$

Total frictional torque,

$$T = \int_0^R 2\pi \mu p r^2 \, dr$$

$$= \frac{2}{3} \mu W R$$

When the shaft rotates at ω rad/s then power lost in frict^n

$$P = T * \omega$$

2. Considering uniform wear

$$P r = C$$

Load transmitted to ring

$$\delta W = p * 2\pi r \, dr$$

$$= \frac{C}{r} * 2\pi r \, dr$$

$$= 2\pi C \, dr$$

Total load transmitted to bearing

$$W = \int_0^R 2\pi C \, dr$$

$$C = W / 2\pi R$$

Frictional torque acting on the ring

$$T_r = 2\pi \mu p r^2 \, dr$$

$$T_r = 2\pi \mu C r \, dr$$

Total frict^{n}al torque on the bearing

$$T = \int_0^R 2\pi \mu C r \, dr$$

$\Rightarrow \pi \mu C R$

$$T = \frac{1}{2} \mu W R$$

Conical pivot bearing

p_n = intensity of pressure normal to the cone

α = Semi angle of cone

μ = coefficient of frictn b/w shaft and bearing

R = radius of the shaft

$$\sin\alpha = dr/dl$$

area of the ring

$$A = 2\pi r \, dl$$

$$A = 2\pi r \, dr \operatorname{cosec}\alpha$$

1. Consider uniform pressure

Normal load acting on the ring

δW_n = Normal pressure * area

$$= p_n * 2\pi r \, dr \operatorname{cosec}\alpha$$

vertical load acting on the ring

δW = vertical component of δW_n

$$= \delta W_n \sin\alpha$$

$$= p_n 2\pi r \, dr \operatorname{cosec}\alpha \sin\alpha$$

$$= p_n 2\pi r \, dr$$