JHARSUGUDA ENGINEERING SCHOOL, JHARSUGUDA

LECTURE NOTES

ON

ANALOG AND DIGITAL COMMUNICATION

(5th Sem. ETC)

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DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATION ENGINEERING

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Syllabus

Unit-1: Elements of Communication Systems: Communication Process- Concept of Elements of Communication System & its Block diagram, Source of information & Communication Channels, Classification of Communication systems (Line & Wireless or Radio), Modulation Process, need of modulation and classify modulation process, Analog and Digital Signals & its conversion, Basic concept of Signals & Signals classification (Analog and Digital), Bandwidth limitation

Unit-2: Amplitude (linear) Modulation System: Amplitude modulation & derive the expression for amplitude modulation signal, power relation in AM wave & find Modulation Index, Generation of Amplitude Modulation(AM)- Linear level AM modulation only, Demodulation of AM waves (liner diode detector, square law detector & PLL), Explain SSB signal and DSBSC signal, Methods of generating & detection SSB-SC signal (Indirect method only), Methods of generation DSB-SC signal (Ring Modulator) and detection of DSB-SC signal (Synchronous detection), Concept of Balanced modulators, Vestigial Side Band Modulation.

Unit-3: Angle Modulation Systems: Concept of Angle modulation & its types (PM & FM), Basic principle of Frequency Modulation & Frequency Spectrum of FM Signal, Expression for Frequency Modulated Signal & Modulation Index and sideband of FM signal, Explain Phase modulation & difference of FM & PM)- working principle with Block Diagram, Compare between AM and FM modulation (Advantages & Disadvantages), Methods of FM Generation (Indirect (Armstrong) method only) working principle with Block Diagram, Methods of FM Demodulator or detector (Forster-Seely & Ratio detector)- working principle with Block Diagram.

Unit-4: AM & FM Transmitter & Receiver: Classification of Radio Receivers, Define the terms Selectivity, Sensitivity, Fidelity and Noise Figure, AM transmitter - working principle with Block Diagram, Concept of Frequency conversion, RF amplifier & IF amplifier, Tuning, S/N ratio, working of super heterodyne radio receiver with Block diagram, Working of FM Transmitter & Receiver with Block Diagram.

Unit-5: Analog to Digital Conversion & Pulse Modulation System: Concept of Sampling Theorem , Nyquist rate & Aliasing, Sampling Techniques (Instantaneous, Natural, Flat Top), Analog Pulse Modulation - Generation and detection of PAM, PWM & PPM system with the help of Block diagram & comparison of all above, Concept of Quantization of signal & Quantization error, Generation & Demodulation of PCM system with Block diagram & its applications, Companding in PCM & Vocoder, Time Division Multiplexing & explain the operation with circuit diagram, Generation & demodulation of Delta modulation with Block diagram, Generation & demodulation of DPCM with Block diagram, Comparison between PCM, DM , ADM & DPCM.

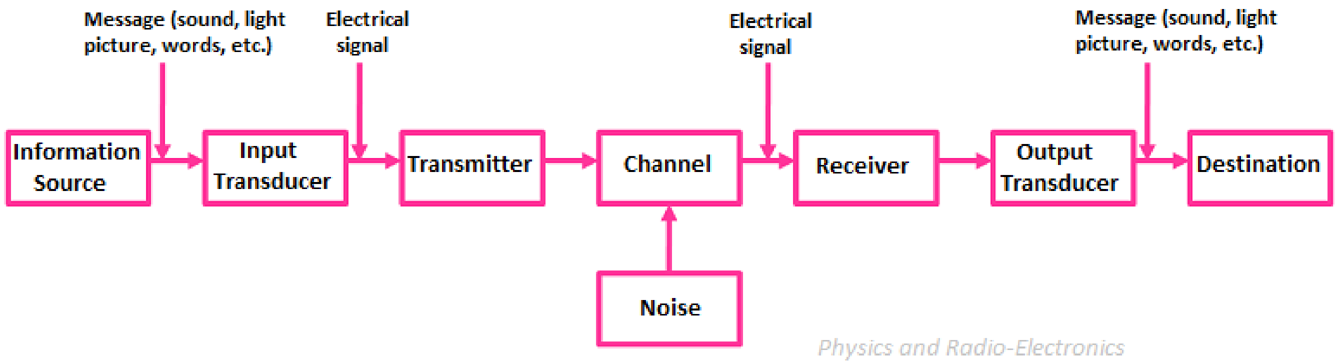
Unit-6: Digital Modulation Techniques: Concept of Multiplexing (FDM & TDM)- (Basic concept , Transmitter & Receiver) & Digital modulation formats, Advantages of digital communication system over Analog system, Digital modulation techniques & types, Generation and Detection of binary ASK, FSK, PSK, QPSK, QAM, MSK, GMSK, Working of T1-Carrier system, Spread Spectrum & its applications, Working operation of Spread Spectrum Modulation Techniques (DS-SS & FH-SS), Define bit, Baud, symbol & channel capacity formula.(Shannon Theorems), Application of Different Modulation Schemes, Types of Modem & its Application.

Unit-1

Elements of Communication Systems

Communication Process: Concept of Elements of Communication System & its Block diagram:

Communication is the process of transmit an information bearing signal, from a source to a destination located at another point. The essential components of a communication system are information source, Input Transducer, Transmitter, Channel, Receiver, Output Transducer and Destination.



(Block diagram of a communication system)

Information Source: The function of information source is to produce required message which has to be transmitted.
Input Transducer: A transducer is a device which converts one form of energy into another form. When the message produced by the information source is not electrical in nature, an input transducer is used to convert it into a time-varying electrical signal.
Transmitter: The function of the transmitter is to process the electrical signal from different aspects. Inside the transmitter, signal processings such as ranging the frequencies, amplification and modulation are achieved.
Channel: Channel means the medium through which the message travels from the transmitter to the receiver. In other words, the function of the channel is to provide a physical connection between the transmitter and the receiver.
Receiver: The main function of the receiver is to reproduce the message signal in electrical form from the distorted received signal. This reproduction of the original signal is accomplished by a process known as the demodulation. Demodulation is the reverce process of modulation carried out in transmitter.
Destination: Destination is the final stage which is used to convert an electrical message signal into its original form.

Source of information:

The telecommunication environment is dominated by the following four important sources of information:

i. Speech
ii. Television
iii. Fax
iv. Personal Computers

Speech: Speech is the primary method for human communication. Basically, the speech communication process involves the transfer of information from a speaker to a listener, which takes place in three successive stages:

a. *Production:* An intended message in the speaker's mind is represented by a speech signal that consists of sounds generated inside the speaker's mouth.
b. *Propagation:* The sound wave propagate through the air, reaching the listener's ears.
c. *Perception:* The incoming sounds are deciphered by the listener into a received message, and thus completing the chain of events that results in the transfer of information from the speaker to the listener.

Television: The second source of information, television refers to the transmission of pictures in motion by means of electrical signals. To accomplish this transmission, each complete picture has to be sequentially scanned. The scanning process is carried out in a TV camera.

Fax: The third source of information is fax machine, is to transmit still pictures over a communication channel. Such a machine provides a highly popular facility for the transmission of hand written or printed text from one point to another. Transmitting text by fax machine is treated simply like transmitting a picture.

Personal Computer: Personal computers are becoming increasingly an important part of our daily lives. We use them for electronic mail, exchange of software, and sharing of resources. The text transmitted by a PC is usually encoded using the American Standard Code for Information Exchange (ASCII), which is the first code developed specifically for computer communications.

Communication channels:

The various communication channels are

i. Telephone channels
ii. Optical fibers
iii. Mobile radio channels
iv. Satellite channels

Telephone Channel: A telephone network makes use of a switching mechanism. This switching mechanism is known as circuit switching and it is used to establish an end-to-end communication link on a temporary basis. The telephone channel supports only the transmission of electrical signals.

Optical fibers: An optical fibre is a dielectric waveguide which transports light signals from one place to another just as a metallic wire pair or a co-axial cable, transports electrical signals.

An optical fiber consists of central core within the propagating electromagnetic field is confined and which is surrounded by a cladding layer, which is itself surrounded by a thin protective jacket. Basically, the core and cladding are both made of pure silica glass, whereas, the jacket is made of plastic.

Mobile Radio channels: Mobile radio channel, extends the capability of the public telecommunications network by introducing mobility into the network by virtue of its ability to broadcast.

The term 'mobile radio' is usually meant to encompass terrestrial situation where a radio transmitter or receiver is capable of being moved, regardless of whether it actually moves or not. The energy reaches the receiving antenna via more than one path. Therefore, in a mobile radio environment, we face a problem of multipath phenomenon in the sense that the various incoming radio waves reach their destination from different directions and with different time delays.

Satellite channel: A satellite channel provides broad-area coverage in a continental as well as intercontinental sense. Moreover, access to remove areas not covered by conventional cable or fiber communications is also a distinct feature of satellites. In almost all satellite communication systems, the satellites are placed in geostationary orbit.

Classification of Communication systems (Line & Wireless or Radio):

Regarding the mode of propagation, communication may be divided in the following two forms:

a. Line Communication
b. Wireless or Radio Communication

Line Communication: In line communication, the medium of transmission is a pair of conductors called transmission line. This is also called as line channel. This means that in line communication, the transmitter and the receiver are connected through a wire or line. However, the installation and maintenance of a transmission line is not only costly and complex but also overcrowds the open space. Apart from this, its message transmission capability is also limited.

Wireless or Radio communication: In wireless or radio communication, a message is transmitted through open space by electromagnetic waves called as radio waves. Radio waves are radiated from the transmitter in open space through a device called antenna. A receiving antenna intercepts the radio waves at the receiver. All the radio, TV and satellite broadcasting are wireless or radio communication. The advantages of wireless communication are cost effectiveness, possible long-distance communication and simplicity.

Modulation Process:

Modulation may be defined as the process by which some characteristics of a signal called carrier is varied in accordance with the instantaneous value of another signal called modulating signal. Signals containing information or intelligence are referred as modulating signals. This information bearing signal is called baseband signal. The carrier frequency is greater than the modulating frequency. The signal resulting from the process of modulation is called modulated signal.

Need of modulation:

In order to carry the low frequency message signal to a longer distance, the high frequency carrier signal is combined with it.

a) Reduction in antenna height
b) Long distance communication
c) Ease of radiation
d) Multiplexing
e) Improve the quality of reception
f) Avoid mixing up of other signals

Classify modulation process:

Modulation is basically of two types

i. Continuous Wave Modulation
ii. Pulse Modulation

Continuous Wave Modulation: When the carrier wave is continuous in nature, the modulation process is known as continuous wave modulation or Analog modulation.

Analog modulation is of two types- Amplitude modulation and Angle modulation.

When amplitude of the carrier is varied in accordance with the message signal, it is known as amplitude modulation. When the angle of the carrier is varied according to the instantaneous value of the modulating signal, it is called angle modulation.

Angle modulation may be further subdivided into Frequency modulation and Phase modulation, in which the instantaneous frequency and phase of the carrier, respectively, are varied in accordance with the message signal.

Pulse Modulation: When the carrier wave is pulse type waveform, the modulation process is known as pulse modulation. In pulse modulation, the carrier consists of a periodic sequence of rectangular pulses. Pulse modulation can be of analog or digital type.

The analog pulse modulation may be of following three types

i. Pulse amplitude modulation (PAM)
ii. Pulse duration modulation (PDM)

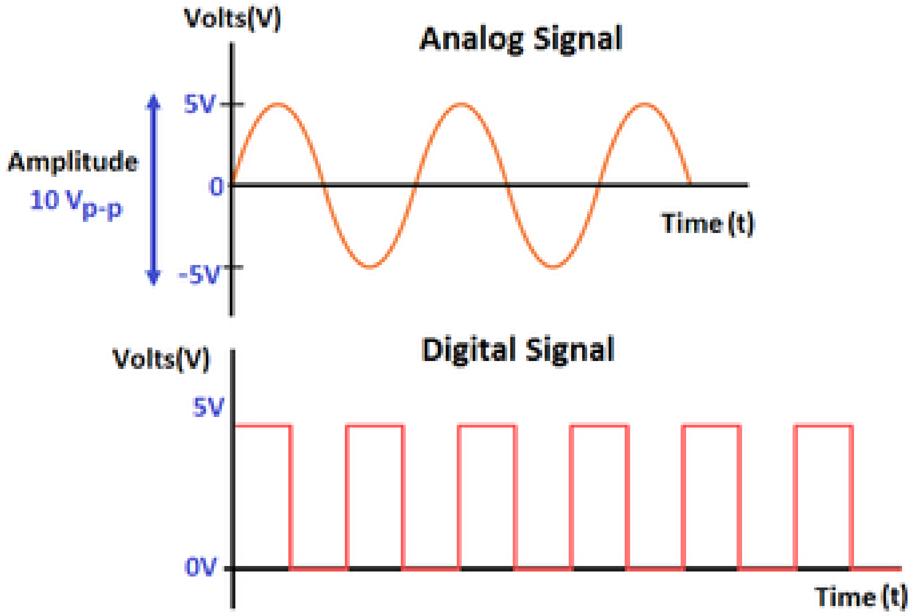
iii. Pulse position modulation (PPM)

The digital form of pulse modulation is known as pulse code modulation (PCM).

Analog and Digital Signals & its conversion:

The analog signal is that type of signal which varies smoothly and continuously with time. This means that analog signals are defined for every value of time and they take on continuous values in a given time interval.

The digital Signal is that type of signal which has a sequence of numbers, each number representing the signal magnitude at an instance of time. The resulting signal is called a digital signal. Digital messages are constructed with a finite number of symbols.



Conversion of Analog signals to Digital signals:

An Analog to Digital Converter (ADC) takes an analog input signal and converts the input, through a mathematical function, into a digital output signal. While there are many ways of implementing an ADC, there are three conceptual steps that occur.

1. The signal is sampled.
2. The sampled signal is quantized.
3. The quantized signal is digitally coded

Sampling

By, sampling we turn a continuous-time function which may take on infinitely many values at different times into a discretized function that may take on infinitely many values at different discrete indices.

Sampling generally is done with a Sample-And-Hold circuit (simple experiments can be done using a capacitor and switch). To be able to reconstruct the signal we must consider the Sampling Theorem which says that a sampling frequency twice the highest frequency we're expecting is needed. In a simple way sampling can be defined as the process of taking samples from the continuous time function x(t) and for the signal to reconstruct we must consider the sampling theorem which states that the sampling frequency must be always greater than or equal to twice the highest frequency.

Quantization

Quantization is the process of taking a continuous voltage signal and mapping it to a discrete number of voltage levels. The number of voltage levels affects the quantization noise that occurs. Since digital computers are binary in nature, the number of quantization levels is usually a power of 2, i.e.,

$$N = 2^n$$

where n is the number of quantization bits.

The signal may be amplified or attenuated before going into the ADC, so that the maximum and minimum voltage levels give the best compromise between resolution of the signal levels and minimization of clipping.

Encoding

Encoding is the process of converting the quantized signals into a digital representation. This encoding is performed by giving each quantization level a unique label. For instance, if four bits are used, the lowest level may be (in binary) 0000, and the next highest level 0001, etc.

Basic concept of Signals & Signals classification (Analog and Digital):

Information converted into an electrical form suitable for transmission is called a signal. There are two types of signals.

i. Analog Signal
ii. Digital Signal

Analog signals are continuous variations of current and voltage whereas digital signals are those that have discrete stepwise value (0 = Low, 1 = High)

Bandwidth limitation:

The bandwidth limitation is a major constraint in a communication system. The frequency range or the band of frequency needed for a particular given transmission is known as bandwidth. This band of frequencies required for a particular is also called channel. The band of frequencies required for a particular transmission is always allocated by some international regulatory agencies. This type of regulation is essential to avoid interference among the signals having same frequency. But for a given transmission, this allocated bandwidth may not be sufficient to convey the entire information.

Thus, we can conclude that bandwidth is a major fundamental limitation of a communication system.

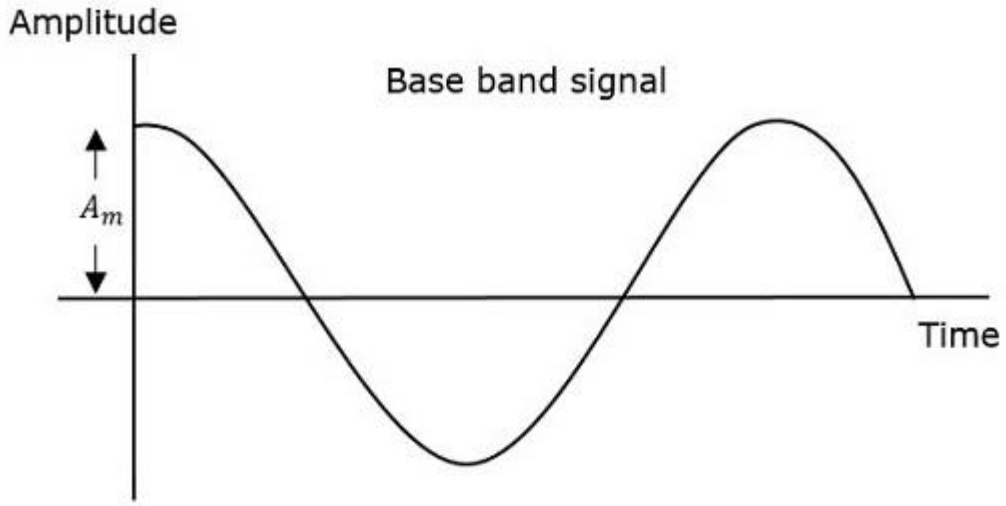
Assignment-1

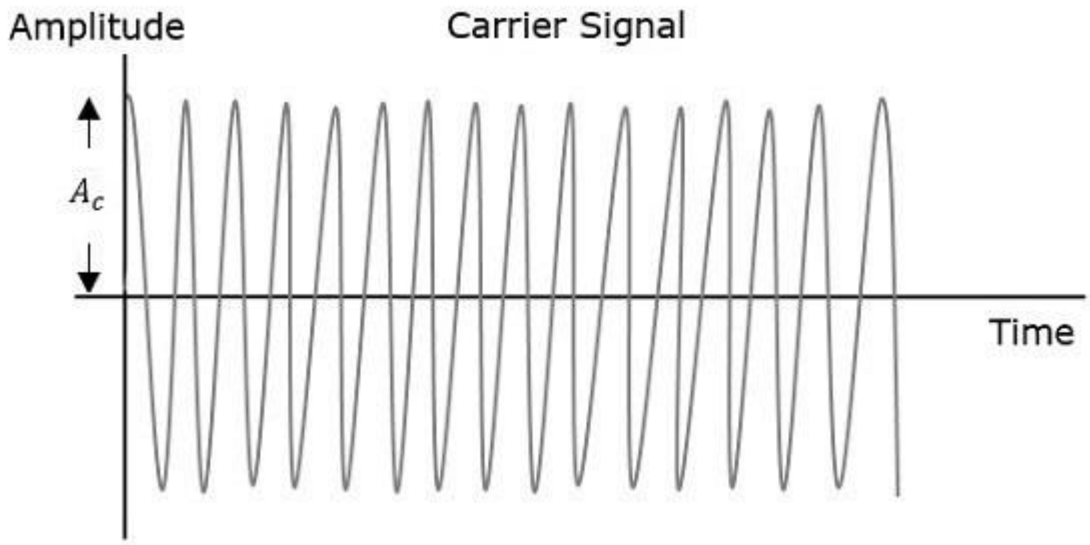
1. Draw the block diagram of communication system and explain the function of each block.
2. What is meant by the term Channel as applied to a communication system?
3. Explain the need for modulation in a communication system?
4. Explain the difference between Analog and Digital Communications.
5. How will you convert an analog signal into digital signal?

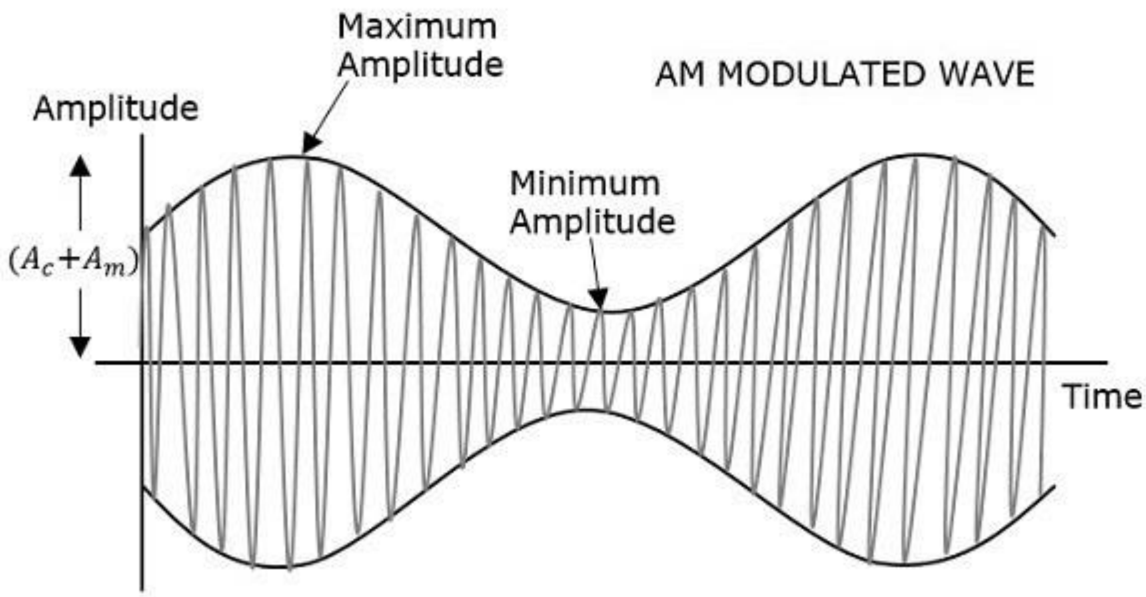
Unit-2
Amplitude (linear) Modulation System

Amplitude modulation & derive the expression for amplitude modulation signal:

According to the standard definition, "The amplitude of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal." Which means, the amplitude of the carrier signal containing no information varies as per the amplitude of the signal containing information, at each instant. This can be well explained by the following figures.







The first figure shows the modulating wave, which is the message signal. The next one is the carrier wave, which is a high frequency signal and contains no information. While, the last one is the resultant modulated wave.

It can be observed that the positive and negative peaks of the carrier wave, are interconnected with an imaginary line. This line helps recreating the exact shape of the modulating signal. This imaginary line on the carrier wave is called as **Envelope**. It is the same as that of the message signal.

Mathematical Expressions

Following are the mathematical expressions for these waves.

Time-domain Representation of the Waves

Let the modulating signal be,

$$m(t) = A_m Cos\ (2\pi f_m t)$$

and the carrier signal be,

$$c(t) = A_c Cos\ (2\pi f_c t)$$

Where,

Am and Ac are the amplitude of the modulating signal and the carrier signal respectively.
fm and fc are the frequency of the modulating signal and the carrier signal respectively.

Then, the equation of Amplitude Modulated wave will be

$s(t)=[Ac+Am\cos(2\pi fmt)]\cos(2\pi fct)$ (Equation 1)

Modulation Index

A carrier wave, after being modulated, if the modulated level is calculated, then such an attempt is called as **Modulation Index** or **Modulation Depth**. It states the level of modulation that a carrier wave undergoes.

Rearrange the Equation 1 as below.

$s(t)=Ac[1+(\frac{Am}{Ac})\cos(2\pi fmt)]\cos(2\pi fct)$
$\Rightarrow s(t)=Ac[1+\mu\cos(2\pi fmt)]\cos(2\pi fct)$ (Equation 2)

Where, μ is Modulation index and it is equal to the ratio of Am and Ac. Mathematically, we can write it as

$$\mu=\frac{Am}{Ac} \qquad \text{(Equation 3)}$$

Hence, we can calculate the value of modulation index by using the above formula, when the amplitudes of the message and carrier signals are known.

Now, let us derive one more formula for Modulation index by considering Equation 1. We can use this formula for calculating modulation index value, when the maximum and minimum amplitudes of the modulated wave are known.

Let Amax and Amin be the maximum and minimum amplitudes of the modulated wave.

We will get the maximum amplitude of the modulated wave, when cos(2πfmt) is 1.

$$\Rightarrow Amax=Ac+Am \qquad \text{(Equation 4)}$$

We will get the minimum amplitude of the modulated wave, when cos(2πfmt) is -1.

$$\Rightarrow Amin=Ac-Am \qquad \text{(Equation 5)}$$

Add Equation 4 and Equation 5.

$$Amax+Amin=Ac+Am+Ac-Am=2Ac$$

$$\Rightarrow Ac=\frac{Amax+Amin}{2} \qquad \text{(Equation 6)}$$

Subtract Equation 5 from Equation 4.

$$Amax-Amin=Ac+Am-(Ac-Am)=2Am$$

$$\Rightarrow Am=\frac{Amax-Amin}{2} \qquad \text{(Equation 7)}$$

The ratio of Equation 7 and Equation 6 will be as follows.

$$\frac{Am}{Ac}=\frac{(Amax-Amin)/2}{(Amax+Amin)/2}$$

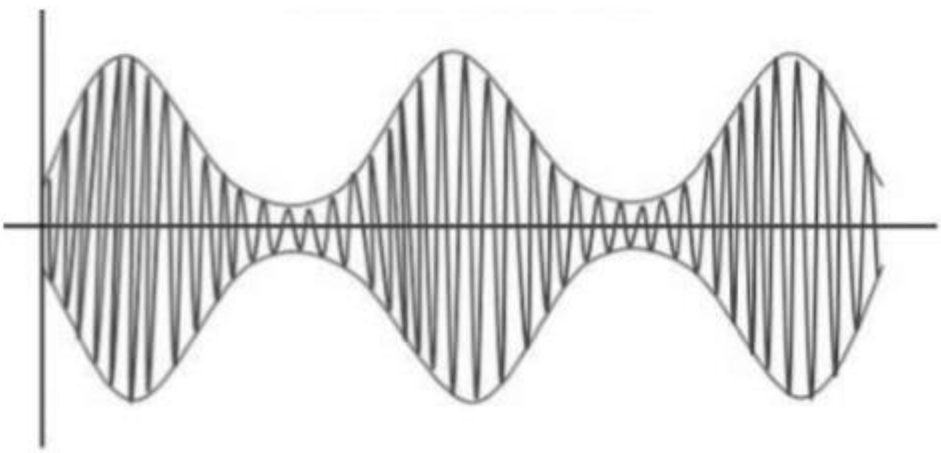
$$\Rightarrow \mu=\frac{Amax-Amin}{Amax+Amin} \qquad \text{(Equation 8)}$$

Therefore, Equation 3 and Equation 8 are the two formulas for Modulation index. The modulation index or modulation depth is often denoted in percentage called as Percentage of Modulation. We will get the **percentage of modulation**, just by multiplying the modulation index value with 100.

For a perfect modulation, the value of modulation index should be 1, which implies the percentage of modulation should be 100%.

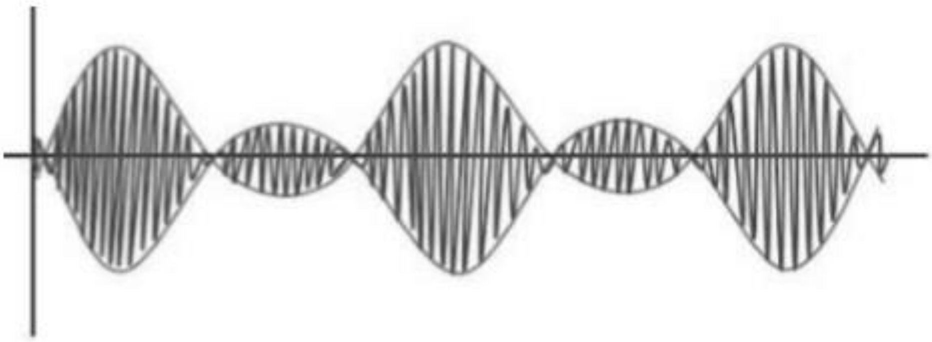
For instance, if this value is less than 1, i.e., the modulation index is 0.5, then the modulated output would look like the following figure. It is called as **Under-modulation**. Such a wave is called as an **under-modulated wave**.

Under-Modulated wave



If the value of the modulation index is greater than 1, i.e., 1.5 or so, then the wave will be an **over-modulated wave**. It would look like the following figure.

Over-Modulated wave



As the value of the modulation index increases, the carrier experiences a 180^{o} phase reversal, which causes additional sidebands and hence, the wave gets distorted. Such an over-modulated wave causes interference, which cannot be eliminated.

Bandwidth of AM Wave

Bandwidth (BW) is the difference between the highest and lowest frequencies of the signal. Mathematically, we can write it as

$$BW = f_{max} - f_{min}$$

Consider the following equation of amplitude modulated wave.

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)]\cos(2\pi f_c t)$$

$$\Rightarrow s(t) = A_c \cos(2\pi f_c t) + A_c \mu \cos(2\pi f_c t)\cos(2\pi f_m t)$$

$$\Rightarrow s(t) = A_c \cos(2\pi f_c t) + \frac{A_c \mu}{2} \cos[2\pi(f_c + f_m)t] + \frac{A_c \mu}{2} \cos[2\pi(f_c - f_m)t]$$

Hence, the amplitude modulated wave has three frequencies. Those are carrier frequency fc, upper sideband frequency fc+fm and lower sideband frequency fc−fm

Here,

$$fmax=fc+fm \text{ and } fmin=fc-fm$$

Substitute, fmax and fmin values in bandwidth formula.

$$BW=fc+fm-(fc-fm)$$

$$\Rightarrow BW=2fm$$

Thus, it can be said that the bandwidth required for amplitude modulated wave is twice the frequency of the modulating signal.

Power Calculations of AM Wave

Consider the following equation of amplitude modulated wave.

$$s(t)=Ac\cos(2\pi fct)+\frac{Ac\,\mu}{2}\cos[2\pi(fc+fm)t]+\frac{Ac\,\mu}{2}\cos[2\pi(fc-fm)t]$$

Power of AM wave is equal to the sum of powers of carrier, upper sideband, and lower sideband frequency components.

$$P_t=P_c+\mathrm{P}P_{USB}+P_{LSB}$$

We know that the standard formula for power of cos signal is

$$P=\frac{{v_{rms}}^2}{R}=\frac{(v_m/\sqrt{2}\,)^2}{2}$$

Where,

v_{rms} is the rms value of cos signal.
$v_m$is the peak value of cos signal.

First, let us find the powers of the carrier, the upper and lower sideband one by one.

Carrier power

$$P=\frac{(A_c/\sqrt{2}\,)^2}{R}=\frac{{A_c}^2}{2R}$$

Upper sideband power

$$P_{USB}=\frac{(A_c\mu/2\sqrt{2}\,)^2}{R}=\frac{{A_c}^2\mu^2}{8R}$$

Similarly, we will get the lower sideband power same as that of the upper side band power.

$$P_{LSB}=\frac{{A_c}^2\mu^2}{8R}$$

Now, let us add these three powers in order to get the power of AM wave.

$$P_t=\frac{{A_c}^2}{2R}+\frac{{A_c}^2\mu^2}{8R}+\frac{{A_c}^2\mu^2}{8R}$$

$$\Rightarrow P_t = \left(\frac{{A_c}^2}{2R}\right)\left(1+\frac{\mu^2}{4}+\frac{\mu^2}{4}\right)$$

$$\Rightarrow P_t = P_c\left(1+\frac{\mu^2}{2}\right)$$

We can use the above formula to calculate the power of AM wave, when the carrier power and the modulation index are known.

If the modulation index μ=1μ=1 then the power of AM wave is equal to 1.5 times the carrier power. So, the power required for transmitting an AM wave is 1.5 times the carrier power for a perfect modulation.

Problem 1

A modulating signal $m(t)=10\cos(2\pi\times10^3 t)$ is amplitude modulated with a carrier signal $c(t)=50\cos(2\pi\times10^5 t)$. Find the modulation index, the carrier power, and the power required for transmitting AM wave.

Solution

Given, the equation of modulating signal as

$$m(t)=10\cos(2\pi\times10^3 t)$$

We know the standard equation of modulating signal as

$$m(t)=A_m\cos(2\pi f_m t)$$

By comparing the above two equations, we will get

Amplitude of modulating signal as Am=10volts

and Frequency of modulating signal as

$$f_m=10^3\,Hz=1KHz$$

Given, the equation of carrier signal is

$$c(t)=50\cos(2\pi\times10^5 t)$$

The standard equation of carrier signal is

$$c(t)=A_c\cos(2\pi f_c t)$$

By comparing these two equations, we will get

Amplitude of carrier signal as Ac=50volts
and Frequency of carrier signal as $f_c=10^5$Hz=100KHz

We know the formula for modulation index as

$$\mu=\frac{Am}{Ac}$$

Substitute, AmAm and AcAc values in the above formula.

$$\mu=\frac{10}{50}=0.2$$

Therefore, the value of **modulation index is 0.2** and percentage of modulation is 20%.

The formula for Carrier power, Pc= is

$$Pc=\frac{{A_c}^2}{2R}$$

Assume R=1Ω and substitute Ac value in the above formula.

$$Pc=\frac{50^2}{2(1)}=1250W$$

Therefore, the **Carrier power**, Pc is **1250 watts**.

We know the formula for **power** required for **transmitting AM** wave is

$$\Rightarrow Pt=Pc(1+\frac{\mu^2}{2})$$

Substitute Pc and μ values in the above formula.

$$Pt=1250(1+\frac{(0.2)^2}{2})=1275W$$

Therefore, the **power required for transmitting AM** wave is **1275 watts**.

Problem 2

The equation of amplitude wave is given by $s(t)=20[1+0.8\cos(2\pi\times10^3t)]\cos(4\pi\times10^5t)$. Find the carrier power, the total sideband power, and the band width of AM wave.

Solution

Given, the equation of Amplitude modulated wave is

$$s(t)=20[1+0.8\cos(2\pi\times10^3t)]\cos(4\pi\times10^5t)$$

Re-write the above equation as

$$s(t)=20[1+0.8\cos(2\pi\times10^3t)]\cos(2\pi\times2\times10^5t)$$

We know the equation of Amplitude modulated wave is

$$s(t)=Ac[1+\mu\cos(2\pi fmt)]\cos(2\pi fct)$$

By comparing the above two equations, we will get

Amplitude of carrier signal as Ac=20volts
Modulation index as μ=0.8
Frequency of modulating signal as $fm=10^3Hz=1KHz$
Frequency of carrier signal as $fc=2\times10^5Hz=200KHz$
The formula for Carrier power, Pc is

$$Pc=\frac{{A_c}^2}{2R}$$

Assume R=1Ω and substitute Ac value in the above formula.

$$Pc=\frac{(20)^2}{2\times1}200W$$

Therefore, the **Carrier power**, Pc is **200watts**.

We know the formula for total side band power is

$$P_{SB}=\frac{P_c\mu^2}{2}$$

Substitute Pc and μ values in the above formula.

$$P_{SB}=\frac{200\times(0.8)^2}{2}=64W$$

Therefore, the **total side band power** is **64 watts.**

We know the formula for bandwidth of AM wave is

$$BW = 2f_m$$

Substitute fm value in the above formula.

$$BW = 2 \times 1K = 2KHz$$

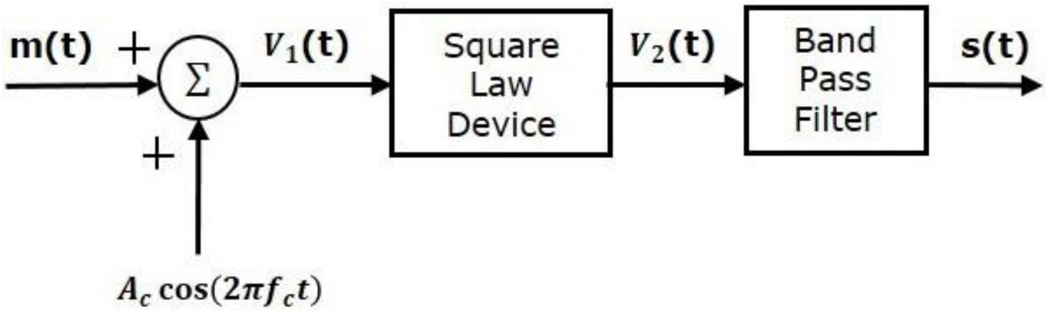
Therefore, the **bandwidth** of AM wave is **2 KHz.**

Generation of Amplitude Modulation (AM):

The modulators, which generate amplitude modulated wave.

Square Law Modulator

Following is the block diagram of the square law modulator



Let the modulating and carrier signals be denoted as m(t) and Acos(2πfct) respectively. These two signals are applied as inputs to the summer (adder) block. This summer block produces an output, which is the addition of the modulating and the carrier signal. Mathematically, we can write it as

$$V_1 t = m(t) + A_c \cos(2\pi f_c t)$$

This signal V1t is applied as an input to a nonlinear device like diode. The characteristics of the diode are closely related to square law.

$$V_2 t = k1 V_1(t) + k2 {V_1}^2(t) \qquad \text{(Equation 1)}$$

Where, k1 and k2 are constants.
Substitute V1(t) in Equation 1

$$V_2 t = \text{k1}[m(t) + A_c \cos(2\pi f_c t)] + \text{k2}[m(t) + A_c \cos(2\pi f_c t)]^2$$

$$\Rightarrow V_2(t) = k_1 m(t) + k_1 A_c \cos(2\pi f_c t) + k_2 m^2(t) + k_2 A_c^2 \cos^2(2\pi f_c t) + 2k_2 m(t) A_c \cos(2\pi f_c t)$$

$$\Rightarrow V_2(t) = k_1 m(t) + k_2 m^2(t) + k_2 A_c^2 \cos^2(2\pi f_c t) + k_1 A_c \left[1 + \left(\frac{2k_2}{k_1}\right) m(t)\right] \cos(2\pi f_c t)$$

The last term of the above equation represents the desired AM wave and the first three terms of the above equation are unwanted. So, with the help of band pass filter, we can pass only AM wave and eliminate the first three terms.

Therefore, the output of square law modulator is

$$s(t)=k1Ac[1+(\frac{2k2}{k1})m(t)] \cos(2\pi fct)$$

The standard equation of AM wave is

$$s(t)=Ac[1+k_a m(t)] \cos(2\pi fct)$$

Where, Ka is the amplitude sensitivity
By comparing the output of the square law modulator with the standard equation of AM wave, we will get the scaling factor as k1 and the amplitude sensitivity ka as $\frac{2k2}{k1}$.

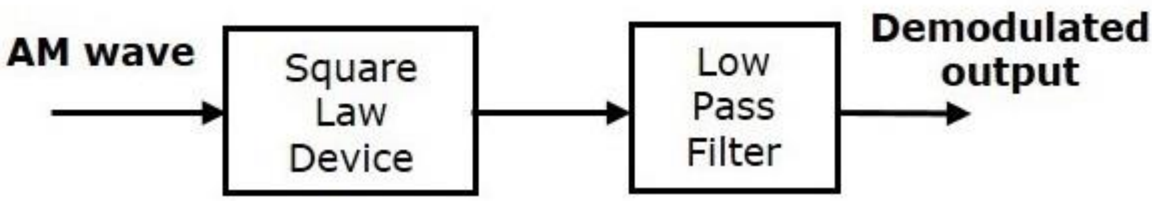
Demodulation of AM waves:

The process of extracting an original message signal from the modulated wave is known as **detection** or **demodulation**. The circuit, which demodulates the modulated wave is known as the **demodulator**. The following demodulators (detectors) are used for demodulating AM wave.

- Square Law Demodulator
- Envelope Detector

Square Law Demodulator

Square law demodulator is used to demodulate low level AM wave. Following is the block diagram of the **square law demodulator**.



This demodulator contains a square law device and low pass filter. The AM wave V1(t) is applied as an input to this demodulator.

The standard form of AM wave is

$$V1(t)=Ac[1+k_a m(t)]\cos(2\pi fct)$$

We know that the mathematical relationship between the input and the output of square law device is

$$V_2 t = k1V_1(t) + k2{V_1}^2(t) \qquad \text{(Equation 1)}$$

Where,

V1(t) is the input of the square law device, which is nothing but the AM wave
V2(t) is the output of the square law device
k1 and k2 are constants
Substitute V1(t) in Equation 1

$$V_2(t) = k_1\left(A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t)\right) + k_2\left(A_c\left[1+k_a m(t)\right]\cos(2\pi f_c t)\right)^2$$

$$\Rightarrow V_2(t) = k_1 A_c \cos(2\pi f_c t) + k_1 A_c k_a m(t)\cos(2\pi f_c t) +$$

$$k_2 {A_c}^2\left[1 + {K_a}^2 m^2(t) + 2k_a m(t)\right]\left(\frac{1+\cos(4\pi f_c t)}{2}\right)$$

$$\Rightarrow V_2(t) = k_1 A_c \cos(2\pi f_c t) + k_1 A_c k_a m(t)\cos(2\pi f_c t) + \frac{K_2 {A_c}^2}{2} +$$

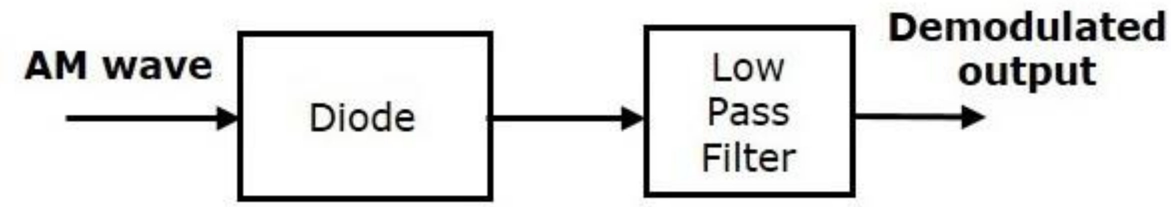
$$\frac{K_2 {A_c}^2}{2}\cos(4\pi f_c t) + \frac{k_2 {A_c}^2 {k_a}^2 m^2(t)}{2} + \frac{k_2 {A_c}^2 {k_a}^2 m^2(t)}{2}\cos(4\pi f_c t) +$$

$$k_2 {A_c}^2 k_a m(t) + k_2 {A_c}^2 k_a m(t)\cos(4\pi f_c t)$$

In the above equation, the term $k_2 {A_c}^2 k_a m(t)$ is the scaled version of the message signal. It can be extracted by passing the above signal through a low pass filter and the DC component $\frac{k_2 {A_c}^2}{2}$ can be eliminated with the help of a coupling capacitor.

Envelope Detector

Envelope detector is used to detect (demodulate) high level AM wave. Following is the block diagram of the envelope detector.



This envelope detector consists of a diode and low pass filter. Here, the diode is the main detecting element. Hence, the envelope detector is also called as the **diode detector**. The low pass filter contains a parallel combination of the resistor and the capacitor.

The AM wave s(t)s(t) is applied as an input to this detector.

We know the standard form of AM wave is

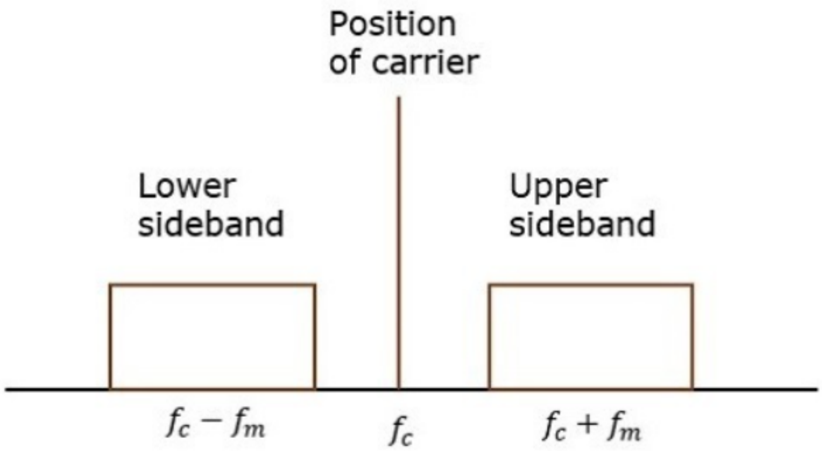
s(t)=Ac[1+k_am(t)] cos(2πfct)

In the positive half cycle of AM wave, the diode conducts and the capacitor charges to the peak value of AM wave. When the value of AM wave is less than this value, the diode will be reverse biased. Thus, the capacitor will discharge through resistor **R** till the next positive half cycle of AM wave. When the value of AM wave is greater than the capacitor voltage, the diode conducts and the process will be repeated.

We should select the component values in such a way that the capacitor charges very quickly and discharges very slowly. As a result, we will get the capacitor voltage waveform same as that of the envelope of AM wave, which is almost similar to the modulating signal.

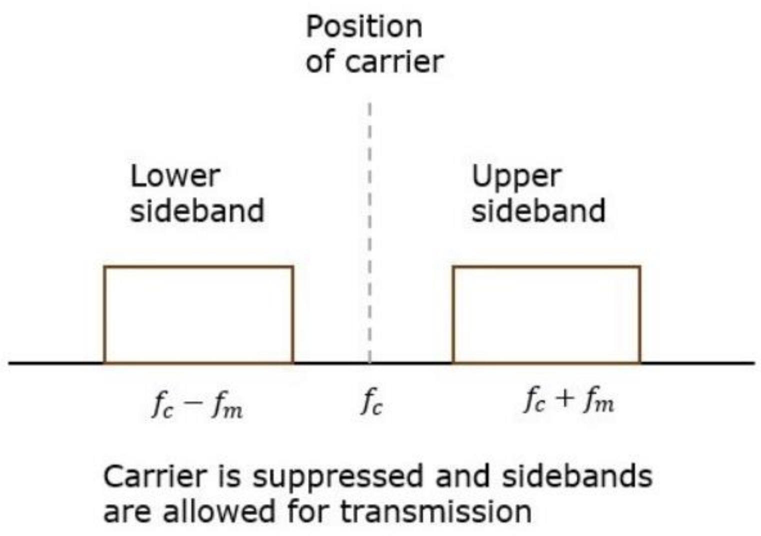
In the process of Amplitude Modulation, the modulated wave consists of the carrier wave and two sidebands. The modulated wave has the information only in the sidebands. **Sideband** is nothing but a band of frequencies, containing power, which are the lower and higher frequencies of the carrier frequency.

The transmission of a signal, which contains a carrier along with two sidebands can be termed as **Double Sideband Full Carrier** system or simply **DSBFC**. It is plotted as shown in the following figure.



However, such a transmission is inefficient. Because, two-thirds of the power is being wasted in the carrier, which carries no information.

If this carrier is suppressed and the saved power is distributed to the two sidebands, then such a process is called as **Double Sideband Suppressed Carrier** system or simply **DSBSC**. It is plotted as shown in the following figure.



Mathematical Expressions

Let us consider the same mathematical expressions for modulating and carrier signals as we have considered in the earlier chapters.

i.e., Modulating signal

$$m(t) = A_m \cos(2\pi f_m t)$$

Carrier signal

$$c(t) = A_c \cos(2\pi f_c t)$$

Mathematically, we can represent the **equation of DSBSC wave** as the product of modulating and carrier signals.

$$s(t) = m(t)\, c(t)$$

$$\Rightarrow s(t) = A_m A_c \cos(2\pi f_m t) \cos(2\pi f_c t)$$

Bandwidth of DSBSC Wave

We know the formula for bandwidth (BW) is

$$BW = f_{max} - f_{min}$$

Consider the equation of DSBSC modulated wave.

$$s(t) = A_m A_c \cos(2\pi f_m t)\cos(2\pi f_c t)$$

$$\Rightarrow s(t) = \frac{A_m A_c}{2}\cos[2\pi(f_c + f_m)t] + \frac{A_m A_c}{2}\cos[2\pi(f_c - f_m)t]$$

The DSBSC modulated wave has only two frequencies. So, the maximum and minimum frequencies are $f_c + f_m$ and $f_c - f_m$ respectively.

i.e.,

$$f_{max} = f_c + f_m \quad \text{and} \quad f_{min} = f_c - f_m$$

Substitute, f_{max} and f_{min} values in the bandwidth formula.

$$BW = f_c + f_m - (f_c - f_m)$$

$$\Rightarrow BW = 2f_m$$

Thus, the bandwidth of DSBSC wave is same as that of AM wave and it is equal to twice the frequency of the modulating signal.

Power Calculations of DSBSC Wave

Consider the following equation of DSBSC modulated wave.

$$s(t) = \frac{A_m A_c}{2}\cos[2\pi(f_c + f_m)t] + \frac{A_m A_c}{2}\cos[2\pi(f_c - f_m)t]$$

Power of DSBSC wave is equal to the sum of powers of upper sideband and lower sideband frequency components.

$$P_t = P_{USB} + P_{LSB}$$

We know the standard formula for power of cos signal is

$$P = \frac{{v_{rms}}^2}{R} = \frac{\left(v_m\sqrt{2}\right)^2}{R}$$

First, let us find the powers of upper sideband and lower sideband one by one.

Upper sideband power

$$P_{USB} = \frac{\left(A_m A_c/2\sqrt{2}\right)^2}{R} = \frac{{A_m}^2{A_c}^2}{8R}$$

Similarly, we will get the lower sideband power same as that of upper sideband power.

$$P_{USB} = \frac{{A_m}^2{A_c}^2}{8R}$$

Now, let us add these two sideband powers in order to get the power of DSBSC wave.

$$P_t = \frac{{A_m}^2 {A_c}^2}{8R} + \frac{{A_m}^2 {A_c}^2}{8R}$$

$$\Rightarrow P_t = \frac{{A_m}^2 {A_c}^2}{4R}$$

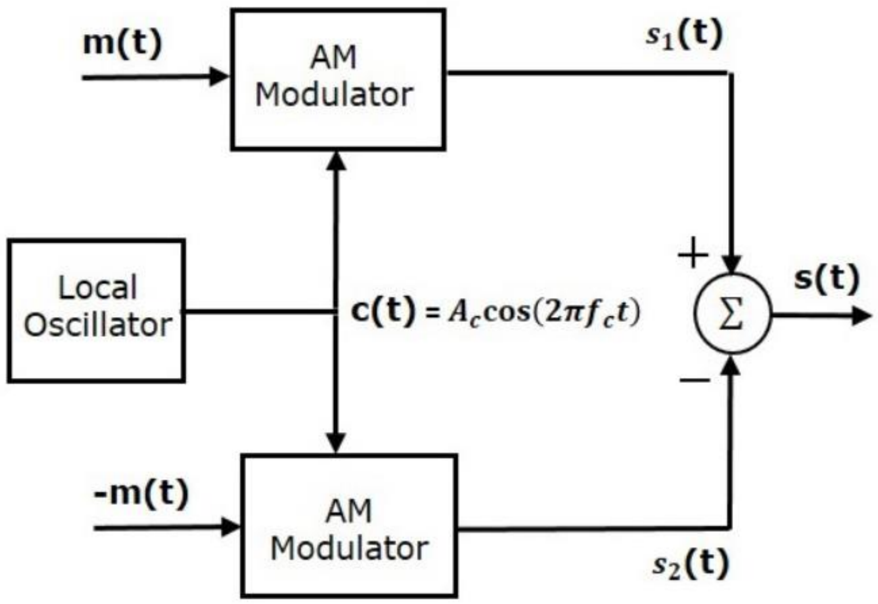
Therefore, the power required for transmitting DSBSC wave is equal to the power of both the sidebands.

The following two modulators generate DSBSC wave.

- Balanced modulator
- Ring modulator

Balanced Modulator

Following is the block diagram of the Balanced modulator.



Balanced modulator consists of two identical AM modulators. These two modulators are arranged in a balanced configuration in order to suppress the carrier signal. Hence, it is called as Balanced modulator.

The same carrier signal $c(t) = A_c \cos(2\pi f_c t)$ is applied as one of the inputs to these two AM modulators. The modulating signal $m(t)$ is applied as another input to the upper AM modulator. Whereas, the modulating signal $m(t)$ with opposite polarity, i.e., $-m(t)$ is applied as another input to the lower AM modulator.

Output of the upper AM modulator is

$$s_1(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

Output of the lower AM modulator is

$$s_2(t) = A_c [1 - k_a m(t)] \cos(2\pi f_c t)$$

We get the DSBSC wave $s(t)$ by subtracting $s_2(t)$ from $s_1(t)$. The summer block is used to perform this operation. $s_1(t)$ with positive sign and $s_2(t)$ with negative sign are applied as inputs to summer block. Thus, the summer block produces an output $s(t)$ which is the difference of $s_1(t)$ and $s_2(t)$.

$$\Rightarrow s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t) - A_c[1 - k_a m(t)]\cos(2\pi f_c t)$$

$$\Rightarrow s(t) = A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t) - A_c\cos(2\pi f_c t) +$$

$$A_c k_a m(t)\cos(2\pi f_c t)$$

$$\Rightarrow s(t) = 2A_c k_a m(t)\cos(2\pi f_c t)$$

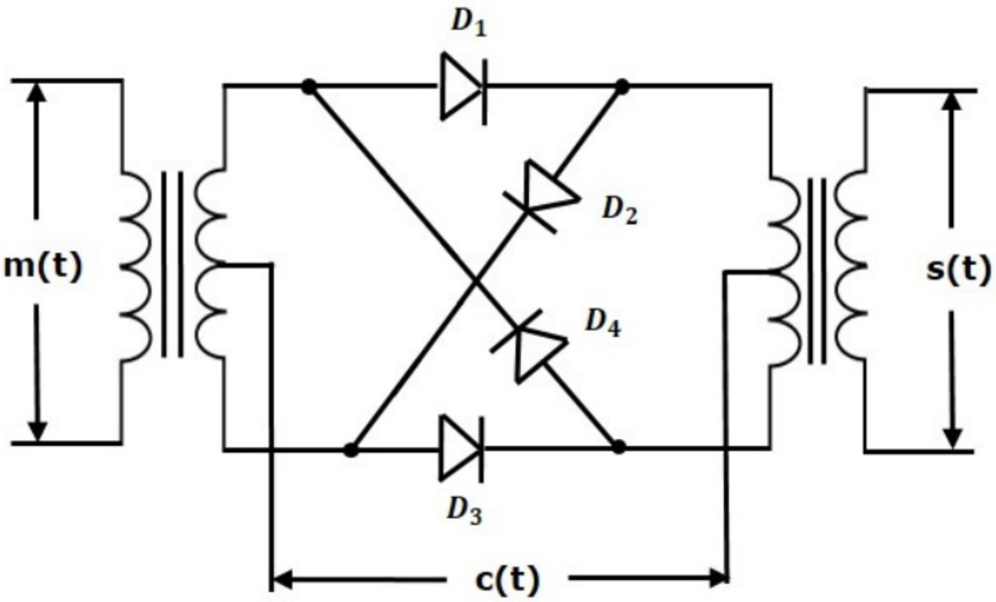
We know the standard equation of DSBSC wave is

$$s(t) = A_c m(t)\cos(2\pi f_c t)$$

By comparing the output of summer block with the standard equation of DSBSC wave, we will get the scaling factor as $2k_a$

Ring Modulator

Following is the block diagram of the Ring modulator.



In this diagram, the four diodes D_1 , D_2 , D_3 and D_4 are connected in the ring structure. Hence, this modulator is called as the **ring modulator**. Two center tapped transformers are used in this diagram. The message signal $m(t)$ is applied to the input transformer. Whereas, the carrier signals $c(t)$ is applied between the two center tapped transformers.

For positive half cycle of the carrier signal, the diodes D_1 and D_3 are switched ON and the other two diodes D_2 and D_4 are switched OFF. In this case, the message signal is multiplied by +1.

For negative half cycle of the carrier signal, the diodes D_2 and D_4 are switched ON and the other two diodes D_1 and D_3 are switched OFF. In this case, the message signal is multiplied by -1. This results in 180^0 phase shift in the resulting DSBSC wave.

From the above analysis, we can say that the four diodes D_1 , D_2 , D_3 and D_4 are controlled by the carrier signal. If the carrier is a square wave, then the Fourier series representation of $c(t)$ is represented as

$$c(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c t(2n-1)]$$

We will get DSBSC wave $s(t)$, which is just the product of the carrier signal $c(t)$ and the message signal $m(t)$ i.e.,

$$s(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[2\pi f_c t(2n-1)]m(t)$$

The above equation represents DSBSC wave, which is obtained at the output transformer of the ring modulator.

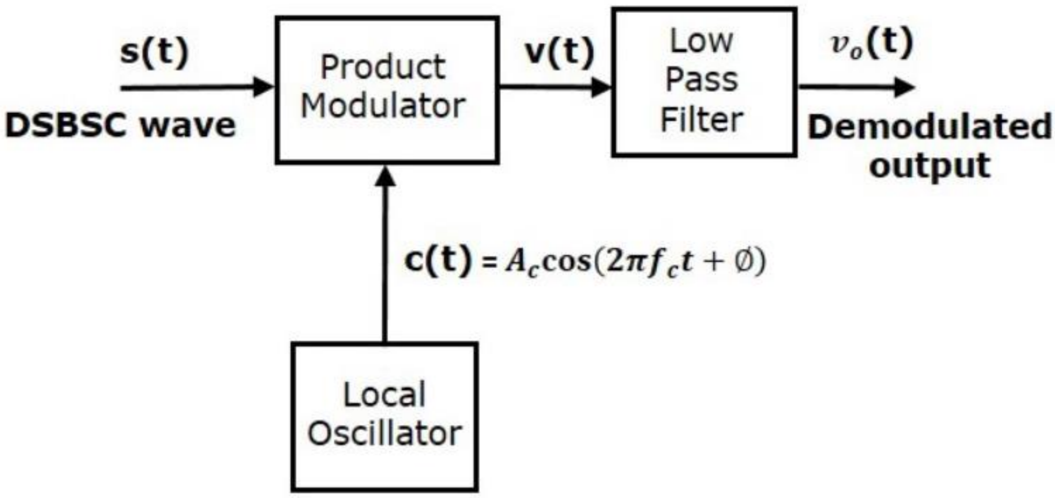
DSBSC modulators are also called as **product modulators** as they produce the output, which is the product of two input signals.

The process of extracting an original message signal from DSBSC wave is known as detection or demodulation of DSBSC. The following demodulators (detectors) are used for demodulating DSBSC wave.

- Coherent Detector
- Costas Loop

Coherent Detector

Here, the same carrier signal (which is used for generating DSBSC signal) is used to detect the message signal. Hence, this process of detection is called as **coherent** or **synchronous detection**. Following is the block diagram of the coherent detector.



In this process, the message signal can be extracted from DSBSC wave by multiplying it with a carrier, having the same frequency and the phase of the carrier used in DSBSC modulation. The resulting signal is then passed through a Low Pass Filter. Output of this filter is the desired message signal.

Let the DSBSC wave be

$$s(t) = A_c \cos(2\pi f_c t) m(t)$$

The output of the local oscillator is

$$c(t) = A_c \cos(2\pi f_c t + \phi)$$

Where, ϕ is the phase difference between the local oscillator signal and the carrier signal, which is used for DSBSC modulation.

From the figure, we can write the output of product modulator as

$$v(t) = s(t)\, c(t)$$

Substitute, $s(t)$ and $c(t)$ values in the above equation.

$$\Rightarrow v(t) = A_c \cos(2\pi f_c t) m(t)\, A_c \cos(2\pi f_c t + \phi)$$

$$= {A_c}^2 \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t)$$

$$= \tfrac{{A_c}^2}{2} \left[\cos(4\pi f_c t + \phi) + \cos\phi\right] m(t)$$

$$v(t) = \frac{{A_c}^2}{2} \cos\phi m(t) + \frac{{A_c}^2}{2} \cos(4\pi f_c t + \phi) m(t)$$

In the above equation, the first term is the scaled version of the message signal. It can be extracted by passing the above signal through a low pass filter.

Therefore, the output of low pass filter is

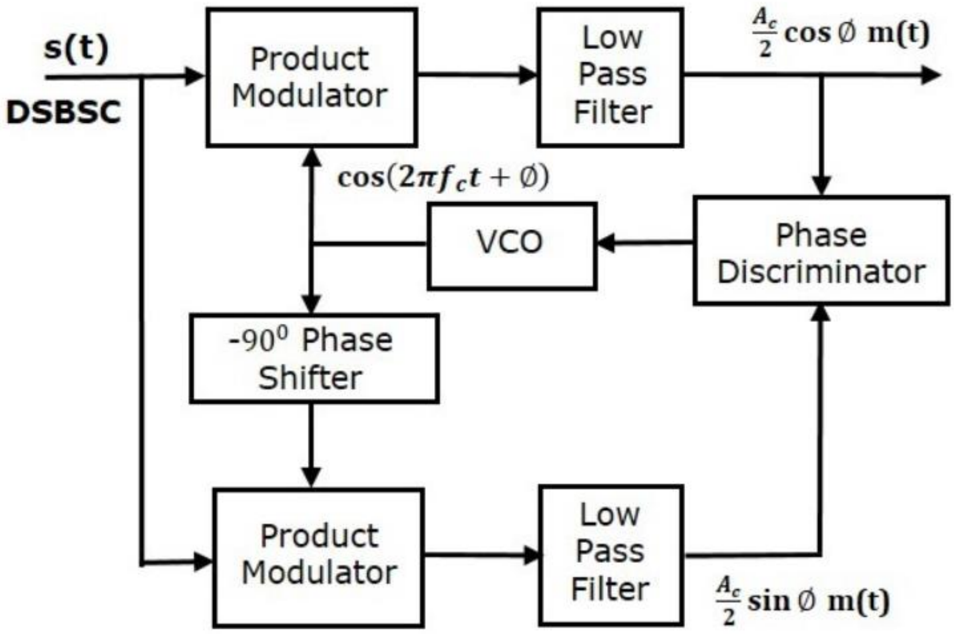
$$v_0 t = \frac{{A_c}^2}{2} \cos \phi m(t)$$

The demodulated signal amplitude will be maximum, when $\phi = 0^0$. That's why the local oscillator signal and the carrier signal should be in phase, i.e., there should not be any phase difference between these two signals.

The demodulated signal amplitude will be zero, when $\phi = \pm 90^0$. This effect is called as **quadrature null effect**.

Costas Loop

Costas loop is used to make both the carrier signal (used for DSBSC modulation) and the locally generated signal in phase. Following is the block diagram of Costas loop.



Costas loop consists of two product modulators with common input $s(t)$, which is DSBSC wave. The other input for both product modulators is taken from **Voltage Controlled Oscillator** (VCO) with

-90^0 phase shift to one of the product modulator as shown in figure.

We know that the equation of DSBSC wave is

$$s(t) = A_c \cos(2\pi f_c t) m(t)$$

Let the output of VCO be

$$c_1(t) = \cos(2\pi f_c t + \phi)$$

This output of VCO is applied as the carrier input of the upper product modulator.

Hence, the output of the upper product modulator is

$$v_1(t) = s(t)\, c_1(t)$$

Substitute, $s(t)$ and $c_1(t)$ values in the above equation.

$$\Rightarrow v_1(t) = A_c \cos(2\pi f_c t) m(t) \cos(2\pi f_c t + \phi)$$

After simplifying, we will get $v_1(t)$ as

$$v_1(t) = \frac{A_c}{2} \cos\phi\, m(t) + \frac{A_c}{2} \cos(4\pi f_c t + \phi) m(t)$$

This signal is applied as an input of the upper low pass filter. The output of this low pass filter is

$$v_{01}(t) = \frac{A_c}{2} \cos\phi\, m(t)$$

Therefore, the output of this low pass filter is the scaled version of the modulating signal.

The output of -90^0 phase shifter is

$$c_2(t) = cos\left(2\pi f_c t + \phi - 90^0\right) = \sin(2\pi f_c t + \phi)$$

This signal is applied as the carrier input of the lower product modulator.

The output of the lower product modulator is

$$v_2(t) = s(t)\, c_2(t)$$

Substitute, $s(t)$ and $c_2(t)$ values in the above equation.

$$\Rightarrow v_2(t) = A_c \cos(2\pi f_c t) m(t) \sin(2\pi f_c t + \phi)$$

After simplifying, we will get $v_2(t)$ as

$$v_2(t) = \frac{A_c}{2} \sin\phi m(t) + \frac{A_c}{2} \sin(4\pi f_c t + \phi) m(t)$$

This signal is applied as an input of the lower low pass filter. The output of this low pass filter is

$$v_{02}(t) = \frac{A_c}{2} \sin\phi m(t)$$

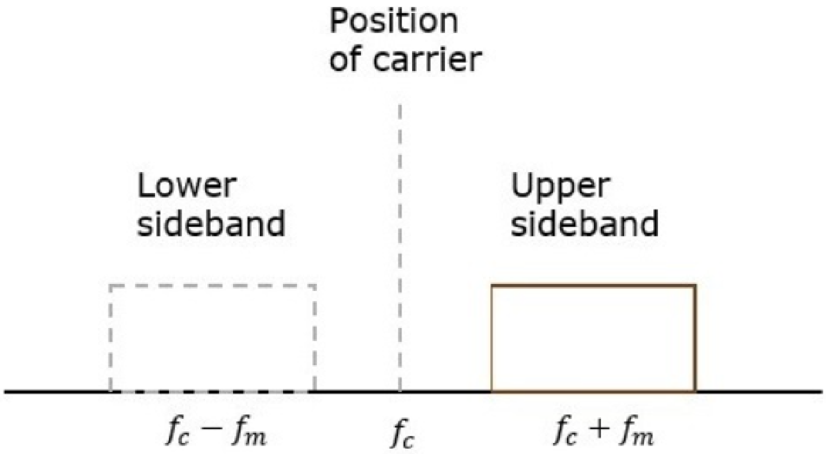
The output of this Low pass filter has -90^0 phase difference with the output of the upper low pass filter.

The outputs of these two low pass filters are applied as inputs of the phase discriminator. Based on the phase difference between these two signals, the phase discriminator produces a DC control signal.

This signal is applied as an input of VCO to correct the phase error in VCO output. Therefore, the carrier signal (used for DSBSC modulation) and the locally generated signal (VCO output) are in phase.

Single Side Band (SSB):

The process of suppressing one of the sidebands along with the carrier and transmitting a single sideband is called as **Single Sideband Suppressed Carrier** system or simply **SSBSC**. It is plotted as shown in the following figure.



Carrier and a sideband are suppressed and a single sideband is allowed for transmission

In the above figure, the carrier and the lower sideband are suppressed. Hence, the upper sideband is used for transmission. Similarly, we can suppress the carrier and the upper sideband while transmitting the lower sideband.

This SSBSC system, which transmits a single sideband has high power, as the power allotted for both the carrier and the other sideband is utilized in transmitting this Single Sideband.

Mathematical Expressions

Let us consider the same mathematical expressions for the modulating and the carrier signals as we have considered in the earlier chapters.

i.e., Modulating signal

$$m(t) = A_m \cos(2\pi f_m t)$$

Carrier signal

$$c(t) = A_c \cos(2\pi f_c t)$$

Mathematically, we can represent the equation of SSBSC wave as

$$s(t) = \frac{A_m A_c}{2} \cos[2\pi (f_c + f_m) t]$$ for the upper sideband

Or

$$s(t) = \frac{A_m A_c}{2} \cos[2\pi (f_c - f_m) t]$$ for the lower sideband

Bandwidth of SSBSC Wave

We know that the DSBSC modulated wave contains two sidebands and its bandwidth is $2f_m$. Since the SSBSC modulated wave contains only one sideband, its bandwidth is half of the bandwidth of DSBSC modulated wave.

i.e., ***Bandwidth of SSBSC modulated wave*** = $\frac{2f_m}{2} = f_m$

Therefore, the bandwidth of SSBSC modulated wave is f_m and it is equal to the frequency of the modulating signal.

Power Calculations of SSBSC Wave

Consider the following equation of SSBSC modulated wave.

$$s(t) = \frac{A_m A_c}{2} \cos[2\pi (f_c + f_m) t]$$ for the upper sideband

Or

$$s(t) = \frac{A_m A_c}{2} \cos[2\pi (f_c - f_m) t]$$ for the lower sideband

Power of SSBSC wave is equal to the power of any one sideband frequency components.

$$P_t = P_{USB} = P_{LSB}$$

We know that the standard formula for power of cos signal is

$$P = \frac{{v_{rms}}^2}{R} = \frac{\left(v_m/\sqrt{2}\right)^2}{R}$$

In this case, the power of the upper sideband is

$$P_{USB} = \frac{\left(A_m A_c/2\sqrt{2}\right)^2}{R} = \frac{{A_m}^2 {A_c}^2}{8R}$$

Similarly, we will get the lower sideband power same as that of the upper side band power.

$$P_{LSB} = \frac{{A_m}^2 {A_c}^2}{8R}$$

Therefore, the power of SSBSC wave is

$$P_t = P_{USB} = P_{LSB} = \frac{{A_m}^2{A_c}^2}{8R}$$

Advantages

- Bandwidth or spectrum space occupied is lesser than AM and DSBSC waves.
- Transmission of more number of signals is allowed.
- Power is saved.
- High power signal can be transmitted.
- Less amount of noise is present.
- Signal fading is less likely to occur.

Disadvantages

- The generation and detection of SSBSC wave is a complex process.
- The quality of the signal gets affected unless the SSB transmitter and receiver have an excellent frequency stability.

Applications

- For power saving requirements and low bandwidth requirements.
- In land, air, and maritime mobile communications.
- In point-to-point communications.
- In radio communications.
- In television, telemetry, and radar communications.
- In military communications, such as amateur radio, etc.

SSBSC Modulators:

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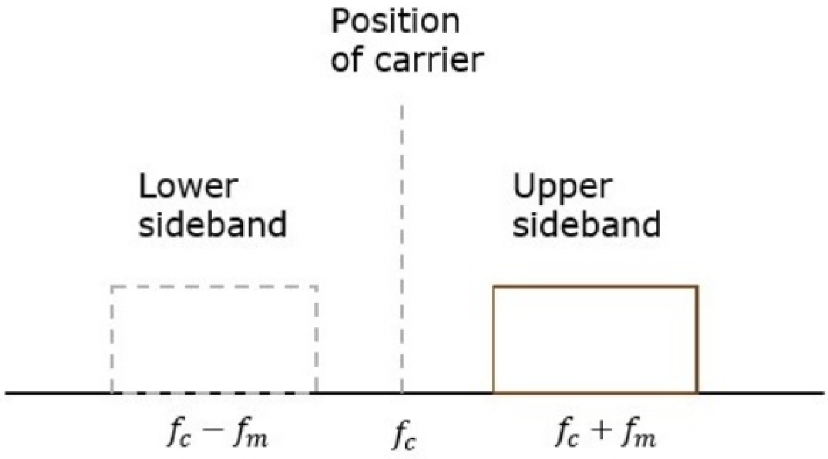

Carrier and a sideband are suppressed and a single sideband is allowed for transmission

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In this case, the power of the upper sideband is

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Similarly, we will get the lower sideband power same as that of the upper side band power.

$$P_{LSB} = \frac{{A_m}^2 {A_c}^2}{8R}$$

Therefore, the power of SSBSC wave is

$$P_t = P_{USB} = P_{LSB} = \frac{{A_m}^2 {A_c}^2}{8R}$$

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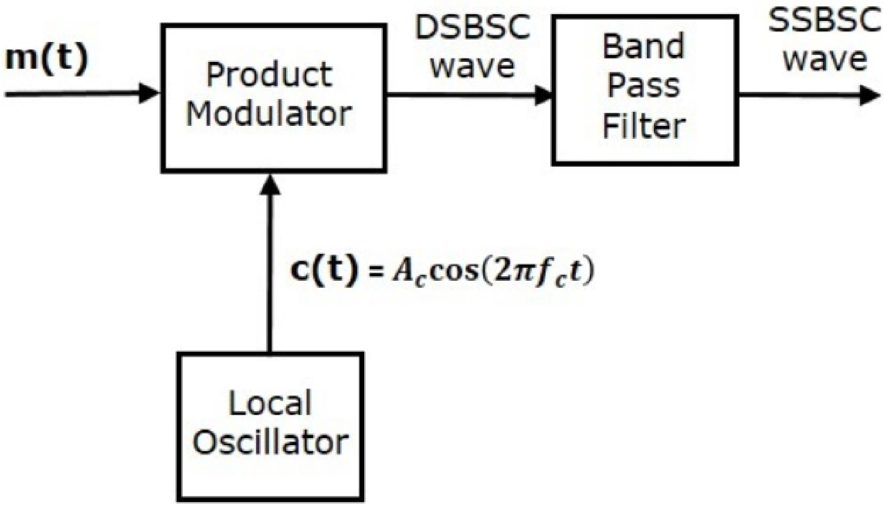
SSBSC Modulators

We can generate the SSBSC wave using two methods

- Frequency discrimination method
- Phase discrimination method

Frequency Discrimination Method

The following figure shows the block diagram of SSBSC modulator using frequency discrimination method.

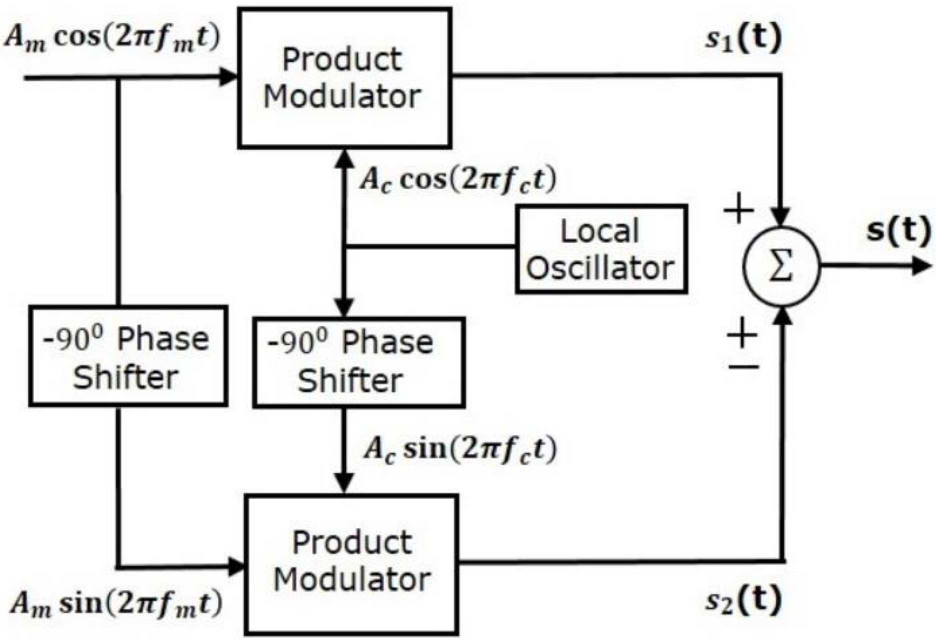


In this method, first we will generate DSBSC wave with the help of the product modulator. Then, apply this DSBSC wave as an input of band pass filter. This band pass filter produces an output, which is SSBSC wave.

Select the frequency range of band pass filter as the spectrum of the desired SSBSC wave. This means the band pass filter can be tuned to either upper sideband or lower sideband frequencies to get the respective SSBSC wave having upper sideband or lower sideband.

Phase Discrimination Method

The following figure shows the block diagram of SSBSC modulator using phase discrimination method.



This block diagram consists of two product modulators, two -90^0 phase shifters, one local oscillator and one summer block. The product modulator produces an output, which is the product of two inputs. The -90^0 phase shifter produces an output, which has a phase lag of -90^0 with respect to the input.

The local oscillator is used to generate the carrier signal. Summer block produces an output, which is either the sum of two inputs or the difference of two inputs based on the polarity of inputs.

The modulating signal $A_m \cos(2\pi f_m t)$ and the carrier signal $A_c \cos(2\pi f_c t)$ are directly applied as inputs to the upper product modulator. So, the upper product modulator produces an output, which is the product of these two inputs.

The output of upper product modulator is

$$s_1(t) = A_m A_c \cos(2\pi f_m t) \cos(2\pi f_c t)$$

$$\Rightarrow s_1(t) = \frac{A_m A_c}{2} \{\cos[2\pi (f_c + f_m) t] + \cos[2\pi (f_c - f_m) t]\}$$

The modulating signal $A_m \cos(2\pi f_m t)$ and the carrier signal $A_c \cos(2\pi f_c t)$ are phase shifted by -90^0 before applying as inputs to the lower product modulator. So, the lower product modulator produces an output, which is the product of these two inputs.

The output of lower product modulator is

$$s_2(t) = A_m A_c \cos\left(2\pi f_m t - 90^0\right) \cos\left(2\pi f_c t - 90^0\right)$$

$$\Rightarrow s_2(t) = A_m A_c \sin(2\pi f_m t) \sin(2\pi f_c t)$$

$$\Rightarrow s_2(t) = \frac{A_m A_c}{2} \{\cos[2\pi (f_c - f_m) t] - \cos[2\pi (f_c + f_m) t]\}$$

Add $s_1(t)$ and $s_2(t)$ in order to get the SSBSC modulated wave $s(t)$ having a lower sideband.

$$s(t) = \frac{A_m A_c}{2} \{\cos[2\pi (f_c + f_m) t] + \cos[2\pi (f_c - f_m) t]\} +$$

$$\frac{A_m A_c}{2} \{\cos[2\pi (f_c - f_m) t] - \cos[2\pi (f_c + f_m) t]\}$$

$$\Rightarrow s(t) = A_m A_c \cos[2\pi (f_c - f_m) t]$$

Subtract $s_2(t)$ from $s_1(t)$ in order to get the SSBSC modulated wave $s(t)$ having a upper sideband.

$$s(t) = \frac{A_m A_c}{2} \{\cos[2\pi (f_c + f_m) t] + \cos[2\pi (f_c - f_m) t]\} -$$

$$\frac{A_m A_c}{2} \{\cos[2\pi (f_c - f_m) t] - \cos[2\pi (f_c + f_m) t]\}$$

$$\Rightarrow s(t) = A_m A_c \cos[2\pi (f_c + f_m) t]$$

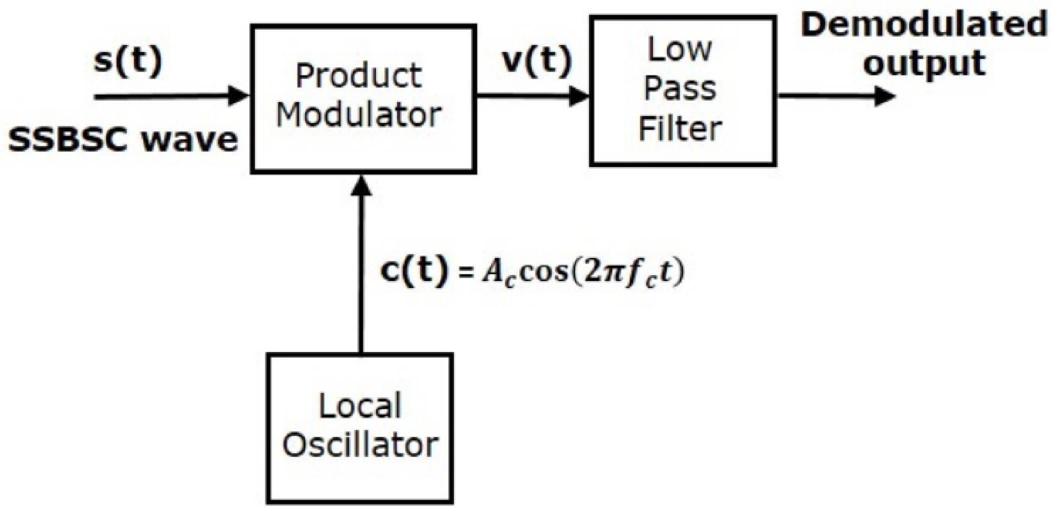
Hence, by properly choosing the polarities of inputs at summer block, we will get SSBSC wave having a upper sideband or a lower sideband.

SSBSC Demodulation

The process of extracting an original message signal from SSBSC wave is known as detection or demodulation of SSBSC. Coherent detector is used for demodulating SSBSC wave.

Coherent Detector

Here, the same carrier signal (which is used for generating SSBSC wave) is used to detect the message signal. Hence, this process of detection is called as **coherent** or **synchronous detection**. Following is the block diagram of coherent detector.



In this process, the message signal can be extracted from SSBSC wave by multiplying it with a carrier, having the same frequency and the phase of the carrier used in SSBSC modulation. The resulting signal is then passed through a Low Pass Filter. The output of this filter is the desired message signal.

Consider the following **SSBSC** wave having a **lower sideband**.

$$s(t) = \frac{A_m A_c}{2} \cos[2\pi (f_c - f_m) t]$$

The output of the local oscillator is

$$c(t) = A_c \cos(2\pi f_c t)$$

From the figure, we can write the output of product modulator as

$$v(t) = s(t)\, c(t)$$

Substitute $s\left(t\right)$ and $c\left(t\right)$ values in the above equation.

$$v\left(t\right)=\frac{A_m A_c}{2}\cos[2\pi\left(f_c-f_m\right)t]A_c\cos(2\pi f_c t)$$

$$=\frac{A_m A_c{}^2}{2}\cos[2\pi\left(f_c-f_m\right)t]\cos(2\pi f_c t)$$

$$=\frac{A_m A_c{}^2}{4}\{\cos[2\pi\left(2f_c-fm\right)]+\cos(2\pi f_m)t\}$$

$$v\left(t\right)=\frac{A_m A_c{}^2}{4}\cos(2\pi f_m t)+\frac{A_m A_c{}^2}{4}\cos[2\pi\left(2f_c-f_m\right)t]$$

In the above equation, the first term is the scaled version of the message signal. It can be extracted by passing the above signal through a low pass filter.

Therefore, the output of low pass filter is

$$v_0\left(t\right)=\frac{A_m A_c{}^2}{4}\cos(2\pi f_m t)$$

Here, the scaling factor is $\frac{A_c{}^2}{4}$.

We can use the same block diagram for demodulating SSBSC wave having an upper sideband. Consider the following **SSBSC** wave having an **upper sideband**.

$$s\left(t\right)=\frac{A_m A_c}{2}\cos[2\pi\left(f_c+f_m\right)t]$$

The output of the local oscillator is

$$c\left(t\right)=A_c\cos(2\pi f_c t)$$

We can write the output of the product modulator as

$$v\left(t\right)=s\left(t\right)c\left(t\right)$$

Substitute $s(t)$ and $c(t)$ values in the above equation.

$$\Rightarrow v(t) = \frac{A_m A_c}{2} \cos[2\pi (f_c + f_m) t] A_c \cos(2\pi f_c t)$$

$$= \frac{A_m A_c^2}{2} \cos[2\pi (f_c + f_m) t] \cos(2\pi f_c t)$$

$$= \frac{A_m A_c^2}{4} \{\cos[2\pi (2f_c + f_m) t] + \cos(2\pi f_m t)\}$$

$$v(t) = \frac{A_m A_c^2}{4} \cos(2\pi f_m t) + \frac{A_m A_c^2}{4} \cos[2\pi (2f_c + f_m) t]$$

In the above equation, the first term is the scaled version of the message signal. It can be extracted by passing the above signal through a low pass filter.

Therefore, the output of the low pass filter is

$$v_0(t) = \frac{A_m A_c^2}{4} \cos(2\pi f_m t)$$

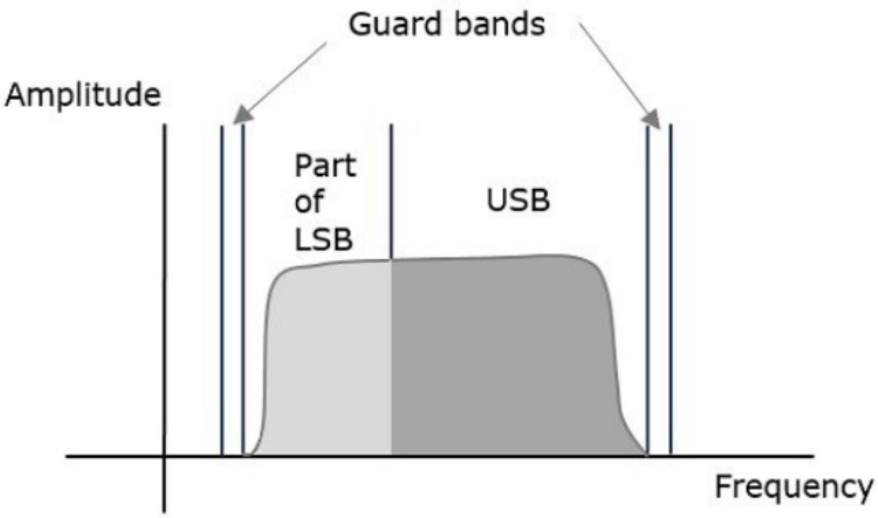
Here too the scaling factor is $\frac{A_c^2}{4}$.

Therefore, we get the same demodulated output in both the cases by using coherent detector.

VSBSC Modulation

To avoid this loss, a technique is chosen, which is a compromise between DSBSC and SSBSC. This technique is known as **Vestigial Side Band Suppressed Carrier (VSBSC)** technique. The word "vestige" means "a part" from which, the name is derived.

VSBSC Modulation is the process, where a part of the signal called as vestige is modulated along with one sideband. The frequency spectrum of VSBSC wave is shown in the following figure.



Along with the upper sideband, a part of the lower sideband is also being transmitted in this technique. Similarly, we can transmit the lower sideband along with a part of the upper sideband. A guard band of very small width is laid on either side of VSB in order to avoid the interferences. VSB modulation is mostly used in television transmissions.

Bandwidth of VSBSC Modulation

We know that the bandwidth of SSBSC modulated wave is f_m . Since the VSBSC modulated wave contains the frequency components of one side band along with the vestige of other sideband, the bandwidth of it will be the sum of the bandwidth of SSBSC modulated wave and vestige frequency f_v .

i.e., ***Bandwidth of VSBSC Modulated Wave =*** $f_m + f_v$

Advantages

Following are the advantages of VSBSC modulation.

- Highly efficient.
- Reduction in bandwidth when compared to AM and DSBSC waves.
- Filter design is easy, since high accuracy is not needed.
- The transmission of low frequency components is possible, without any difficulty.
- Possesses good phase characteristics.

Disadvantages

Following are the disadvantages of VSBSC modulation.

- Bandwidth is more when compared to SSBSC wave.
- Demodulation is complex.

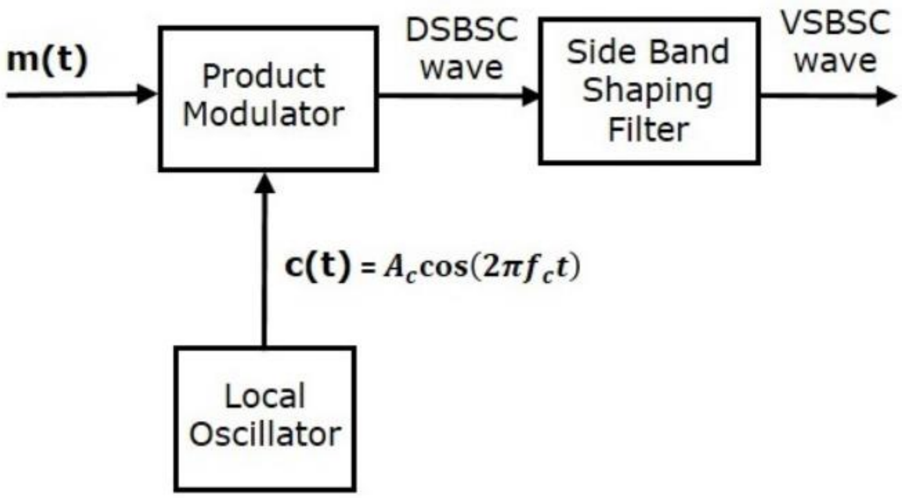
Applications

The most prominent and standard application of VSBSC is for the transmission of television signals. Also, this is the most convenient and efficient technique when bandwidth usage is considered.

Now, let us discuss about the modulator which generates VSBSC wave and the demodulator which demodulates VSBSC wave one by one.

Generation of VSBSC

Generation of VSBSC wave is similar to the generation of SSBSC wave. The VSBSC modulator is shown in the following figure.



In this method, first we will generate DSBSC wave with the help of the product modulator. Then, apply this DSBSC wave as an input of sideband shaping filter. This filter produces an output, which is VSBSC wave.

The modulating signal $m(t)$ and carrier signal $A_c \cos(2\pi f_c t)$ are applied as inputs to the product modulator. Hence, the product modulator produces an output, which is the product of these two inputs.

Therefore, the output of the product modulator is

$$p(t) = A_c \cos(2\pi f_c t) m(t)$$

Apply Fourier transform on both sides

$$P(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

The above equation represents the equation of DSBSC frequency spectrum.

Let the transfer function of the sideband shaping filter be $H(f)$. This filter has the input $p(t)$ and the output is VSBSC modulated wave $s(t)$. The Fourier transforms of $p(t)$ and $s(t)$ are $P(t)$ and $S(t)$ respectively.

Mathematically, we can write $S(f)$ as

$$S(t) = P(f) H(f)$$

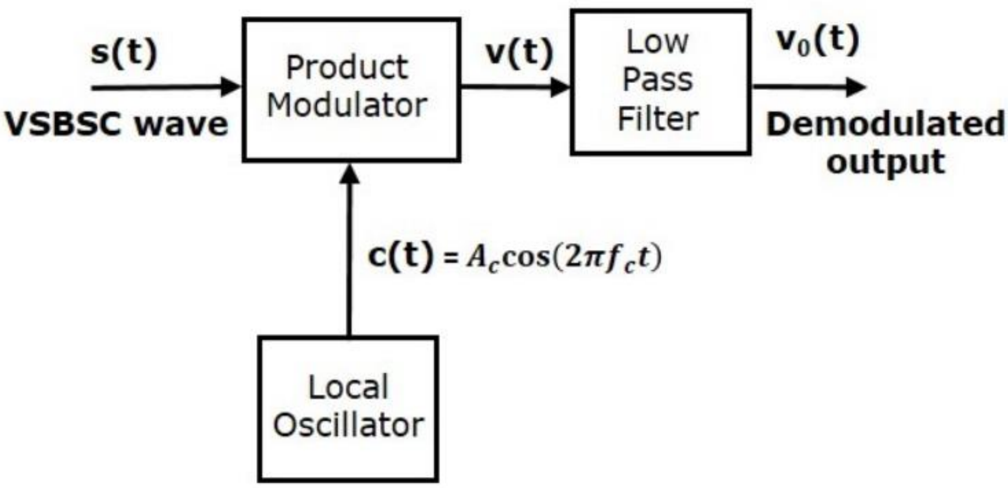
Substitute $P(f)$ value in the above equation.

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)] H(f)$$

The above equation represents the equation of VSBSC frequency spectrum.

Demodulation of VSBSC

Demodulation of VSBSC wave is similar to the demodulation of SSBSC wave. Here, the same carrier signal (which is used for generating VSBSC wave) is used to detect the message signal. Hence, this process of detection is called as **coherent** or **synchronous detection**. The VSBSC demodulator is shown in the following figure.



In this process, the message signal can be extracted from VSBSC wave by multiplying it with a carrier, which is having the same frequency and the phase of the carrier used in VSBSC modulation. The resulting signal is then passed through a Low Pass Filter. The output of this filter is the desired message signal.

Let the VSBSC wave be $s(t)$ and the carrier signal is $A_c \cos(2\pi f_c t)$.

From the figure, we can write the output of the product modulator as

$$v(t) = A_c \cos(2\pi f_c t) s(t)$$

Apply Fourier transform on both sides

$$V(f) = \frac{A_c}{2}[S(f - f_c) + S(f + f_c)]$$

We know that $S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]H(f)$

From the above equation, let us find $S(f - f_c)$ and $S(f + f_c)$.

$$S(f - f_c) = \frac{A_c}{2}[M(f - f_c - f_c) + M(f - f_c + f_c)]H(f - f_c)$$

$$\Rightarrow S(f - f_c) = \frac{A_c}{2}[M(f - 2f_c) + M(f)]H(f - f_c)$$

$$S(f + f_c) = \frac{A_c}{2}[M(f + f_c - f_c) + M(f + f_c + f_c)]H(f + f_c)$$

$$\Rightarrow S(f + f_c) = \frac{A_c}{2}[M(f) + M(f + 2f_c)]H(f + f_c)$$

Substitute, $S(f - f_c)$ and $S(f + f_c)$ values in $V(f)$.

$$V(f) = \frac{A_c}{2}[\frac{A_c}{2}[M(f - 2f_c) + M(f)]H(f - f_c) +$$

$$\frac{A_c}{2}[M(f) + M(f + 2f_c)]H(f + f_c)]$$

$$\Rightarrow V(f) = \frac{{A_c}^2}{4} M(f) \left[H(f - f_c) + H(f + f_c) \right]$$

$$+ \frac{{A_c}^2}{4} \left[M(f - 2f_c) H(f - f_c) + M(f + 2f_c) H(f + f_c) \right]$$

In the above equation, the first term represents the scaled version of the desired message signal frequency spectrum. It can be extracted by passing the above signal through a low pass filter.

$$V_0(f) = \frac{{A_c}^2}{4} M(f) \left[H(f - f_c) + H(f + f_c) \right]$$

Unit-3

Angle Modulation Systems

There are two forms of angle modulation that may be distinguished – phase modulation and frequency modulation

Basic Definitions: Phase Modulation (PM) and Frequency Modulation (FM)

Let $\theta_i(t)$ denote the angle of modulated sinusoidal carrier, which is a function of the message. The resulting angle-modulated wave is expressed as

$$s(t) = A_c cos[\theta_i(t)] \ldots\ldots\ldots\ldots\ldots (1)$$

Where A_c is the carrier amplitude. A complete oscillation occurs whenever $\theta_i(t)$ changes by 2π radians. If $\theta_i(t)$ increases monotonically with time, the average frequency in Hz, over an interval from t to $t+\Delta t$, is given by

$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t} \ldots\ldots\ldots\ldots\ldots (2)$$

Thus the instantaneous frequency of the angle-modulated wave $s(t)$ is defined as

$$f_i(t) = \lim_{\Delta t\to 0} f_{\Delta t}(t)$$

$$f_i(t) = \lim_{\Delta t\to 0} [\frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}]$$

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} \ldots\ldots\ldots\ldots\ldots (3)$$

Thus, according to equation (1), the angle modulated wave s(t) is interpreted as a rotating Phasor of length Ac and angle $\theta_i(t)$. The angular velocity of such a Phasor is $d\theta_i(t)/dt$, in accordance with equ (3).In the simple case of an unmodulated carrier, the angle $\theta_i(t)$ is

$$\theta_i(t) = 2\pi f_c t + \emptyset_c$$

And the corresponding Phasor rotates with a constant angular velocity equal to $2\pi f_c$.The constant ϕ_c is the value of $\theta_i(t)$ at t=0.

There are an infinite number of ways in which the angle $\theta_i(t)$ may be varied in some manner with the baseband signal.

But the 2 commonly used methods are **Phase modulation** and **Frequency modulation.**

Phase Modulation (PM) is that form of angle modulation in which the angle $\theta_i(t)$ is varied linearly with the baseband signal m(t), as shown by

$$\theta_i(t) = 2\pi f_c t + k_p m(t) \ldots\ldots\ldots\ldots\ldots\ldots..(4)$$

The term $2\pi f_c t$ represents the angle of the unmodulated carrier, and the constant k_p represents the ***phase sensitivity*** of the modulator, expressed in radians per volt.

The phase-modulated wave *s(t)* is thus described in time domain by

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)] \ldots\ldots\ldots\ldots\ldots(5)$$

Frequency Modulation (FM) is that form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the baseband signal *m(t)*, as shown by

$$f_i(t) = f_c + k_f m(t) \ldots\ldots\ldots\ldots\ldots\ldots.(6)$$

The term f_c represents the frequency of the unmodulated carrier, and the constant k_f represents the ***frequency sensitivity*** of the modulator, expressed in hertz per volt.

Integrating equ.(6) with respect to time and multiplying the result by 2π, we get

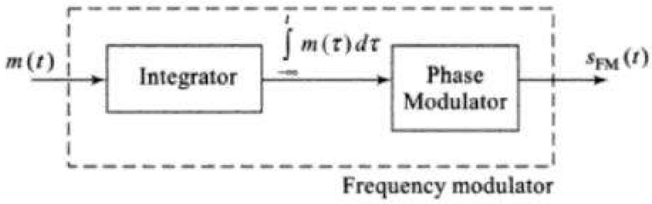
$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\ dt \ldots\ldots\ldots\ldots\ldots.(7)$$

Where, for convenience it is assumed that the angle of the unmodulated carrier wave is zero at t=0. The frequency modulated wave is therefore described in the time domain by

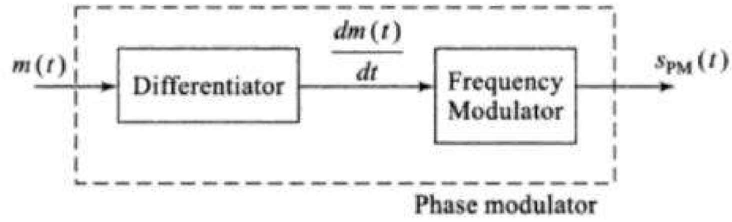
$$s(t) = A_c \cos[\ 2\pi f_c t + 2\pi k_f \int_0^t m(t)\ dt] \ldots\ldots\ldots\ldots\ldots\ldots..(8)$$

Relationship between PM and FM

Comparing equ (5) with (8) reveals that an FM wave may be regarded as a PM wave in which the modulating wave is $\int_0^t m(t)dt$ in place of *m(t)*.



A PM wave can be generated by first differentiating *m(t)* and then using the result as the input to a frequency modulator.


m(t)
Differentiator
dm(t)/dt
Frequency Modulator
s_PM(t)
Phase modulator

Thus the properties of PM wave can be deduced from those of FM waves and vice versa

Single tone Frequency modulation

Consider a sinusoidal modulating wave defined by

$$m(t) = A_m \cos(2\pi f_m t) \dots\dots\dots\dots\dots (1)$$

The instantaneous frequency of the resulting FM wave is

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t)$$

$$f_i(t) = f_c + \Delta f \cos(2\pi f_m t) \dots\dots\dots\dots\dots (2)$$

Where $$\Delta f = k_f A_m \dots\dots\dots\dots\dots (3)$$

The quantity Δf is called the ***frequency deviation***, representing the maximum departure of the instantaneous frequency of the FM wave from the carrier frequency f_c.

Fundamental characteristic of an FM wave is that the frequency deviation Δf is proportional to the amplitude of the modulating wave, and is independent of the modulation frequency.

Using equation (2), the angle $\theta_i(t)$ of the FM wave is obtained as

$$\theta_i(t) = 2\pi \int_0^t f_i(t)dt$$

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \dots\dots\dots\dots\dots\dots\dots (4)$$

The ratio of the frequency deviation Δf *to* the modulation frequency f_m is commonly called the ***modulation index*** of the FM wave. Modulation index is denoted by $\boldsymbol{\beta}$ and is given as

$$\mathrm{Q} = \frac{\Delta f}{f_m} \dots\dots\dots\dots\dots\dots\dots (5)$$

And

$$\theta_i(t) = 2\pi f_c t + \mathrm{Q} \sin(2\pi f_m t) \dots\dots\dots\dots\dots\dots\dots (6)$$

In equation (6) the parameter $\boldsymbol{\beta}$ represents the phase deviation of the FM wave, that is, the maximum departure of the angle $\theta_i(t)$ from the angle $2\pi f_c t$ of the unmodulated carrier.

The FM wave itself is given by

$$s(t) = A_c \cos[2\pi f_c t + \text{Q} sin(2\pi f_m t)] \ldots\ldots\ldots\ldots\ldots\ldots..(7)$$

Depending on the value of modulation index β, we may distinguish two cases of frequency modulation. Narrow-band FM for which β is small and Wide-band FM for which β is large, both compared to one radian.

Narrow-Band Frequency modulation

Consider the Single tone FM wave

$$s(t) = A_c \cos[2\pi f_c t + Q \sin(2\pi f_m t)] \quad \ldots\ldots\ldots\ldots(1)$$

Expanding this relation we get

$$s(t) = A_c \cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)] \quad \ldots\ldots(2)$$

Assuming that the modulation index β is small compared to one radian, we may use the following approximations:

$$\cos[\beta \sin(2\pi f_m t)] \simeq 1$$

and

$$\sin[\beta \sin(2\pi f_m t)] \simeq \beta \sin(2\pi f_m t)$$

Hence, Equation (2) simplifies to

$$s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t) \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Equation (3) defines the approximate form of a narrowband FM signal produced by a sinusoidal modulating signal $A_m \cos(2\pi f_m t)$. From this representation we deduce the modulator shown in block diagram form in Figure . This modulator involves splitting the carrier wave $A_c \cos(2\pi f_c t)$ into two paths. One path is direct; the other path contains a -90 degree phase-shifting network and a product modulator, the combination of which generates a DSB-SC modulated signal. The difference between these two signals produces a narrowband FM signal, but with some distortion.

Ideally, an FM signal has a constant envelope and, for the case of a sinusoidal modulating signal of frequency f_m, the angle $\theta_i(t)$ is also sinusoidal with the same frequency.

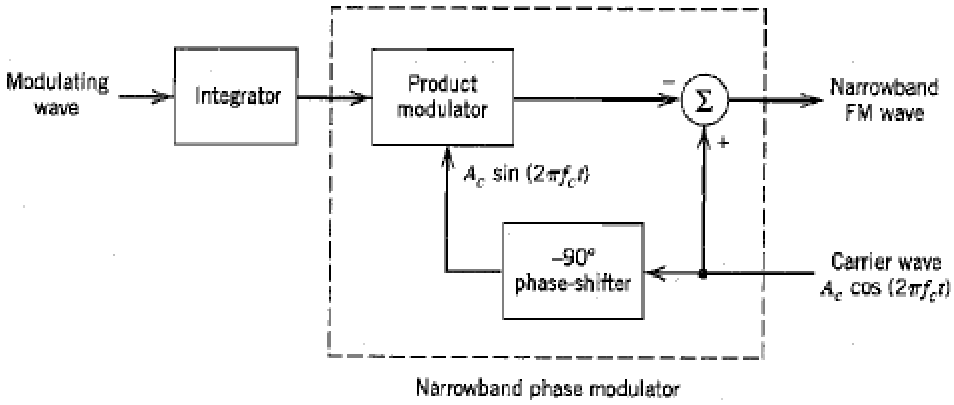


FIGURE Block diagram of a method for generating a narrowband FM signal.

But the modulated signal produced by the narrowband modulator of Figure differs from this ideal condition in two fundamental respects:

1. The envelope contains a *residual* amplitude modulation and, therefore, varies with time.
2. For a sinusoidal modulating wave, the angle $\theta_i(t)$ contains *harmonic distortion* in the form of third- and higher-order harmonics of the modulation frequency f_m.

However, by restricting the modulation index to $\beta \leq 0.3$ radians, the effects of residual AM and harmonic PM are limited to negligible levels.

Returning to Equation (3), we may expand it as follows:

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c \{\cos[2\pi(f_c + f_m)t] - \cos[2\pi(f_c - f_m)t]\} \text{........(4)}$$

This expression is somewhat similar to the corresponding one defining an AM signal, which is as follows:

$$s_{AM}(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \mu A_c \{\cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - f_m)t]\} \text{.........(5)}$$

where μ is the modulation factor of the AM signal. Comparing Equations (4) and (5), we see that in the case of sinusoidal modulation, the basic difference between an AM signal and a narrowband FM signal is that the algebraic sign of the lower side frequency in the narrowband FM is reversed. Thus, a narrowband FM signal requires essentially the same transmission bandwidth (i.e., $2f_m$) as the AM signal.

We may represent the narrowband FM signal with a phasor diagram as shown in Figure a , where we have used the carrier phasor as reference. We see that the resultant

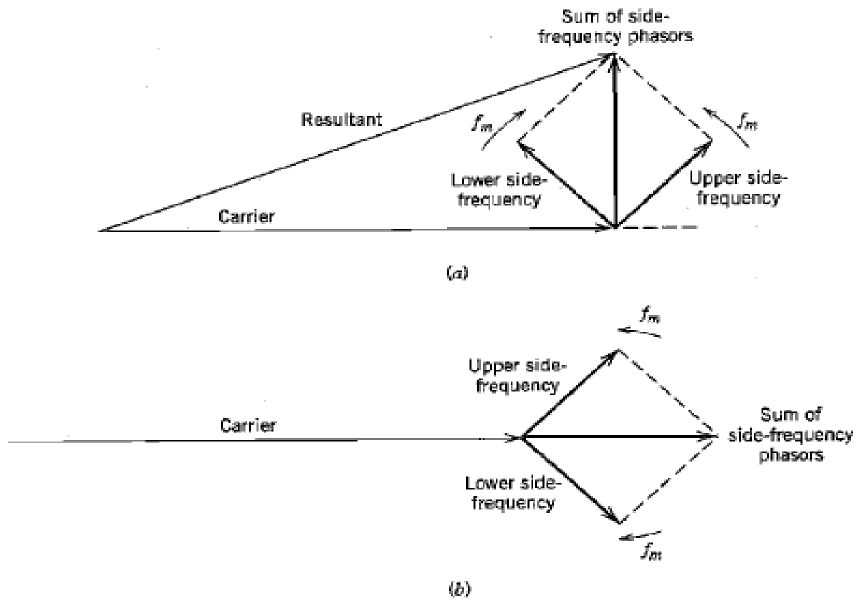


FIGURE A phasor comparison of narrowband FM and AM waves for sinusoidal modulation. (*a*) Narrowband FM wave. (*b*) AM wave.

of the two side-frequency phasors is always at right angles to the carrier phasor. The effect of this is to produce a resultant phasor representing the narrowband FM signal that is approximately of the same amplitude as the carrier phasor, but out of phase with respect to it. This phasor diagram should be contrasted with that of Figure *(b)* , representing an AM signal. In this latter case we see that the resultant phasor representing the AM signal has an amplitude that is different from that of the carrier phasor but always in phase with it.

Wide band frequency Modulation

The spectrum of the signle-tone FM wave of equation

$$\boldsymbol{s(t) = A_c \cos[2\pi f_c t + Q\sin(2\pi f_m t)]} \; \dots\dots\dots\dots (1)$$

For an arbitrary vale of the modulation index Q is to be determined.

An FM wave produced by a sinusoidal modulating wave as in equation (1) is by itself nonperiodic, unless the carrier frequency f_c is an integral multiple of the modualtion frequency f_m. Rewriting the equation in the form

$$\begin{aligned} s(t) &= \mathrm{Re}[A_c \exp(j2\pi f_c t + j\beta \sin(2\pi f_m t))] \\ &= \mathrm{Re}[\tilde{s}(t) \exp(j2\pi f_c t)] \qquad \dots\dots\dots\dots(2) \end{aligned}$$

where $\tilde{s}(t)$ is the *complex envelope* of the FM signal $s(t)$, defined by

$$\tilde{s}(t) = A_c \exp[j\beta \sin(2\pi f_m t)] \qquad \dots\dots\dots\dots\dots(3)$$

$\tilde{s}(t)$ is periodic function of time,with a fundamental frequency equal to the modulation frequency f_m. $\tilde{s}(t)$ in the form of complex Fourier series is as follows

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_m t) \qquad \dots\dots\dots\dots\dots(4)$$

where the complex Fourier coefficient c_n is defined by

$$\begin{aligned} c_n &= f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) \exp(-j2\pi n f_m t)\, dt \\ &= f_m A_c \int_{-1/2f_m}^{1/2f_m} \exp[j\beta \sin(2\pi f_m t) - j2\pi n f_m t]\, dt \qquad \dots\dots\dots\dots(5) \end{aligned}$$

Define a new variable:

$$x = 2\pi f_m t \qquad \dots\dots\dots\dots\dots\dots\dots\dots(6)$$

Hence, we may rewrite Equation (5) in the new form

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]\, dx \qquad \dots\dots\dots\dots\dots(7)$$

The integral on the RHS of equation (7) is recognizedasthe nth order Bessel Function of the first kind and argument Q. This function is commonly denoted by the symbol J_n(Q), that is

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp[j(\beta \sin x - nx)]\, dx \qquad \text{.........(8)}$$

Accordingly, we may reduce Equation (7) to

$$c_n = A_c J_n(\beta) \qquad \text{................(9)}$$

Substituting Equation (9) in (5), we get, in terms of the Bessel function $J_n(\beta)$, the following expansion for the complex envelope of the FM signal:

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \exp(j2\pi n f_m t) \qquad \text{..............(10)}$$

Next, substituting Equation (10) in (2), we get

$$s(t) = A_c \cdot \text{Re}\left[\sum_{n=-\infty}^{\infty} J_n(\beta) \exp[j2\pi(f_c + nf_m)t]\right] \qquad \text{.............(11)}$$

Interchanging the order of summation and evaluation of the real part in the right-hand side of Equation (11), we finally get

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + nf_m)t] \qquad \text{................(12)}$$

Equ. (12) is the Fourier series representation of the single-tone FM wave *s(t)* for an arbitrary value of Q.

The discrete spectrum of *s(t)* is obtained by taking the Fourier transform of both sides of equation (12); thus

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)] \text{.............(13)}$$

In the figure below, we have plotted the Bessel function J_n(Q) versus the modulation index Q for different positive integer value of n.

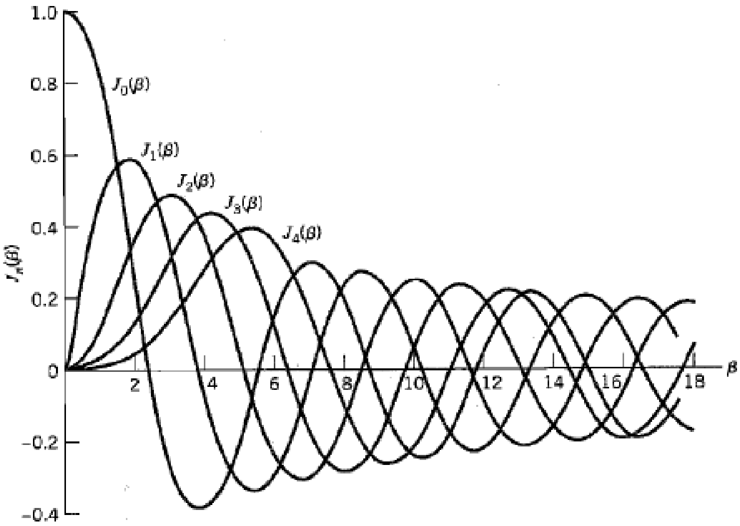


FIGURE Plots of Bessel functions of the first kind for varying order.

Properties of Bessel Function

1. $J_n(\beta) = (-1)^n J_{-n}(\beta)$ for all n, both positive and negative **.........(14)**

2. For small values of the modulation index β, we have

$$\left.\begin{aligned} J_0(\beta) &\simeq 1 \\ J_1(\beta) &\simeq \frac{\beta}{2} \\ J_n(\beta) &\simeq 0, \qquad n > 2 \end{aligned}\right\} \qquad \textbf{...............(15)}$$

3. $$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1 \qquad \textbf{...................(16)}$$

Thus using equations (13) through (16) and the curves in the above figure, following observations are made

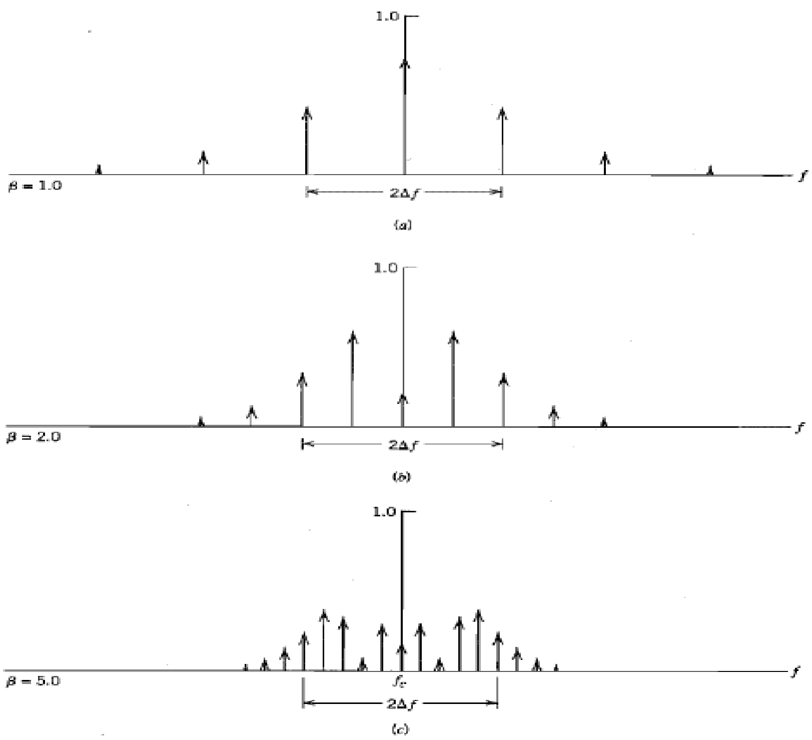
1. The spectrum of an FM signal contains a carrier component and an infinite set of side frequencies located symmetrically on either side of the carrier at frequency separations of f_m, $2f_m$, $3f_m$, $\cdots$. In this respect, the result is unlike that which prevails in an AM system, since in an AM system a sinusoidal modulating signal gives rise to only one pair of side frequencies.
2. For the special case of β small compared with unity, only the Bessel coefficients $J_0(\beta)$ and $J_1(\beta)$ have significant values, so that the FM signal is effectively composed of a carrier and a single pair of side frequencies at $f_c \pm f_m$. This situation corresponds to the special case of narrowband FM that was considered earlier.
3. The amplitude of the carrier component varies with β according to $J_0(\beta)$. That is, unlike an AM signal, the amplitude of the carrier component of an FM signal is dependent on the modulation index β. The physical explanation for this property is that the envelope of an FM signal is constant, so that the average power of such a signal developed across a 1-ohm resistor is also constant, as shown by

$$P = \frac{1}{2} A_c^2 \qquad \text{..................(17)}$$

When the carrier is modulated to generate the FM signal, the power in the side frequencies may appear only at the expense of the power originally in the carrier, thereby making the amplitude of the carrier component dependent on β. Note that the average power of an FM signal may also be determined from Equation (**12**), obtaining

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta) \qquad \text{......................(18)}$$

Spectrum Analysis of Sinusoidal FM Wave using Bessel functions



The above figure shows the Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of varying frequency and fixed amplitude. Only the spectra for positive frequencies are shown.

Transmission Bandwidth of FM waves

In theory, an FM signal contains an infinite number of side frequencies so that the bandwidth required to transmit such a signal is similarly infinite in extent. In practice, however, we find that the FM signal is effectively limited to a finite number of significant side frequencies compatible with a specified amount of distortion. We may therefore specify an effective bandwidth required for the transmission of an FM signal. Consider first the

case of an FM signal generated by a single-tone modulating wave of frequency f_m. In such an FM signal, the side frequencies that are separated from the carrier frequency f_c by an amount greater than the frequency deviation Δf decrease rapidly toward zero, so that the bandwidth always exceeds the total frequency excursion, but nevertheless is limited. Specifically, for large values of the modulation index β, the bandwidth approaches, and is only slightly greater than, the total frequency excursion $2\Delta f$ in accordance with the situation shown in Figure 2.25. On the other hand, for small values of the modulation index β, the spectrum of the FM signal is effectively limited to the carrier frequency f_c and one pair of side frequencies at $f_c \pm f_m$, so that the bandwidth approaches $2f_m$. We may thus define an approximate rule for the transmission bandwidth of an FM signal generated by a single-tone modulating signal of frequency f_m as follows:

$$B_T \simeq 2\Delta f + 2f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

This relation is known as **Carson's rule.**

Generation of FM Signal

Direct methods for FM generation

Reactance modulator:

The direct method of FM generation using the reactance modulator involves providing a voltage-variable reactance across the tank circuit of an oscillator. Though the varactor diode modulator can be called a reactance modulator, the term is generally applied to those modulators in which an active device is made to behave as a variable reactance. In Fig. we show the basic FET capacitive reactance modulator.

Under certain conditions, the impedance z, across terminals AA' is almost entirely reactive. The circuit can be made either inductive or capacitive by a simple component change and its reactance can be shown to be proportional to the transconductance of the device, which in turn can be made to depend on the variations in the gate bias. The circuit impedance is: $z = \frac{v}{i_d}$, where v is the voltage applied across AA' and i_d is the resulting drain current. For the impedance across AA' to be capacitive, the following conditions have to be met:

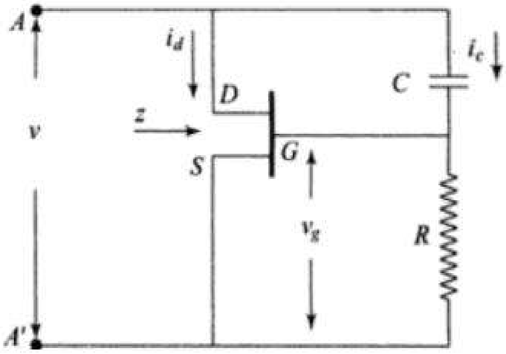


Fig. *FET capacitive reactance modulator*

- The bias current i_c must be negligible compared to the drain current i_d.
- The drain-to-gate impedance (X_c) must be greater than gate-to-source impedance (R here) by at least a factor of five.

$$v_g = i_c R = \frac{vR}{(R - jX_c)}$$

Drain current: $i_d = g_m v_g = \dfrac{g_m vR}{(R - jX_c)}$

Since $X_c \gg R$, we have:

$$z = \frac{v}{i_d} \approx -\frac{jX_c}{g_m R}$$

This is clearly a capacitive reactance with equivalent impedance:

$$X_{eq} = \frac{X_c}{g_m R} = \frac{1}{2\pi f C g_m R} = \frac{1}{2\pi f C_{eq}}$$

Hence, under these assumptions, the impedance looking into AA' is a pure reactance given by:

$$C_{eq} = g_m CR$$

Since C_{eq} depends on the transconductance g_m, it can be varied with bias voltage.

Now $X_c \gg R$. Let $X_c = nR$ at the carrier frequency. Then, $\dfrac{1}{\omega C} = nR$.

Therefore, $$C = \frac{1}{\omega n R} = \frac{1}{2\pi f n R}$$

$$C_{eq} = g_m CR = \frac{g_m}{2\pi f n}$$

What would have happened if the positions of C and R are interchanged and if $R \gg X_c$?

$$v_g = i_c R = \left[\frac{v(-jX_c)}{(R - jX_c)}\right]$$

Drain current: $i_d = g_m v_g = \left[\frac{g_m v(-jX_c)}{(R - jX_c)}\right]$

$$z = \frac{v}{i_d} = \frac{(R - jX_c)}{(-jX_c\, g_m)}$$

$$= \left(\frac{1}{X_c\, g_m}\right)[X_c + jR] \approx \frac{jR}{(X_c\, g_m)}$$

Clearly, the impedance is inductive and can be written as:

$$X_{eq} = \frac{R}{X_c\, g_m} = \frac{(2\pi f CR)}{g_m} = 2\pi f L_{eq}, \text{ where } L_{eq} = \frac{CR}{g_m}$$

Thus, the FET reactance modulator behaves as a three-terminal reactive element (either inductive or capacitive) that may be connected across the tank circuit of the oscillator to be frequency modulated. The reactance appears between the drain and the source and its value may be controlled by a signal at the third terminal, i.e., the gate.

A disadvantage of the above methods of FM generation is that they do not provide carrier frequency stability. For attaining the required order of carrier frequency stability in the 88-108 MHz range used for FM transmission, it is necessary to use crystal oscillators. However, the high Q of crystal oscillators permits direct modulation only in some narrowband applications[2]. For wideband FM generation using crystal oscillators, the indirect method is adopted.

Indirect Method for WBFM Generation (ARMSTRONG'S Method):

In this method, an NBFM signal is generated using an integrator and a phase modulator. The NBFM signal is then converted to WBFM using a frequency multiplier. Let us first consider the principle behind a frequency multiplier.

A frequency multiplier is an amplifier whose output signal frequency is an integer multiple of the input frequency. If the input to the frequency multiplier is $A\cos\theta(t)$ then the output is $A\cos[n\theta(t)]$, where n is an integer. This can be achieved by feeding a signal frequency that is rich in harmonic distortion (e.g. from a class C amplifier) into an LC tank circuit tuned to n times the input frequency. This arrangement results in the n^{th} harmonic being the only significant output. For larger multiplication factors, a cascade of doublers and triplers are used. For example $n = 1000$ could be approximated by a cascade of ten doublers ($2^{10} = 1024$).

Effect of frequency multiplication on a NBFM signal

Consider single tone NBFM:

$s_{NBFM}(t) = A_c\cos(\omega_c' t + \beta'\sin\omega_m t)$ where $\beta' < 1$ rad and ω_c' is a stable sub-carrier frequency.

When applied to a frequency multiplier, the NBFM signal is converted to a WBFM signal given by:

$$s_{FM}(t) = A_c\cos[n(\omega_c' t + \beta'\sin\omega_m t)]$$
$$= A_c\cos[n\omega_c' t + n\beta'\sin\omega_m t]$$
$$s_{FM}(t) = A_c\cos[\omega_c t + \beta\sin\omega_m t]$$

where $\omega_c = n\omega_c'$ is the final carrier frequency and $\beta = n\beta'$ is the final β.

Note that ω_m is unaffected by frequency multiplication.

The maximum frequency deviation of the NBFM signal also gets multiplied by a factor of n since $\beta = n\beta'$ implies $(\Delta\omega/\omega_m) = n(\Delta\omega'/\omega_m)$ or $\Delta\omega = n\,\Delta\omega'$.

Detection of FM Signal

Balanced slope detector. Figure **(a)** shows the schematic diagram for a *balanced slope detector.* A single-ended slope detector is a tuned-circuit frequency discriminator, and a balanced slope detector is simply two single-ended slope detectors connected in parallel and fed 180° out of phase. The phase inversion is accomplished by center tapping the tuned secondary windings of transformer T_1. In Figure **(a)**, the tuned circuits (L_a, C_a, and L_b, C_b) perform the FM-to-AM conversion, and the balanced peak detectors (D_1, C_1, R_1, and D_2, C_2, R_2) remove the information from the AM envelope. The top tuned circuit (L_a and C_a) is tuned to a frequency (f_a) that is above the IF center frequency (f_o) by approximately $1.33 \times \Delta f$ (for the FM broadcast band, this is approximately 1.33×75 kHz = 100 kHz). The lower tuned circuit (L_b and C_b) is tuned to a frequency (f_b) that is below the IF center frequency by an equal amount.

Circuit operation is quite simple. The output voltage from each tuned circuit is proportional to the input frequency, and each output is rectified by its respective peak detector. Therefore, the closer the input frequency is to the tank-circuit resonant frequency, the greater the tank-circuit output voltage. The IF center frequency falls exactly halfway between the resonant frequencies of the two tuned circuits. Therefore, at the IF center frequency, the output voltages from the two tuned circuits are equal in amplitude but opposite in polarity. Consequently, the rectified output voltage across $R1$ and $R2$,

Balanced Slope Detector

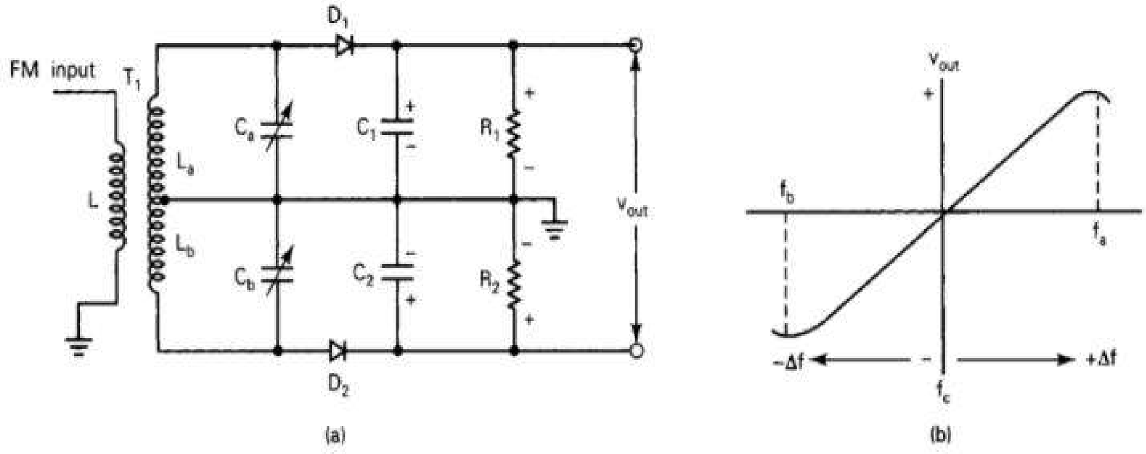


FIGURE Balanced slope detector: (a) schematic diagram; (b) voltage-versus-frequency response curve

when added, produce a differential output voltage V_{out} 5 0 V. When the IF deviates above resonance, the top tuned circuit produces a higher output voltage than the lower tank circuit, and Vout goes positive. When the IF deviates below resonance, the output voltage from the lower tank circuit is larger than the output voltage from the upper tank circuit, and V_{out} goes negative. The output-versus-frequency response curve is shown in Figure **(b).**

Although the slope detector is probably the simplest FM detector, it has several inherent disadvantages, which include poor linearity, difficulty in tuning, and lack of provisions for limiting. Because limiting is not provided, a slope detector produces an output voltage that is proportional to amplitude, as well as frequency variations in the input signal and, consequently, must be preceded by a separate limiter stage. A balanced slope detector is aligned by injecting a frequency equal to the IF center frequency and tuning C_a and C_b for 0 V at the output. Then frequencies equal to f_a and f_b are alternately injected while C_a and C_b are tuned for maximum and equal output voltages with opposite polarities.

Phase Locked Loop

Basically, the *phase-locked loop* consists of three major components: a *multiplier*, a *loop filter*, and a *voltage-controlled oscillator* (VCO) connected together in the form of a feedback system, as shown in Figure below. The VCO is a sinusoidal generator whose frequency is determined by a voltage applied to it from an external source. In effect, any frequency modulator may serve as a VCO. We assume that initially we have adjusted the VCO so that when the control voltage is zero, two conditions are satisfied:

1. The frequency of the VCO is precisely set at the unmodulated carrier frequency f_c.
2. The VCO output has a 90 degree phase-shift with respect to the unmodulated carrier wave.

Suppose then that the input signal applied to the phase-locked loop is an FM signal defined by

$$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)]$$

where A_c is the carrier amplitude. With a modulating signal $m(t)$, the angle $\phi_1(t)$ is related to $m(t)$ by the integral

$$\phi_1(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

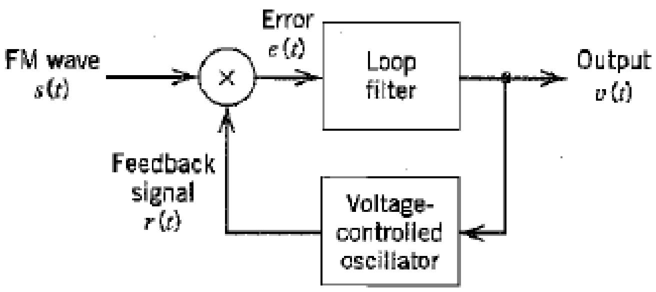


FIGURE Phase-locked loop.

where k_f is the frequency sensitivity of the frequency modulator. Let the VCO output in the phase-locked loop be defined by

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)]$$

where A_v is the amplitude. With a control voltage $v(t)$ applied to the VCO input, the angle $\phi_2(t)$ is related to $v(t)$ by the integral

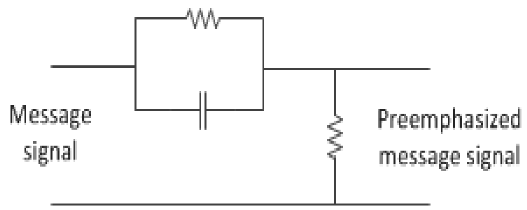
$$\phi_2(t) = 2\pi k_v \int_0^t v(\tau)\, d\tau$$

where k_v is the frequency sensitivity of the VCO, measured in Hertz per volt. The object of the phase-locked loop is to generate a VCO output $r(t)$ that has the same phase angle (except for the fixed difference of 90 degrees) as the input FM signal $s(t)$. The time-varying phase angle $\phi_1(t)$ characterizing $s(t)$ may be due to modulation by a message signal $m(t)$, in which case we wish to recover $\phi_1(t)$ and thereby produce an estimate of $m(t)$. In other applications of the phase-locked loop, the time-varying phase angle $\phi_1(t)$ of the incoming signal $s(t)$ may be an unwanted phase shift caused by fluctuations in the communication channel; in this latter case, we wish to *track* $\phi_1(t)$ so as to produce a signal with the same phase angle for the purpose of coherent detection (synchronous demodulation).

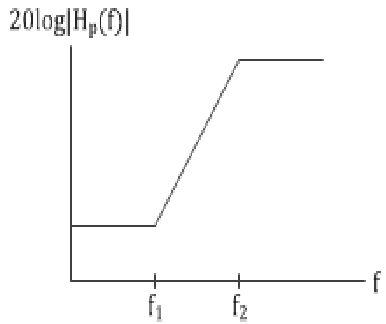
PRE-EMPHASIS AND DE-EMPHASIS NETWORKS

In FM, the noise increases linearly with frequency. By this, the higher frequency components of message signal are badly affected by the noise. To solve this problem, we can use a pre-emphasis filter of transfer function $H_p(f)$ at the transmitter to boost the higher frequency components before modulation. Similarly, at the receiver, the de-emphasis filter of transfer function $H_d(f)$can be used after demodulator to attenuate the higher frequency components thereby restoring the original message signal.

The pre-emphasis network and its frequency response are shown in Figure (a) and (b) respectively. Similarly, the counter part for de-emphasis network is shown in Figure below.



(a)



(b)

Figure (a) Pre-emphasis network. (b) Frequency response of pre-emphasis network.

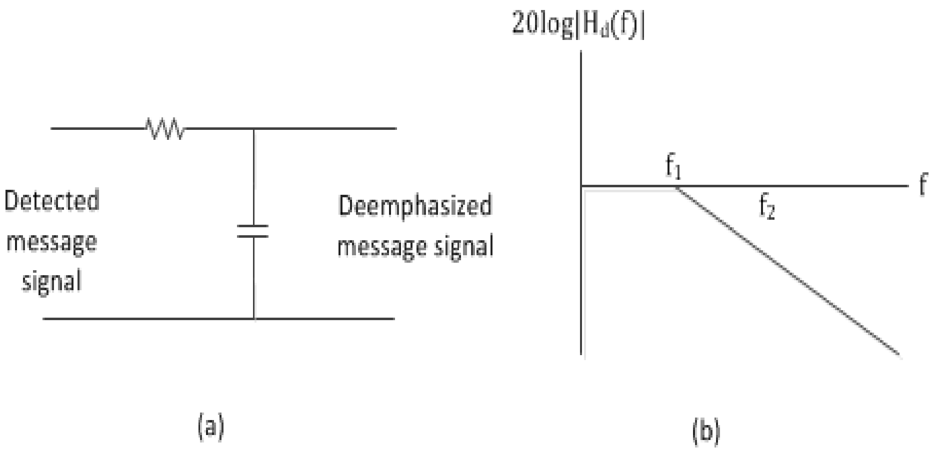


Figure (a) De-emphasis network. (b) Frequency response of De-emphasis network.

Comparison of AM and FM

S.NO	AMPLITUDE MODULATION	FREQUENCY MODULATION
1.	Band width is very small which is one of the biggest advantage	It requires much wider channel (7 to 15 times) as compared to AM.
2.	The amplitude of AM signal varies depending on modulation index.	The amplitude of FM signal is constant and independent of depth of the modulation.
3.	Area of reception is large	The area of reception is small since it is limited to line of sight.
4.	Transmitters are relatively simple & cheap.	Transmitters are complex and hence expensive.
5.	The average power in modulated wave is greater than carrier power. This added power is provided by modulating source.	The average power in frequency modulated wave is same as contained in un-modulated wave.
6.	More susceptible to noise interference and has low signal to noise ratio, it is more difficult to eliminate effects of noise.	Noise can be easily minimized amplitude variations can be eliminated by using limiter.
7.	It is not possible to operate without interference.	It is possible to operate several independent transmitters on same frequency.
8.	The maximum value of modulation index = 1, otherwise over-modulation would result in distortions.	No restriction is placed on modulation index.

Unit-4

AM & FM Transmitter & Receiver

Radio Transmitters

There are two approaches in generating an AM signal. These are known as low and high level modulation. They're easy to identify: A low level AM transmitter performs the process of modulation near the beginning of the transmitter. A high level transmitter performs the modulation step last, at the last or "final" amplifier stage in the transmitter. Each method has advantages and disadvantages, and both are in common use.

Low-Level AM Transmitter:

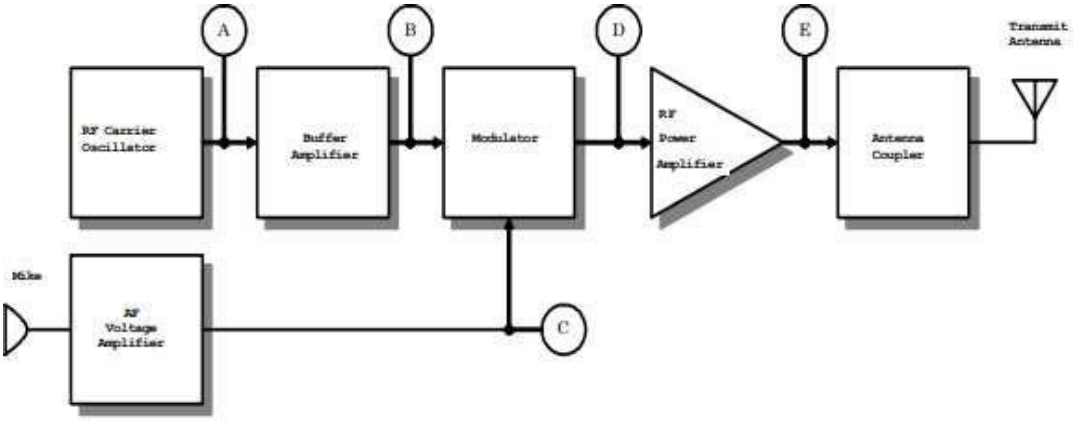


Fig.1. Low-Level AM Transmitter Block Diagram

There are two signal paths in the transmitter, audio frequency (AF) and radio frequency (RF). The RF signal is created in the RF carrier oscillator. At test point A the oscillator's output signal is present. The output of the carrier oscillator is a fairly small AC voltage, perhaps 200 to 400 mV RMS. The oscillator is a critical stage in any transmitter. It must produce an accurate and steady frequency. Every radio station is assigned a different carrier frequency. The dial (or display) of a receiver displays the carrier frequency. If the oscillator drifts off frequency, the receiver will be unable to receive the transmitted signal without being readjusted. Worse yet, if the oscillator drifts onto the frequency being used by another radio station, interference will occur. Two circuit techniques are commonly used to stabilize the oscillator, buffering and voltage regulation.

The buffer amplifier has something to do with buffering or protecting the oscillator. An oscillator is a little like an engine (with the speed of the engine being similar to the oscillator's frequency). If the load on the engine is increased (the engine is asked to do more work), the engine will respond by slowing down. An oscillator acts in a very similar fashion. If the current drawn from the oscillator's output is increased or decreased, the oscillator may speed up or slow down slightly.

Buffer amplifier is a relatively low-gain amplifier that follows the oscillator. It has a constant input impedance (resistance). Therefore, it always draws the same amount of current from the oscillator. This helps to prevent "pulling" of the oscillator frequency. The buffer amplifier is needed because of what's happening "downstream" of the oscillator. Right after this stage is the modulator. Because the modulator is a nonlinear amplifier, it may not have a constant input resistance -- especially when information is passing into it. But since there is a buffer amplifier between the oscillator and modulator, the oscillator sees a steady load resistance, regardless of what the modulator stage is doing.

Voltage Regulation: An oscillator can also be pulled off frequency if its power supply voltage isn't held constant. In most transmitters, the supply voltage to the oscillator is regulated at a constant value. The regulated voltage value is often between 5 and 9 volts; zener diodes and three-terminal regulator ICs are commonly used voltage regulators. Voltage regulation is especially important when a transmitter is being powered by batteries or an automobile's electrical system. As a battery discharges, its terminal voltage falls. The DC supply voltage in a car can be anywhere between 12 and 16 volts, depending on engine RPM and other electrical load conditions within the vehicle.

Modulator: The stabilized RF carrier signal feeds one input of the modulator stage. The modulator is a variable-gain (nonlinear) amplifier. To work, it must have an RF carrier signal and an AF information signal. In a low-level transmitter, the power levels are low in the oscillator, buffer, and modulator stages; typically, the modulator output is around 10 mW (700 mV RMS into 50 ohms) or less.

AF Voltage Amplifier: In order for the modulator to function, it needs an information signal. A microphone is one way of developing the intelligence signal, however, it only produces a few millivolts of signal. This simply isn't enough to operate the modulator, so a voltage amplifier is used to boost the microphone's signal. The signal level at the output of the AF voltage amplifier is usually at least 1 volt RMS; it is highly dependent upon the transmitter's design. Notice that the AF amplifier in the transmitter is only providing a voltage gain, and not necessarily a current gain for the microphone's signal. The power levels are quite small at the output of this amplifier; a few mW at best.

RF Power Amplifier: At test point D the modulator has created an AM signal by impressing the information signal from test point C onto the stabilized carrier signal from test point B at the buffer amplifier output. This signal (test point D) is a complete AM signal, but has only a few milliwatts of power. The RF power amplifier is normally built with several stages. These stages increase both the voltage and current of the AM signal. We say that power amplification occurs when a circuit provides a current gain. In order to accurately amplify the tiny AM signal from the modulator, the RF power amplifier stages must be linear. You might recall that amplifiers are divided up into "classes," according to the conduction angle of the active device within. Class A and class B amplifiers are considered tobe linear amplifiers, so the RF power amplifier stages will normally be constructed using one or both of these type of amplifiers. Therefore, the signal at test point E looks just like that of test point D; it's just much bigger in voltage and current.

Antenna Coupler: The antenna coupler is usually part of the last or final RF power amplifier, and as such, is not really a separate active stage. It performs no amplification, and has no active devices. It performs two important jobs: Impedance matching and filtering. For an RF power amplifier to function correctly, it must be supplied with a load resistance equal to that for which it was designed.

The antenna coupler also acts as a low-pass filter. This filtering reduces the amplitude of harmonic energies that may be present in the power amplifier's output. (All amplifiers generate harmonic distortion, even "linear" ones.) For example, the transmitter may be tuned to operate on 1000 kHz. Because of small nonlinearities in the amplifiers of the transmitter, the transmitter will also produce harmonic energies on 2000 kHz (2nd harmonic), 3000 kHz (3rd harmonic), and so on. Because a low-pass filter passes the fundamental frequency (1000 kHz) and rejects the harmonics, we say that harmonic attenuation has taken place.

High-Level AM Transmitter:

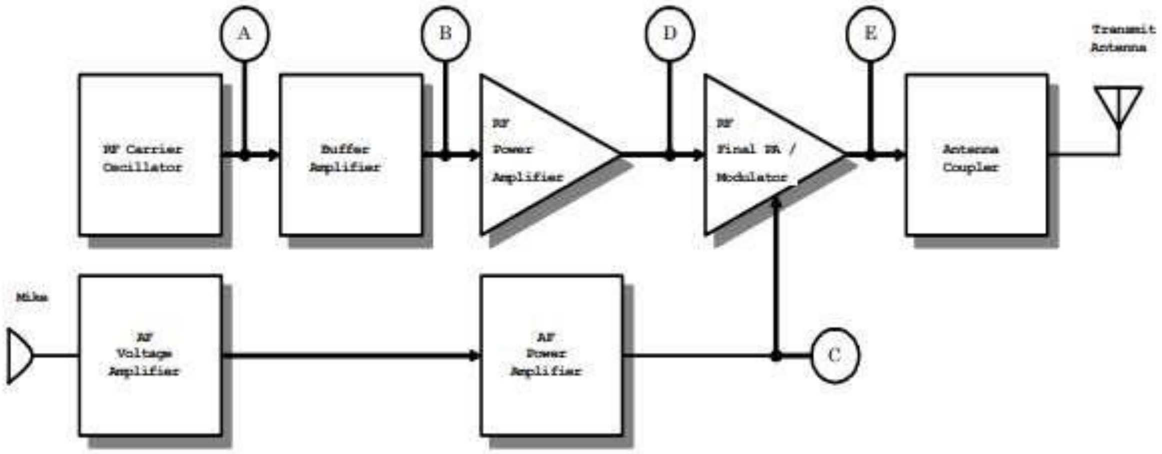


Fig.2. High-Level AM Transmitter Block Diagram

The high-level transmitter of Figure 9 is very similar to the low-level unit. The RF section begins just like the low-level transmitter; there is an oscillator and buffer amplifier. The difference in the high level transmitter is where the modulation takes place. Instead of adding modulation immediately after buffering, this type of transmitter amplifies the unmodulated RF carrier signal first. Thus, the signals at points A, B, and D in Figure 9 all look like unmodulated RF carrier waves. The only difference is that they become bigger in voltage and current as they approach test point D.

The modulation process in a high-level transmitter takes place in the last or final power amplifier. Because of this, an additional audio amplifier section is needed. In order to modulate an amplifier that is running at power levels of several watts (or more), comparable power levels of information are required. Thus, an audio power amplifier is required. The final power amplifier does double-duty in a high-level transmitter. First, it provides power gain for the RF carrier signal, just like the RF power amplifier did in the low-level transmitter. In addition to providing power gain, the final PA also performs the task of modulation. The final power amplifier in a high-level transmitter usually operates in class C, which is a highly nonlinear amplifier class.

Comparison:

Low Level Transmitters

- Can produce any kind of modulation; AM, FM, or PM.
- Require linear RF power amplifiers, which reduce DC efficiency and increases production costs.

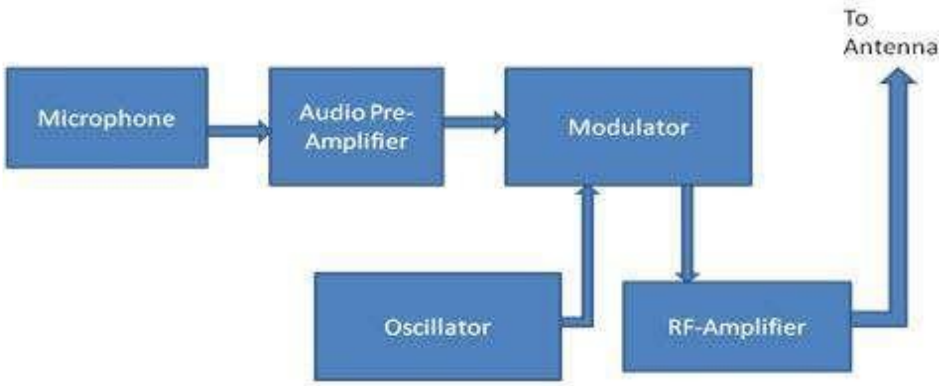
High Level Transmitters

- Have better DC efficiency than low-level transmitters, and are very well suited for battery operation.
- Are restricted to generating AM modulation only.

FM Transmitter

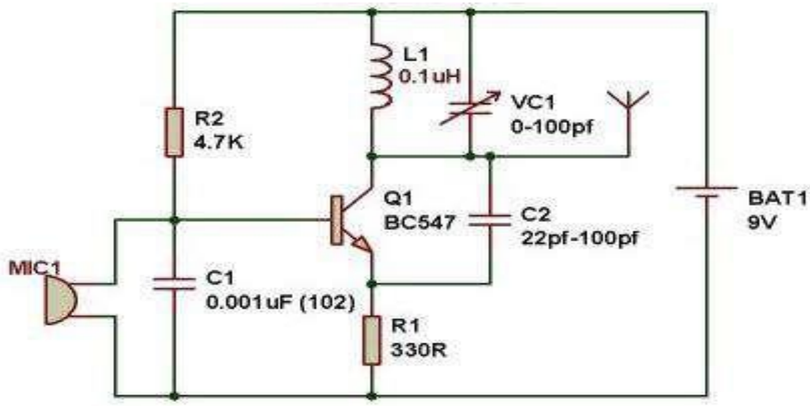
The FM transmitter is a single transistor circuit. In the telecommunication, the frequency modulation (FM)transfers the information by varying the frequency of carrier wave according to the message signal. Generally, the FM transmitter uses VHF radio frequencies of 87.5 to 108.0 MHz to transmit & receive the FM signal. This transmitter accomplishes the most excellent range with less power. The performance and working of the wireless audio transmitter circuit is depends on the induction coil & variable capacitor. This article will explain about the working of the FM transmitter circuit with its applications.

The FM transmitter is a low power transmitter and it uses FM waves for transmitting the sound, this transmitter transmits the audio signals through the carrier wave by the difference of frequency. The carrier wave frequency is equivalent to the audio signal of the amplitude and the FM transmitter produce VHF band of 88 to 108MHZ.Plese follow the below link for: Know all About Power Amplifiers for FM Transmitter


To
Antenna
Microphone
Audio Pre-
Amplifier
Modulator
Oscillator
RF-Amplifier

Working of FM Transmitter Circuit

The following circuit diagram shows the FM transmitter circuit and the required electrical and electronic components for this circuit is the power supply of 9V, resistor, capacitor, trimmer capacitor, inductor, mic, transmitter, and antenna. Let us consider the microphone to understand the sound signals and inside the mic there is a presence of capacitive sensor. It produces according to the vibration to the change of air pressure and the AC signal.



The formation of the oscillating tank circuit can be done through the transistor of 2N3904 by using the inductor and variable capacitor. The transistor used in this circuit is an NPN transistor used for general purpose amplification. If the current is passed at the inductor L1 and variable capacitor then the tank circuit will oscillate at the resonant carrier frequency of the FM modulation. The negative feedback will be the capacitor C2 to the oscillating tank circuit.

To generate the radio frequency carrier waves the FM transmitter circuit requires an oscillator. The tank circuit is derived from the LC circuit to store the energy for oscillations. The input audio signal from the mic penetrated to the base of the transistor, which **modulates the LC tank circuit** carrier frequency in FM format. The variable capacitor is used to change the resonant frequency for fine modification to the FM frequency band. The modulated signal from the antenna is radiated as radio waves at the FM frequency band and the antenna is nothing but copper wire of 20cm long and 24 gauge. In this circuit the length of the antenna should be significant and here you can use the 25-27 inches long copper wire of the antenna.

Application of FM Transmitter

- The FM transmitters are used in the homes like sound systems in halls to fill the sound with the audio source.
- These are also used in the cars and fitness centres.
- The correctional facilities have used in the FM transmitters to reduce the prison noise in common areas.

Advantages of the FM Transmitters

- The FM transmitters are easy to use and the price is low
- The efficiency of the transmitter is very high
- It has a large operating range
- This transmitter will reject the noise signal from an amplitude variation.

Receivers

Introduction to Radio Receivers:

In radio communications, a **radio receiver** (**receiver** or simply **radio**) is an electronic device that receives **radio** waves and converts the information carried by them to a usable form.

Types of Receivers:

The TRF (Tuned Radio Frequency) Receiver and Superheterodyne Receiver are the two main configurations of the receivers, they have real practical or commercial significance. Most of the present day receivers use superheterodyne configuration. But the TRF receivers are simple and easy to understand.

Tuned Radio Frequency Receiver:

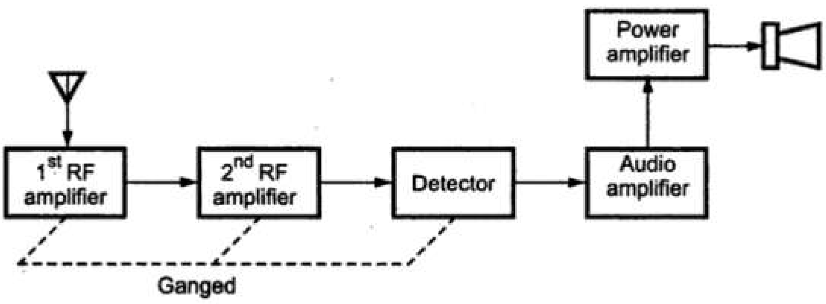


Fig. 1 shows the block diagram of TRF receiver.

It consists of two or three stages of RF amplifiers, detector, audio amplifier and power amplifier. The RF amplifier stages placed between the antenna and detector are used to increase the strength of the received signal before it is applied to the detector. These RF amplifiers are tuned to fix frequency, amplify the desired band of frequencies. Therefore, they provide amplification for selected band of frequencies and rejection for all others. As selection and amplification process is carried out in two or three stages and each stage must amplify the same band of frequencies, the ganged tuning is provided.

The amplified signal is then demodulated using detector to recover the modulating signal. The recovered signal is amplified further by the audio amplifier followed by power amplifier which provides sufficient gain to operate a loudspeaker. The TRF receivers suffered from number of annoying problems. These are listed in the next section.

Problems in TRF Receivers:

1. Tracking of Tuned Circuit

In a receiver, tuned circuits are made variable so that they can be set to the frequency of the desired signal. In most of the receivers, the capacitors in the tuned circuits are made variable. These capacitors are 'ganged' between the stages so that they all can be changed simultaneously when the tuning knob is rotated. To have perfect tuning the capacitor values between the stages must be exactly same but this is not the case. The differences in the capacitors cause the resonant frequency of each tuned circuit to be slightly different, thereby increasing the pass band.

2. Instability

As high gain is achieved at one frequency by a multistage amplifier, there are more chances of positive feedback (of getting back the small part of output of the last stage at the input to the first with the correct polarity) through some stray path, resulting in oscillations. These oscillations are unavoidable at high frequencies.

3. Variable Bandwidth

TRF receivers suffer from a variation in bandwidth over the tuning range. Consider a medium wave receiver required to tune over 535 kHz to 1640 kHz and it provides the necessary bandwidth of 10 kHz at 535 kHz. Let us calculate Q of this circuit.

$$Q = \frac{f}{\text{Bandwidth}} = \frac{535\,\text{kHz}}{10\,\text{kHz}} = 53.5$$

Now consider the frequency at the other end of the broadcast band, i.e. 1640 kHz. At 1640 kHz, Q of the coil should be 164 (1640 kHz / 10 kHz). However, in practice due to various losses depending on frequency, we will not set so large increase in Q. Let us assume that at 1640 kHz frequency Q is increased to value 100 instead of 164. With this Q of the tuned circuit bandwidth can be calculated as follows

$$\text{Bandwidth} = \frac{f}{Q} = \frac{1640\,\text{kHz}}{100} = 16.4 \text{ kHz}$$

We know, necessary bandwidth is 10 kHz. This increase in bandwidth of tuned circuit, pick up the adjacent stations along with station it is tuned for, providing insufficient adjacent frequency rejection. In other words we can say that in TRF receivers the bandwidth of the tuned circuit varies over the frequency range, resulting in poor selectivity of the receiver.

Because of the problems of tracking, instability and bandwidth variation, the TRF receivers have almost been replaced by superheterodyne receivers.

Superheterodyne Receivers

To solve basic problems of TRF receivers, in these receivers, first all the incoming RF frequencies are converted to a fix lower frequency called **intermediate frequency** (IF). Then this fix intermediate frequency is amplified and detected to reproduce the original information. Since the characteristics of the IF amplifier are independent of the frequency to which the receiver is tuned, the selectivity and sensitivity of superheterodyne receivers are fairly uniform throughout its tuning range.

Mixer circuit is used to produce the frequency translation of the incoming signal down to the IF. The incoming signals are mixed with the local oscillator frequency signal in such a way that a constant frequency difference is maintained between the local oscillator and the incoming signals. This is achieved by using ganged tuning capacitors.

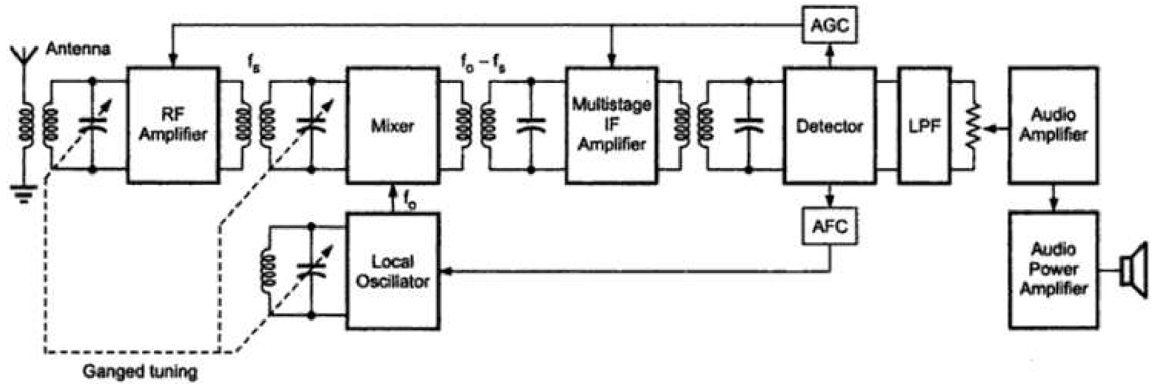


Fig.2. Block diagram of Super heterodyne Receiver.

Fig. 2 shows the block diagram of superheterodyne receiver. As shown in the Fig. 2 antenna picks up the weak radio signal and feeds it to the RF amplifier. The RF amplifier provides some initial gain and selectivity. The output of the RF amplifier is applied to the input of the mixer. The mixer also receives an input from local oscillator.

The output of the mixer circuit is difference frequency ($f_o - f_s$) commonly known as IF (Intermediate Frequency). The signal at this intermediate frequency contains the same modulation as the original carrier. This signal is amplified by one or more IF amplifier stages, and most of the receiver gain is obtained in these IF stages.

The highly amplified IF signal is applied to detector circuits to recover the original modulating information. Finally, the output of detector circuit is fed to audio and power amplifier which provides a sufficient gain to operate a speaker.

Another important circuit in the superheterodyne receiver are AGC and AFC circuit. AGC is used to maintain a constant output voltage level over a wide range of RF input signal levels.

It derives the dc bias voltage from the output of detector which is proportional to the amplitude of the received signal. This dc bias voltage is feedback to the IF amplifiers, and sometimes to the RF amplifier, to control the gain of the receiver. As a result, it provides a constant output voltage level over a wide range of RF input signal levels. AFC circuit generates AFC signal which is used to adjust and stabilize the frequency of the local oscillator.

Characteristics of Radio Receiver:

The performance of the radio receiver can be measured in terms of following receiver characteristics

- Selectivity
- Sensitivity
- Fidelity
- Image frequency and its rejection
- Double spotting

Selectivity

Selectivity refers to the ability of a receiver to select a signal of a desired frequency while reject all others. Selectivity in a receiver is obtained by using tuned circuits. These are LC circuits tuned to resonate at a desired signal frequency. The Q of these tuned circuits determines the selectivity. Selectivity shows the attenuation that the receiver offers to signals at frequencies near to the one to which it is tuned. A good receiver isolates the desired signal in the RF spectrum and eliminate all other signals.

Recall that Q is the ratio of inductive reactance to resistance ($Q = X_L/R$), and we know that bandwidth of the tuned circuit is given by

$$B_w = \frac{f_r}{Q}$$

where f_r is the resonant frequency. The bandwidth of a tuned circuit is measure of the selectivity. Narrower the bandwidth the better selectivity. To have narrower bandwidth and better selectivity the Q of the tuned circuit must be high.

Sensitivity

The sensitivity of a communication receiver refers to the receivers ability to pick up weak signals, and amplify it. It is often defined in terms of the voltage that must be applied to the receiver input terminals to give a standard output power, measured at the output terminals. The more gain that a receiver has, the smaller the input signal necessary to produce desired output power. Therefore, sensitivity is a primary function of the overall receiver gain. It is often expressed in microvolts or in decibels. The sensitivity of receiver mostly depends on the gain of the IF amplifiers. Good communication receiver has sensitivity of 0.2 to 1μV.

Fidelity

Fidelity refers to the ability of the receiver to reproduce all the modulating frequencies equally. Fig. 3 shows the typical fidelity curve for radio receiver.

The fidelity at the lower modulating frequencies is determined by the low frequency response of the IF amplifier and the fidelity at the higher modulating frequencies is determined by the high frequency response of the IF amplifier. Fidelity is difficult to obtain in AM receiver because good fidelity requires more bandwidth of IF amplifier resulting in poor selectivity.

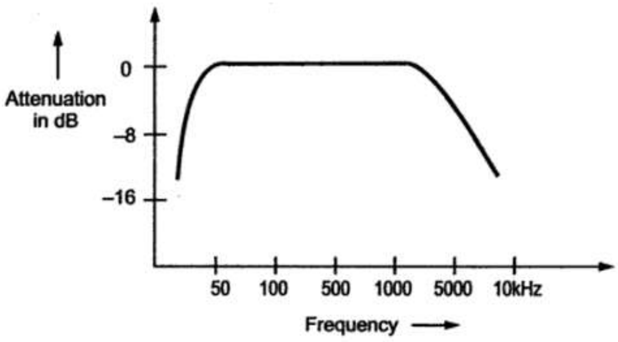


Fig.3. Typical Fidelity curve

Image Frequency and its Rejection

In standard broadcast receiver the local oscillator frequency is made higher than the signal frequency by an amount equal to intermediate frequency (IF). Therefore $f_o = f_s + f_i$. When f_o and f_s are mixed, the difference frequency, which is one of the by products, is equal to f_i only f_i is passed and amplified by the IF stage.

If a frequency f_{si} $(f_o + f_i)$, i.e. $f_{si} = f_s + 2\ f_i$, appears at the input of the mixer then it will produce the sum and difference frequencies regardless of the inputs. Therefore, the mixer output will be the difference frequency at the IF value. The terms f_{si} is called the **image** frequency and is defined as the signal frequency plus twice the intermediate frequency. Unfortunately, this image frequency signal is also amplified by the IF amplifiers resulting in interference. This has the effect of two stations being received simultaneously and is naturally undesirable.

The rejection of an image frequency by a single tuned circuit is the ratio of the gain at the signal frequency to the gain at the image frequency. It is given by

$$\alpha = \sqrt{1+Q^2\rho^2}$$

where $\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$

and Q = loaded Q of tuned circuit

If the receiver has an RF stage, then there are two tuned circuits, both tuned to f_s ; the rejection of each will be calculated by the same formula, and the total rejection will be the product of two.

The image rejection depends on the selectivity of the RF amplifier and tuned circuits and must be achieved before the IF stage. Once the spurious frequency enters the first IF amplifier, it becomes impossible to remove it from the wanted signal.

Double Spotting

The phenomenon of double spotting occurs at higher frequencies due to poor front end selectivity of the receivers. In this, receiver picks up same short-wave station at two nearby points on the receiver dial.

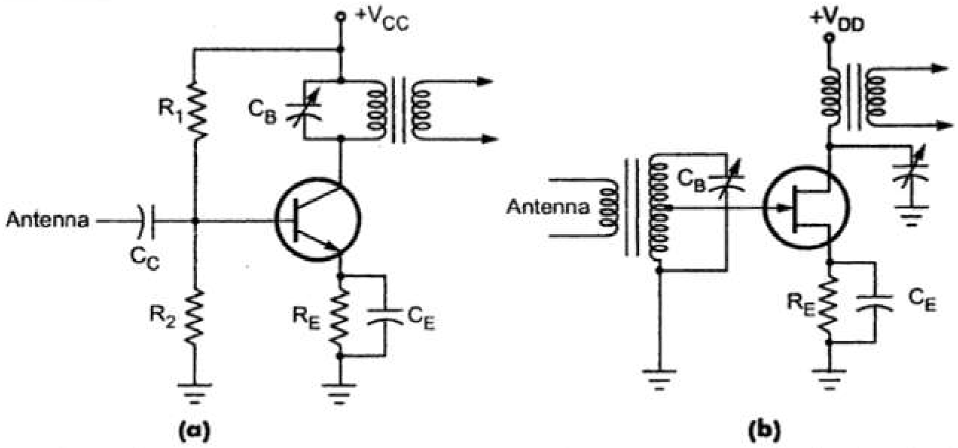
When the receiver is tuned across the band, a strong signal appears to be at two different frequencies, once at the desired frequency and again when the receiver is tuned to 2 times IF below the desired frequency. In this second case, the signal becomes the image, reduced in strength by the image rejection, thus making it appear that the signal is located at two frequencies in the band.

Blocks in Super heterodyne Receiver:

- **Basic principle**
  - Mixing
  - Intermediate frequency of 455 KHz
  - Ganged tuning
- **RF section**
  - Tuning circuits – reject interference and reduce noise figure
  - Wide band RF amplifier
- **Local Oscillator**
  - 995 KHz to 2105 KHz
  - Tracking
- **IF amplifier**
  - Very narrow band width Class A amplifier – selects 455 KHz only
  - Provides much of the gain
  - Double tuned circuits
- **Detector**
  - RF is filtered to ground

1. RF Amplifier:

RF amplifier provides initial gain and selectivity. Fig. 4 shows the RF amplifier circuits. It is a tuned circuit followed by an amplifier The RF amplifier is usually a simple class A circuit. A typical bipolar circuit is shown in Fig. 4. (a) , and a typical FET circuit is shown in Fig. 4. (b).



The values of resistors R_1 and R_2 in the bipolar circuit are adjusted such that the amplifier works as class A amplifier. The antenna is connected through coupling capacitor to the base of the transistor. This makes the circuit very broad band as the transistor will amplify virtually any signal picked up by the antenna. However the collector is tuned with a parallel resonant circuit to provide the initial selectivity for the mixer input.

The FET circuit shown in Fig. 4 (b) is more effective than the transistor circuit. Their high input impedance minimizes the loading on tuned circuits, thereby permitting the Q of the circuit to be higher and selectivity to be sharper.

The receiver having an RF amplifier stage has following advantages :

1. It provides greater gain, i.e. better sensitivity.
2. It improves image-frequency rejection.
3. It improves signal to noise ratio.
4. It improves rejection of adjacent unwanted signals, providing better selectivity.
5. It provides better coupling of the receiver to the antenna.
6. It prevents spurious frequencies from entering the mixer and heterodyning there to produce an interfering frequency equal to the IF from the desired signal.
7. It also prevents reradiation of the local oscillator through the antenna of the receiver.

2. Mixer

The frequency converter is a nonlinear resistance having two sets of input terminals and one set of output terminal. The two inputs to the frequency converter are the input signal along with any modulation and the input from a local oscillator (LO). The output contains several frequencies including the difference between the input frequencies. The difference frequency is called intermediate frequency and output circuit of the mixer is tuned for the intermediate frequency.

Separately Excited Mixer:

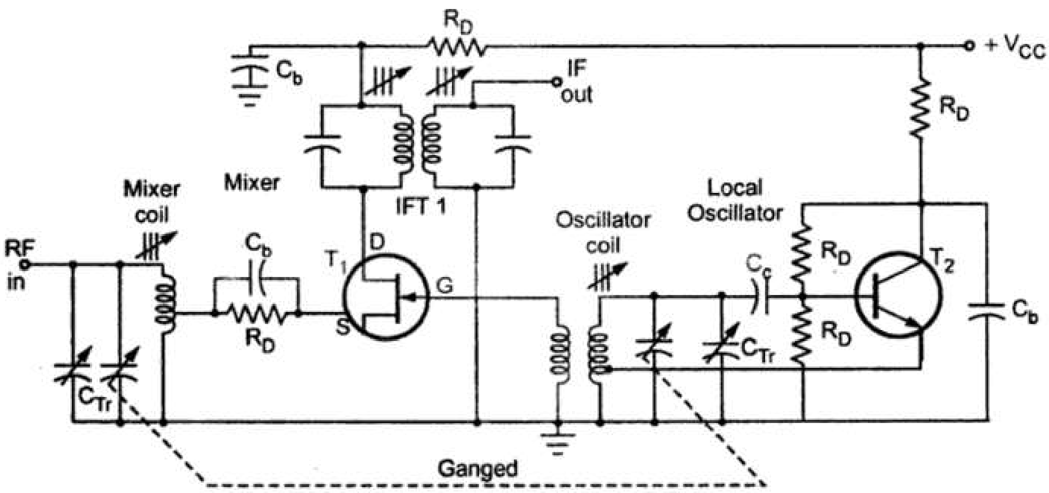


Fig.5 Separately Excited FET Mixer

Fig. 5 shows the separately excited mixer using FET. Here, one device acts as a mixer while the other supplies the necessary oscillations. The bipolar transistor T_2, forms the Hartley oscillator circuit. It oscillates with local frequency (f_o). FET T_1, is a mixer, whose gate is fed with the output of local oscillator and its bias is adjusted such that it operates in a nonlinear portion of its characteristic. The local oscillator varies the gate bias of the FET to vary its transconductance in a nonlinear manner, resulting intermediate frequency (IF) at the output. The output is taken through double tuned transformer in the drain of the mixer and fed to the IF amplifier. The ganged tuning capacitor allows simultaneous tuning of mixer and local oscillator.

The C_{Tr}, a small trimmer capacitors across each of the tuning capacitors are used for fine adjustments.

Self Excited Mixer:

It is possible to combine the function of the mixer and local oscillator in one circuit. The circuit is commonly known as self excited mixer. Fig. 6 shows self excited bipolar transistor mixer. The circuit oscillates and the transconductance of the transistor is varied in a nonlinear manner at the local oscillator rate. This variable transconductance (g_m) is used by the transistor to amplify the incoming RF signal.

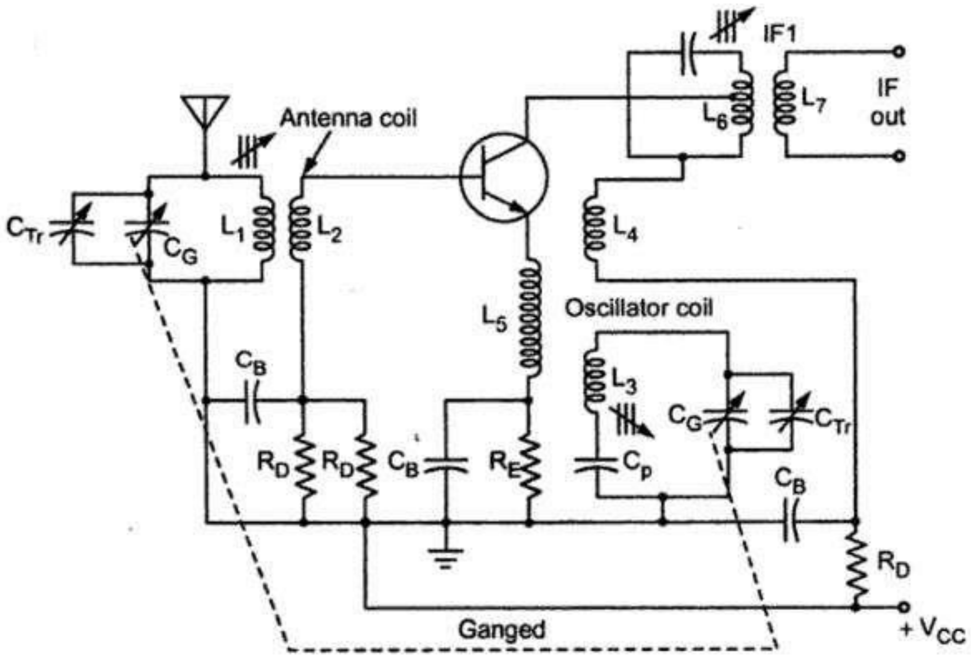


Fig.6. Self Excited Mixer

3. Tracking

The superheterodyne receiver has number of tunable circuits which must all be tuned correctly if any given station is to be received. The ganged tuning is employed to do this work, which mechanically couples all tuning circuits so that only one tuning control or dial is required. Usually, there are three tuned circuits : Antenna or RF tuned circuit, mixer tuned circuit and local oscillator tuned circuit. All these circuits must be tuned to get proper RF input and to get IF frequency at the output of mixer. The process of tuning circuits to get the desired output is called **Tracking**. Any error that exists in the frequency difference will result in an incorrect frequency being fed to the IF amplifier. Such errors are known as **'Tracking Errors'** and these must be avoided.

To avoid tracking errors standard capacitors are not used, and ganged capacitors with identical sections are used. A different value of inductance and special extra capacitors called trimmers and padders are used to adjust the capacitance of the oscillator to the proper range. There are three common methods used for tracking. These are

- Padder tracking
- Trimmer tracking

4. Local Oscillator

In shortwave broadcasting, the operating limit for receivers is 36 MHz. For such operating limit local oscillators such as Armstrong, Hartley, Colpitts, Clapp or ultra-audion are used. The Colpitts, Clapp and ultra-audion oscillators are used at the top of the operating limit, whereas Hartley oscillator is used for frequencies below 120 MHz. All these oscillators are LC oscillators and each employs only one tuned circuit to defermine its frequency. When higher frequency stability of local oscillator is required, the circuits like AFC (Automatic Frequency Control) are used.

5. IF Amplifier

IF amplifiers are tuned voltage amplifiers tuned for the fixed frequency. Its important function is to amplify only tuned frequency signal and reject all others. As we know, most of the receiver gain is provided by the IF amplifiers, to obtain required gain, usually two or more stages of IF amplifiers are required.

Fig. 7 shows the two stage IF amplifier. Two stages are transformer coupled and all IF transformers are single tuned, i.e. tuned for single frequency.

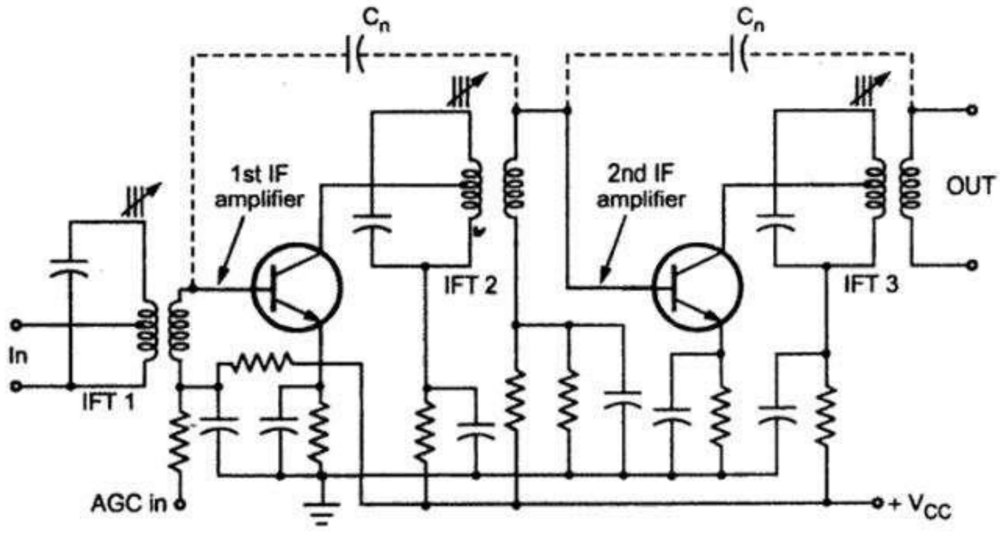


Fig.7 Two Stage IF Amplifier

Choice of Intermediate Frequency:

Selection of the intermediate frequency depends on various factors. While choosing the intermediate frequency it is necessary to consider following factors.

1. Very high intermediate frequency will result in poor selectivity and poor adjacent channel rejection.
2. A high value of intermediate frequency increases tracking difficulties.
3. At low values of intermediate frequency, image frequency rejection is poor.
4. At very low values of intermediate frequency, selectivity is too sharp. Cutting off the sidebands.
5. At very low IF, the frequency stability of the local oscillator must be correspondingly high because any frequency drift is now a larger proportion of the low IF than of a high IF.
6. The IF must not fall in the tuning range of the receiver, otherwise instability will occur and heterodyne whistles will be heard, making it impossible to tune to the frequency band immediately adjacent to the intermediate frequency.

With the above considerations the standard broadcast AM receivers [tuning to 540 to 1650 kHz] use an IF within the 438 kHz to 465 kHz range. The 465 kHz IF is most commonly used.

6. Automatic Gain Control

Automatic Gain Control is a system by means of which the overall gain of a radio receiver is varied automatically with the variations in the strength of the receiver signal, to maintain the output substantially constant. AGC circuitry derives the dc bias voltage from the output of the detector. It applies this derived dc bias voltage to a selected number of RF, IF and mixer stages to control their gains. When the average signal level increases, the size of the AGC bias increases, and the gain of the controlled stages decreases. When there is no signal, there is a minimum AGC bias, and the amplifiers produce maximum gain. There are two types of AGC circuits in use : Simple AGC and Delayed AGC

Simple AGC

In simple AGC receivers the AGC bias starts to increase as soon as the received signal level exceeds the background noise level. As a result receiver gain starts falling down, reducing the sensitivity of the receiver.

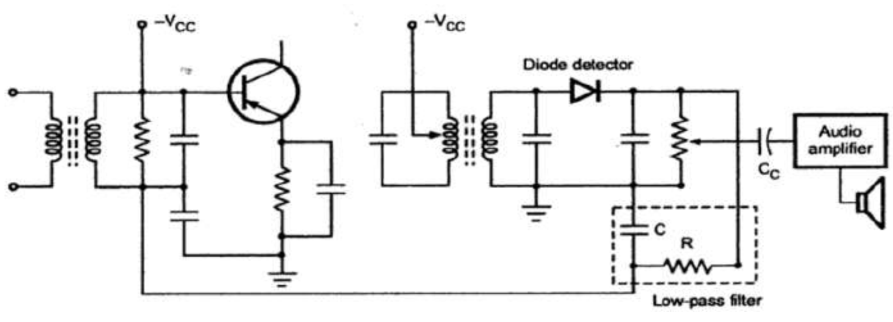


Fig.8. Simple AGC circuit

Fig. 8 shows the simple AGC circuit. In this circuit, dc bias produced by half wave rectifier as a AM detector, is used to control the gain of RF or IF amplifier. Before application of this voltage to the base of the RF and / or IF stage amplifier the audio signal is removed by the lowpass filter. The time constant of the filter is kept at least 10 times longer than the period of the lowest modulation frequency received. If the time constant is kept longer, it will give better filtering, but it will cause an annoying delay in the application of the AGC control when tuning from one signal to another. The recovered signal is then passed through C_C to remove the dc. The resulting ac signal is further amplified and applied to the loudspeaker.

Delayed AGC

Simple AGC is clearly an improvement over no AGC at all. Unfortunately, in simple AGC circuit, the unwanted weak signals(noise signals) are amplified with high gain. To

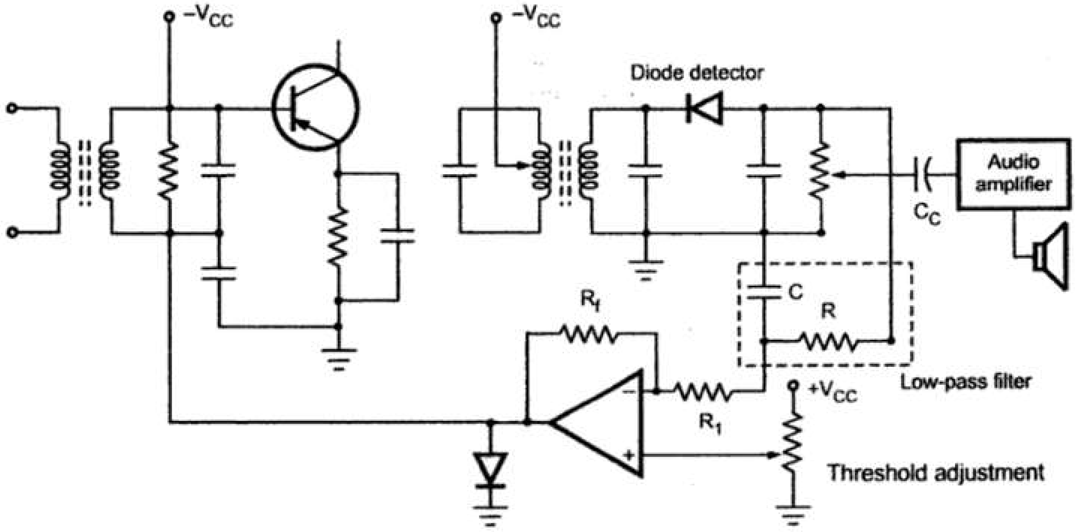


Fig.9. Delayed AGC circuit

avoid this, in delayed AGC circuits, AGC bias is not applied to amplifiers until signal strength has reached a predetermined level, after which AGC bias is applied as with simple AGC, but more strongly.

Here, AGC output is applied to the difference amplifier. It gives negative dc AGC only when AGC output generated by diode detector is above certain dc threshold voltage. This threshold voltage can be adjusted by adjusting the voltage at the positive input of the operational amplifier.

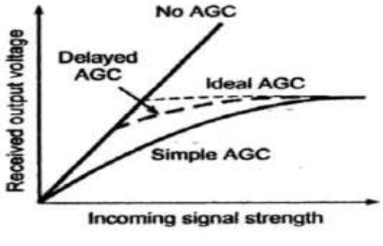


Fig.10. Response of receiver with various AGC circuits.

FM Receiver:

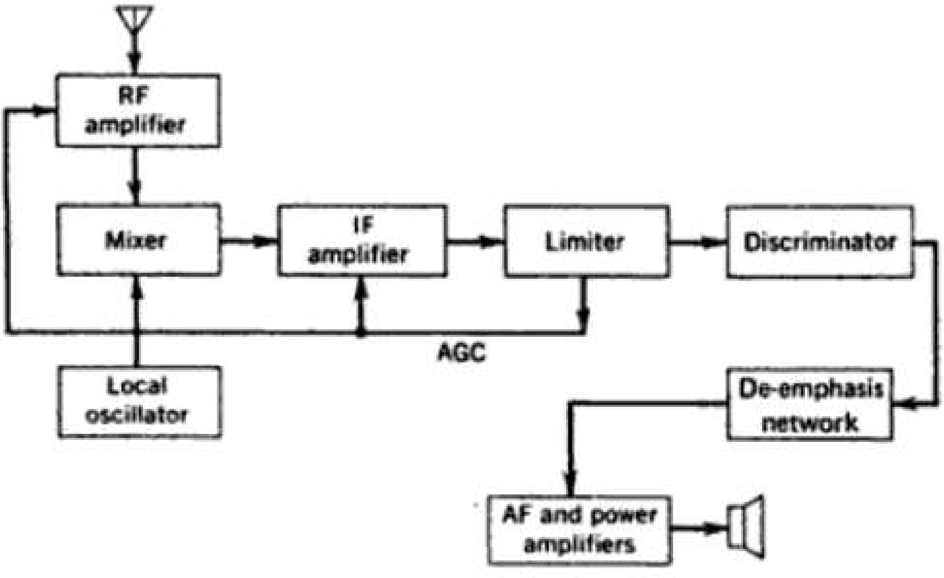


Fig.11. FM Receiver Block diagram

The FM receiver is a superheterodyne receiver, and the block diagram of Figure 11 shows just how similar it is to an AM receiver. The basic differences are as follows:

1. Generally much higher operating frequencies in FM
2. Need for limiting and de-emphasis in FM
3. Totally different methods of demodulation
4. Different methods of obtaining AGC

Comparisons with AM Receivers

A number of sections of the FM receiver correspond exactly to those of other receivers already discussed. The same criteria apply in the selection of the intermediate frequency, and IF amplifiers are basically similar. A number of concepts have very similar meanings so that only the differences and special applications need be pointed out.

RF amplifiers An RF amplifier is always used in an FM receiver. Its main purpose is to reduce the noise figure, which could otherwise be a problem because of the large bandwidths needed for FM. It is also required to match the input impedance of the receiver to that of the antenna. To meet the second requirement, grounded gate (or base) or cascode amplifiers are employed. Both types have the property of low input impedance and matching the antenna, while neither requires neutralization. This is because the input electrode is grounded on either type of amplifier, effectively isolating input from output. A typical FET grounded-gate RF amplifier is shown in Figure
It has all the good points mentioned and the added features of low distortion and simple operation.

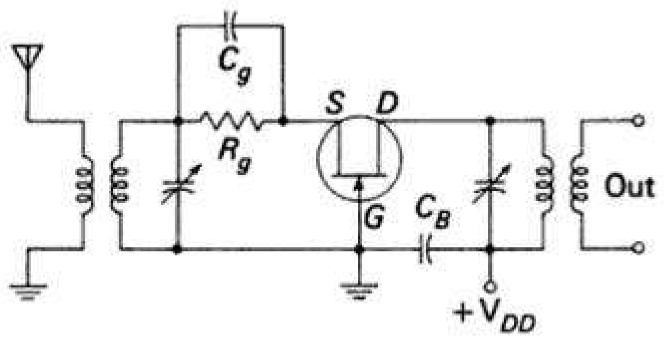


FIGURE **Grounded-gate FET RF amplifier.**

Oscillators and mixers The oscillator circuit takes any of the usual forms, with the Colpitts and Clapp predominant, being suited to VHF operation. Tracking is not normally much of a problem in FM broadcast receivers. This is because the tuning frequency range is only 1.25:1, much less than in AM broadcasting.

A very satisfactory arrangement for the front end of an FM receiver consists of FETs for the RF amplifier and mixer, and a bipolar transistor oscillator. As implied by this statement, separately excited oscillators are normally used

Intermediate frequency and IF amplifiers Again, the types and operation do not differ much from their AM counterparts. It is worth noting, however, that the intermediate frequency and the bandwidth required are far higher than in AM broadcast receivers. Typical figures for receivers operating in the 88- to 108-MHz band are an IF of 10.7 MHz and a bandwidth of 200 kHz. As a consequence of the large bandwidth, gain per stage may be low. Two IF amplifier stages are often provided, in which case the shrinkage of bandwidth as stages are cascaded must be taken into account.

Double limiter

A double limiter consists of two amplitude limiters in cascade, an arrangement that increases the limiting range very satisfactorily. Numerical values given to illustrate limiter performance showed an output voltage (all values peak-to-peak, as before) of 5 V for any input within the 0.4- to 4-V range, above which output gradually decreases. It is quite possible that an output of 0.6 V is not reached until the input to the first limiter is about 20 V. If the range of the second limiter is 0.6 to 6 V, it follows that all voltages between 0.4 and 20 V fed to the double limiter will be limited. The use of the double limiter is seen to have increased the limiting range quite considerably.

Automatic gain control (AGC)

A suitable alternative to the second limiter is automatic gain control. This is to ensure that the signal fed to the limiter is within its limiting range, regardless of the input signal strength, and also to prevent overloading of the last IF amplifier. If the limiter used has leak-type bias, then this bias voltage will vary in proportion to the input voltage (as shown in Figure 6-31) and may therefore be used for AGC. Sometimes a separate AGC detector is used, which takes part of the output of the last IF amplifier and rectifies and filters it in the usual manner.

Amplitude Limiter:

In order to make full use of the advantages offered by FM, a demodulator must be preceded by an amplitude limiter, on the grounds that any amplitude changes in the signal fed to the FM demodulator are spurious

They must therefore be removed if distortion is to be avoided. The point is significant, since most FM demodulators react to amplitude changes as well as frequency changes. The limiter is a form of clipping device, a circuit whose output tends to remain constant despite changes in the input signal. Most limiters behave in this fashion, provided that the input voltage remains within a certain range. The common type of limiter uses two separate electrical effects to provide a relatively constant output. There are leak-type bias and early (collector) saturation.

Operation of the amplitude limiter Figure shows a typical FET amplitude limiter. Examination of the dc conditions shows that the drain supply voltage has been dropped through resistor R_D. Also, the bias on the gate is leak-type bias supplied by the parallel $R_g - C_g$ combination. Finally, the FET is shown neutralized by means of capacitor C_N, in consideration of the high frequency of operation.

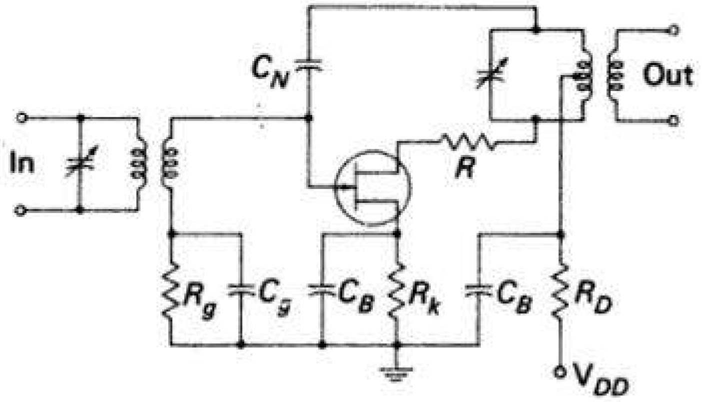


FIGURE Amplitude limiter.

Leak-type bias provides limiting, as shown in Figure . When input signal voltage rises, current flows in the $R_g - C_g$ bias circuit, and a negative voltage is developed across the capacitor. It is seen that the bias on the FET is increased in proportion to the size of the input voltage. As a result, the gain of the amplifier is lowered, and the output voltage tends to remain constant.

Although some limiting is achieved by this process, it is insufficient by itself, the action just described would occur only with rather large input voltages. To overcome this, early saturation of the output current is used, achieved by means of a low drain supply voltage. This is the reason for the drain dropping resistor of Figure . The supply voltage for a limiter is typically one-half of the normal dc drain voltage. The result of early saturation is to ensure limiting for conveniently low input voltages.

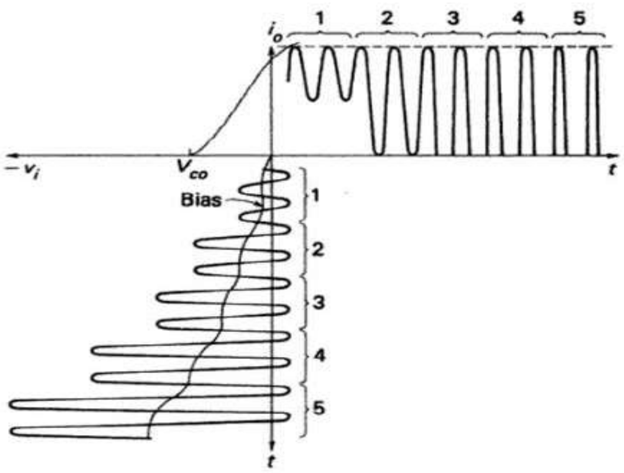


FIGURE Amplitude limiter transfer characteristic.

It is possible for the gate-drain section to become forward-biased under saturation conditions, causing a short circuit between input and output. To avert this, a resistance of a few hundred ohms is placed between the drain and its tank. This is *R* of Figure

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Unit-5

Analog to Digital Conversion & Pulse Modulation System

Introduction to Pulse Modulation

Carrier is a train of pulses

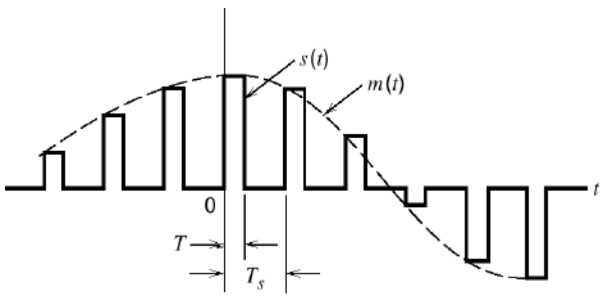
Example: Pulse Amplitude Modulation (PAM), Pulse width modulation (PWM) , Pulse Position Modulation (PPM)

Types of Pulse Modulation:

- The immediate result of sampling is a pulse-amplitude modulation (PAM) signal
- PAM is an analog scheme in which the amplitude of the pulse is proportional to the amplitude of the signal at the instant of sampling
- Another analog pulse-forming technique is known as **pulse-duration modulation (PDM).** This is also known as **pulse-width modulation (PWM)**
- **Pulse-position modulation** is closely related to PDM

Pulse Amplitude Modulation:

In PAM, amplitude of pulses is varied in accordance with instantaneous value of modulating signal.



PAM Generation:

The carrier is in the form of narrow pulses having frequency fc. The uniform sampling takes place in multiplier to generate PAM signal. Samples are placed Ts sec away from each other.

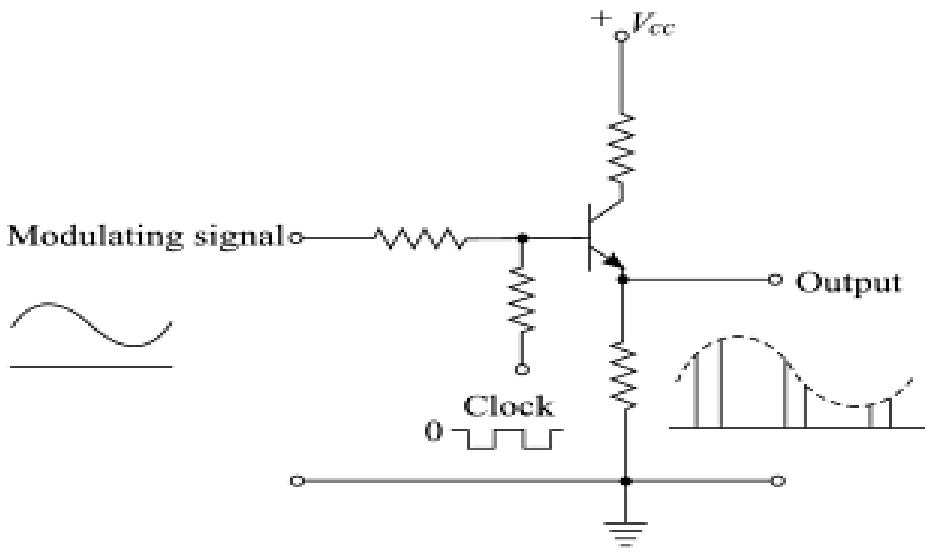


Figure PAM Modulator

- The circuit is simple emitter follower.
- In the absence of the clock signal, the output follows input.
- The modulating signal is applied as the input signal.
- Another input to the base of the transistor is the clock signal.
- The frequency of the clock signal is made equal to the desired carrier pulse train frequency.
- The amplitude of the clock signal is chosen the high level is at ground level(0v) and low level at some negative voltage sufficient to bring the transistor in cutoff region.
- When clock is high, circuit operates as emitter follower and the output follows in the input modulating signal.
- When clock signal is low, transistor is cutoff and output is zero.
- Thus the output is the desired PAM signal.

PAM Demodulator:

- The PAM demodulator circuit which is just an envelope detector followed by a second order op-amp low pass filter (to have good filtering characteristics) is as shown below

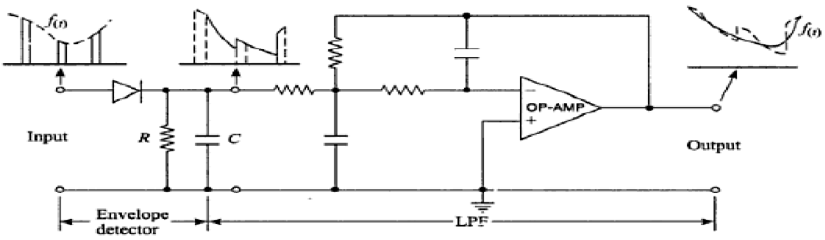
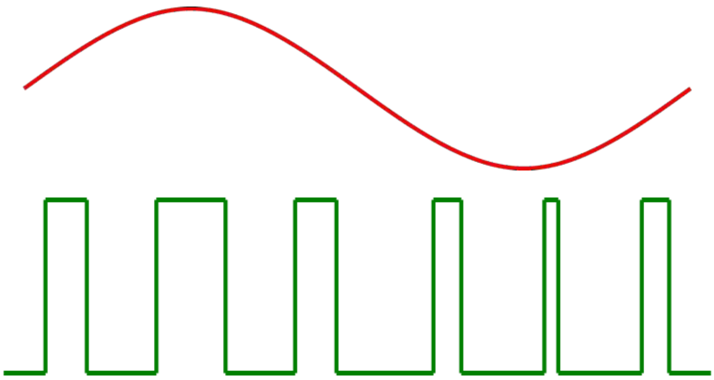


Figure PAM Demodulator

Pulse Width Modulation:

- In this type, the amplitude is maintained constant but the width of each pulse is varied in accordance with instantaneous value of the analog signal.



- In PWM information is contained in width variation. This is similar to FM.
- In pulse width modulation (PWM), the width of each pulse is made directly proportional to the amplitude of the information signal.

Pulse Position Modulation:

- In this type, the sampled waveform has fixed amplitude and width whereas the position of each pulse is varied as per instantaneous value of the analog signal.
- PPM signal is further modification of a PWM signal.

PPM & PWM Modulator:

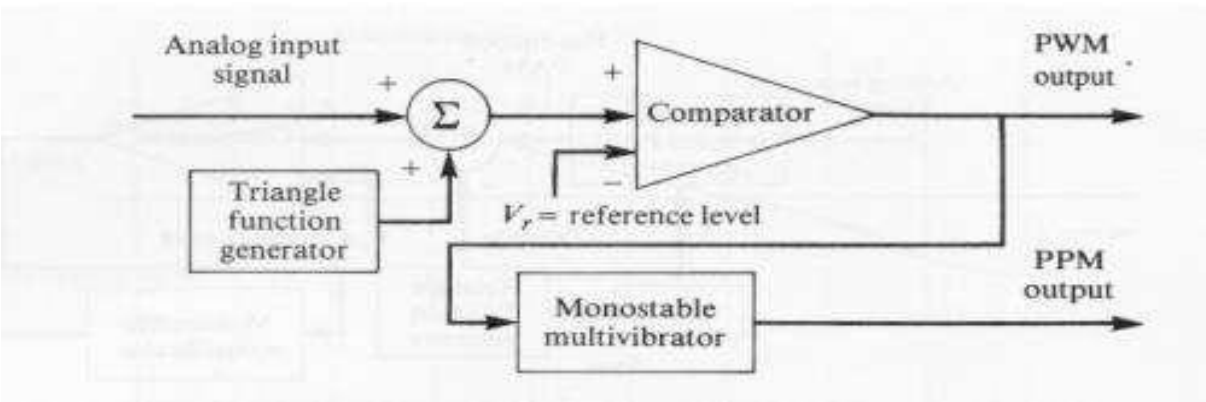


Figure PWM & PPM Modulator

- The PPM signal can be generated from PWM signal.
- The PWM pulses obtained at the comparator output are applied to a mono stable multi

vibrator which is negative edge triggered.

- Hence for each trailing edge of PWM signal, the monostable output goes high. It remains high for a fixed time decided by its RC components.
- Thus as the trailing edges of the PWM signal keeps shifting in proportion with the modulating signal, the PPM pulses also keep shifting.
- Therefore all the PPM pulses have the same amplitude and width. The information is conveyed via changing position of pulses.

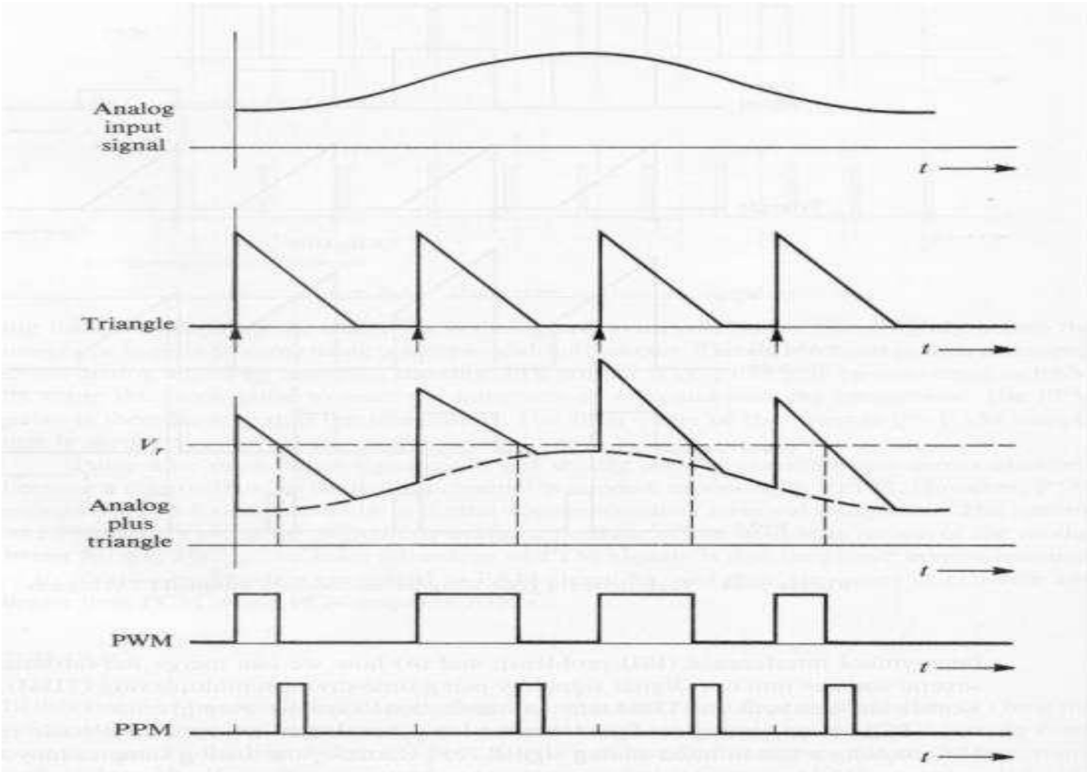


Figure PWM & PPM Modulation waveforms

PWM Demodulator:

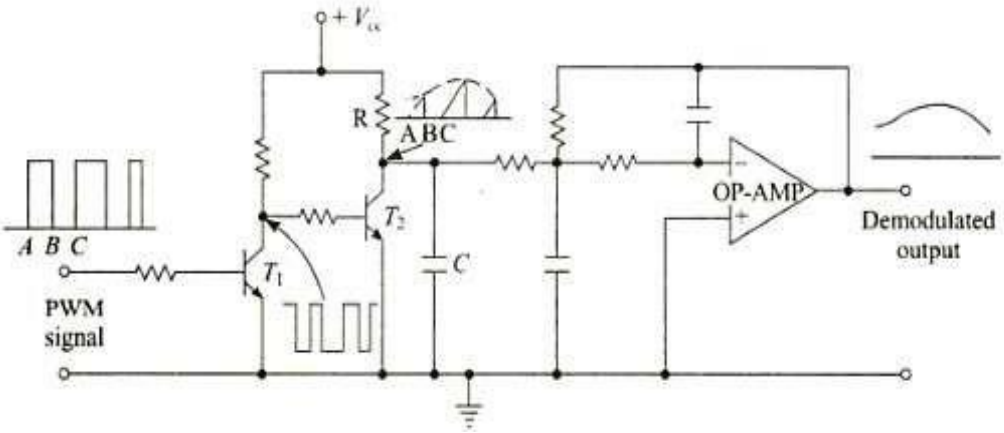


Figure PWM Demodulator

- Transistor T1 works as an inverter.
- During time interval A-B when the PWM signal is high the input to transistor T2 is low.
- Therefore, during this time interval T2 is cut-off and capacitor C is charged through an R-C combination.
- During time interval B-C when PWM signal is low, the input to transistor T2 is high, and it gets saturated.
- The capacitor C discharges rapidly through T2. The collector voltage of T2 during B-C is low.
- Thus, the waveform at the collector of T2is similar to saw-tooth waveform whose envelope is the modulating signal.
- Passing it through 2nd order op-amp Low Pass Filter, gives demodulated signal.

PPM Demodulator:

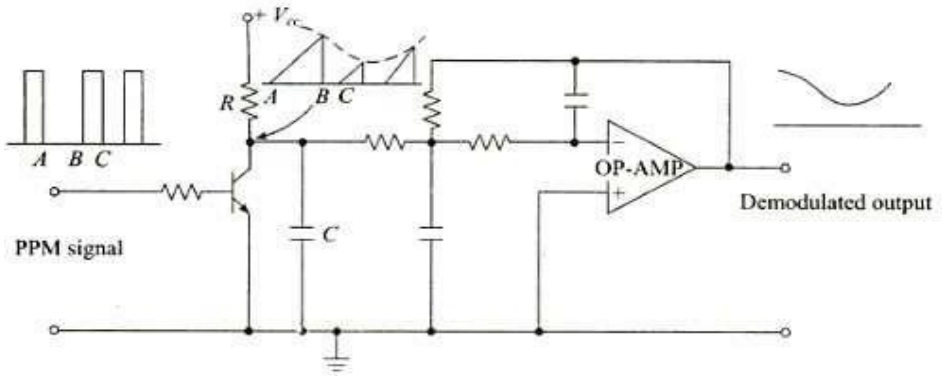


Figure PPM Demodulator

- The gaps between the pulses of a PPM signal contain the information regarding the modulating signal.
- During gap A-B between the pulses the transistor is cut-off and the capacitor C gets charged through R-C combination.
- During the pulse duration B-C the capacitor discharges through transistor and the collector voltage becomes low.
- Thus, waveform across collector is saw-tooth waveform whose envelope is the modulating signal.
- Passing it through 2nd order op-amp Low Pass Filter, gives demodulated signal.

Multiplexing

Multiplexing is the set of techniques that allows the simultaneous transmission of multiple signals across a single common communications channel.

Multiplexing is the transmission of analog or digital information from one or more sources to one or more destination over the same transmission link.

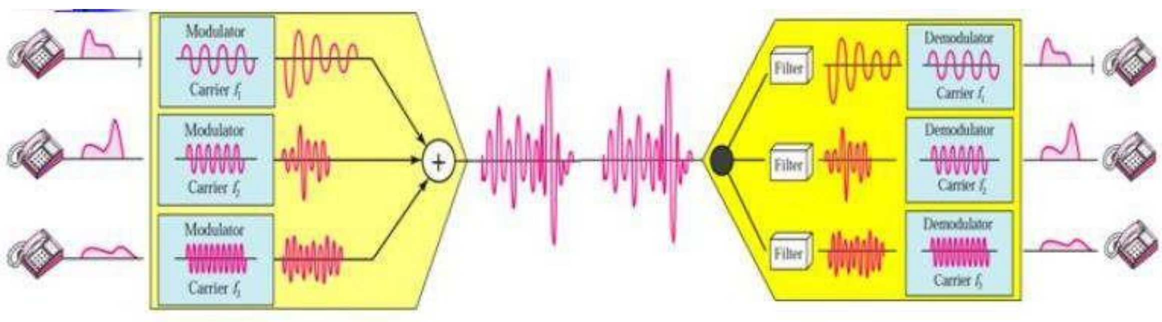
Although transmissions occur on the same transmitting medium, they do not necessarily occupy the same bandwidth or even occur at the same time.

Frequency Division Multiplexing

Frequency division multiplexing (FDM) is a technique of multiplexing which means combining more than one signal over a shared medium. In FDM, signals of different frequencies are combined for concurrent transmission.

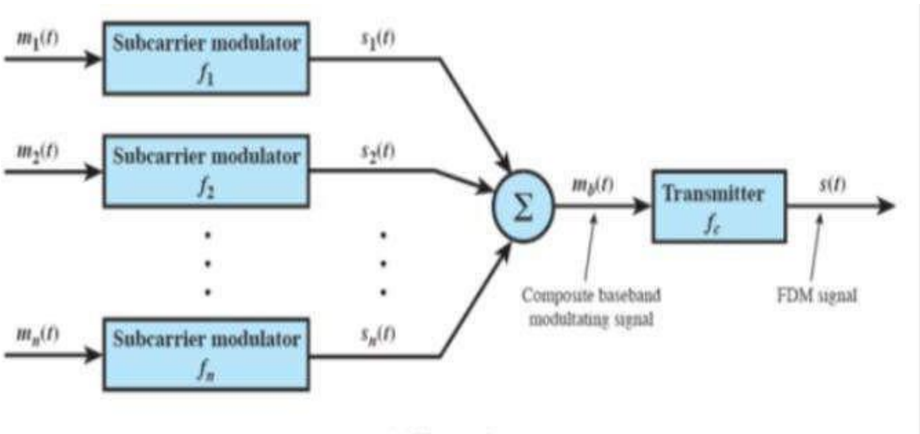
In FDM, the total bandwidth is divided to a set of frequency bands that do not overlap. Each of these bands is a carrier of a different signal that is generated and modulated by one of the sending devices. The frequency bands are separated from one another by strips of unused frequencies called the guard bands, to prevent overlapping of signals.

The modulated signals are combined together using a multiplexer (MUX) in the sending end. The combined signal is transmitted over the communication channel, thus allowing multiple independent data streams to be transmitted simultaneously. At the receiving end, the individual signals are extracted from the combined signal by the process ofdemultiplexing (DEMUX).



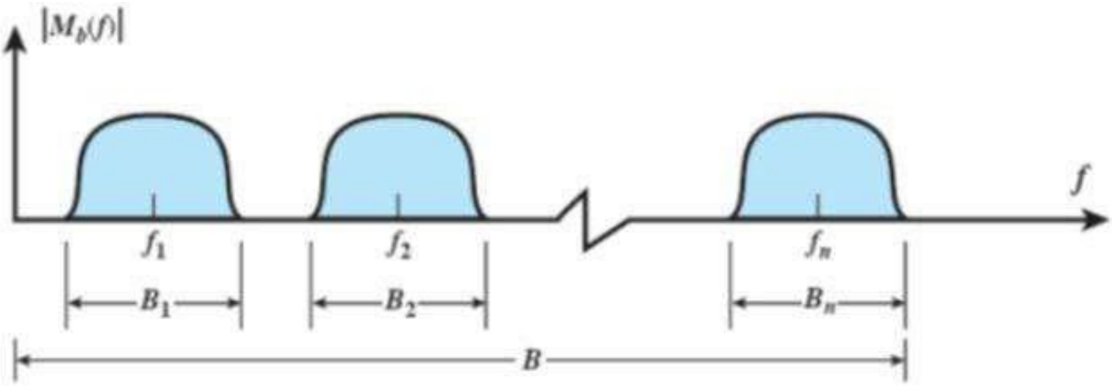
FDM system Transmitter

- Analog or digital inputs: m_i (t); i = 1,2, ... n
- Each input modulates a subcarrier of frequency fi; i=1, 2, ... n
- Signals are summed to produce a composite baseband signal denoted as $m_b(t)$
- f_i is chosen such that there is no overlap.



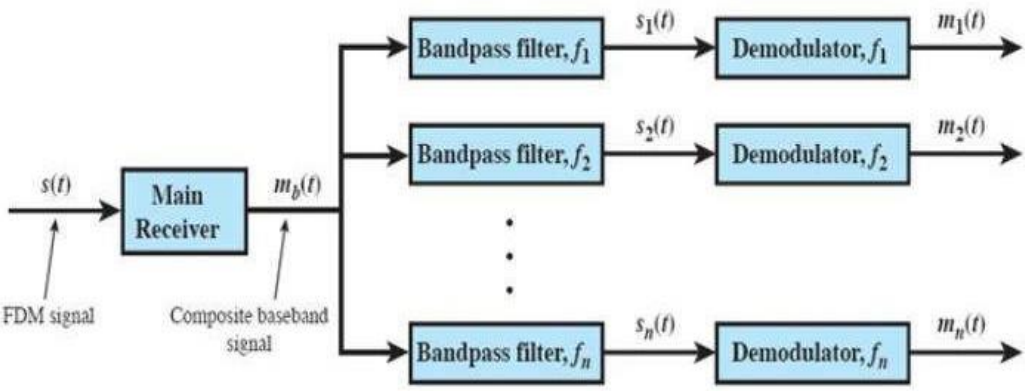
(a) Transmitter

Spectrum of composite baseband modulating signal



FDM system Receiver

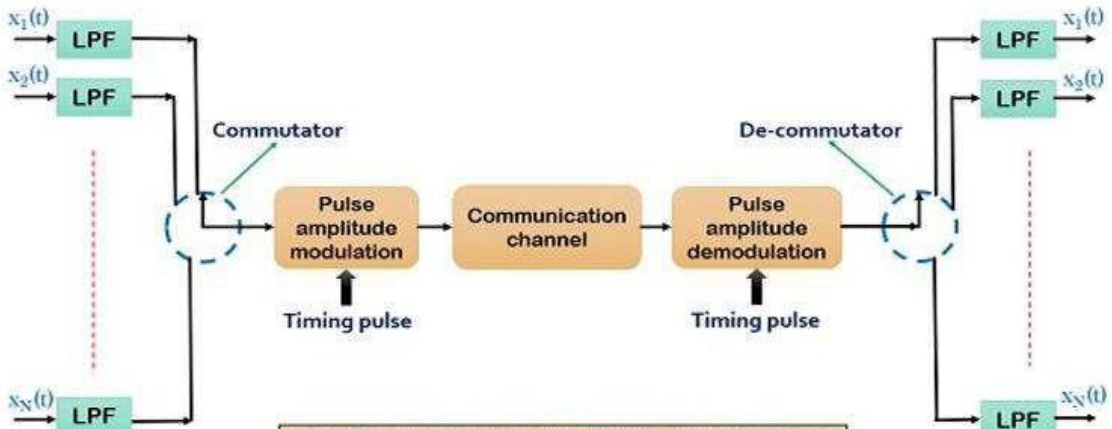
- The Composite base band signal $m_b(t)$ is passed through n band pass filters with response centred on f_i
- Each $s_i(t)$ component is demodulated to recover the original analog/digital data.



Time Division Multiplexing

TDM technique combines time-domain samples from different message signals (sampled at same rate) and transmits them together across the same channel.

The multiplexing is performed using a commutator (switch). At the receiver a decommutator (switch) is used in synchronism with the commutator to demultiplex the data.



Block diagram of PAM/TDM system

The input signals, all band limited to fm (max) by the LPFs are sequentially sampled at the transmitter by a commutator.

The Switch makes one complete revolution in Ts,(1/fs) extracting one sample from each input. Hence the output is a PAM waveform containing the individual message sampled periodically interlaced in time.

A set of pulses consisting of one sample from each input signal is called a frame.

At the receiver the de-commutator separates the samples and distributes them to a bank of LPFs, which in turn reconstruct the original messages.

Synchronizing is provided to keep the de-commutator in step with the commutator.

Elements of Digital Communication Systems

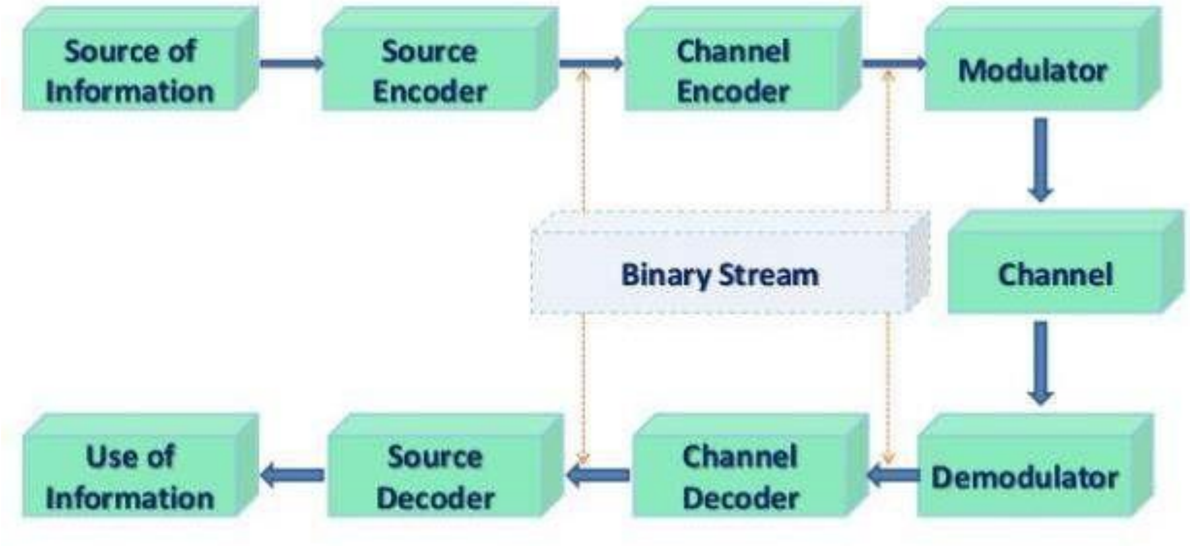


Figure Elements of Digital Communication Systems

1. Information Source and Input Transducer:

The source of information can be analog or digital, e.g. analog: audio or video signal, digital: like teletype signal. In digital communication the signal produced by this source is converted into digital signal which consists of 1′s and 0′s. For this we need a source encoder.

2. Source Encoder:

In digital communication we convert the signal from source into digital signal as mentioned above. The point to remember is we should like to use as few binary digits as possible to represent the signal. In such a way this efficient representation of the source output results in little or no redundancy. This sequence of binary digits is called ***information sequence.***

Source Encoding or Data Compression: the process of efficiently converting the output of whether analog or digital source into a sequence of binary digits is known as source encoding.

3. Channel Encoder:

The information sequence is passed through the channel encoder. The purpose of the channel encoder is to introduce, in controlled manner, some redundancy in the binary information sequence that can be used at the receiver to overcome the effects of noise and interference encountered in the transmission on the signal through the channel.

For example take k bits of the information sequence and map that k bits to unique n bit sequence called code word. The amount of redundancy introduced is measured by the ratio *n/k* and the reciprocal of this ratio (k/n) is known as *rate of code or code rate.*

4. Digital Modulator:

The binary sequence is passed to digital modulator which in turns convert the sequence into electric signals so that we can transmit them on channel (we will see channel later). The digital modulator maps the binary sequences into signal wave forms , for example if we represent 1 by sin x and 0 by cos x then we will transmit sinx for 1 and cos x for 0. (a case similar to BPSK)

5. Channel:

The communication channel is the physical medium that is used for transmitting signals from transmitter to receiver. In wireless system, this channel consists of atmosphere , for traditional telephony, this channel is wired , there are optical channels, under water acoustic channels etc.We further discriminate this channels on the basis of their property and characteristics, like AWGN channel etc.

6. Digital Demodulator:

The digital demodulator processes the channel corrupted transmitted waveform and reduces the waveform to the sequence of numbers that represents estimates of the transmitted data symbols.

7. Channel Decoder:

This sequence of numbers then passed through the channel decoder which attempts to reconstruct the original information sequence from the knowledge of the code used by the channel encoder and the redundancy contained in the received data

Note: The average probability of a bit error at the output of the decoder is a measure of the performance of the demodulator – decoder combination.

8. Source Decoder:

At the end, if an analog signal is desired then source decoder tries to decode the sequence from the knowledge of the encoding algorithm. And which results in the approximate replica of the input at the transmitter end.

9. Output Transducer:

Finally we get the desired signal in desired format analog or digital.

Advantages of digital communication

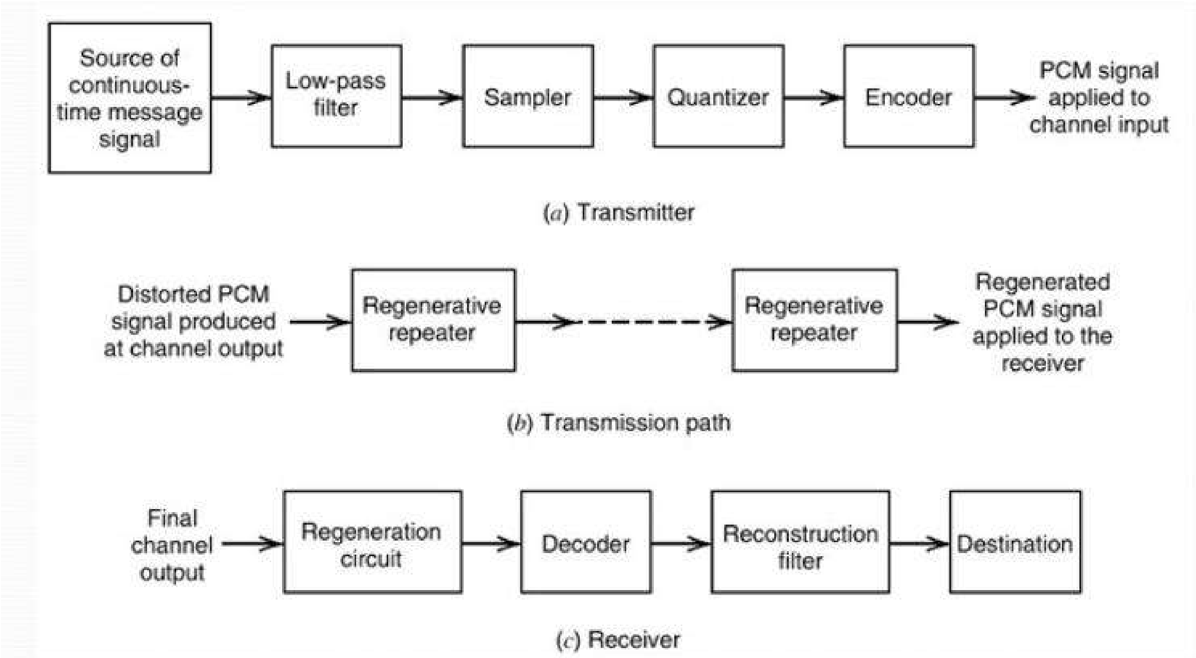
- Can **withstand channel noise and distortion** much better as long as the noise and the distortion are within limits.
- **Regenerative repeaters** prevent accumulation of noise along the path.
- Digital **hardware implementation is flexible.**
- Digital signals **can be coded** to yield extremely **low error rates**, **high fidelity** and well as **privacy**.
- Digital communication is inherently more efficient than analog in realizing the exchange of SNR for bandwidth.
- It is easier and more **efficient to multiplex** several digital signals.

- Digital signal **storage is relatively easy** and **inexpensive.**
- **Reproduction** with digital messages is extremely reliable **without deterioation.**
- The **cost** of digital hardware continues to halve every two or three years while **performance or capacity doubles** over the same time period.

Disadvantages

- TDM digital transmission is not compatible with FDM
- A Digital system requires large bandwidth.

Elements of PCM System



Sampling:

- Process of converting analog signal into discrete signal.
- Sampling is common in all pulse modulation techniques
- The signal is sampled at regular intervals such that each sample is proportional to amplitude of signal at that instant
- Analog signal is sampled every $T_s\ Secs$, called sampling interval. f_s=1/T_S is called sampling rate or sampling frequency.
- f_s=2f_m is Min. sampling rate called **Nyquist rate.** Sampled spectrum (ω) is repeating periodically without overlapping.
- Original spectrum is centered at ω=0 and having bandwidth of ω_m. Spectrum can be recovered by passing through low pass filter with cut-off ω_m.
- For f_s<2f_m sampled spectrum will overlap and cannot be recovered back. This is called **aliasing.**

Sampling methods:

- Ideal – An impulse at each sampling instant.
- Natural – A pulse of Short width with varying amplitude.
- Flat Top – Uses sample and hold, like natural but with single amplitude value.

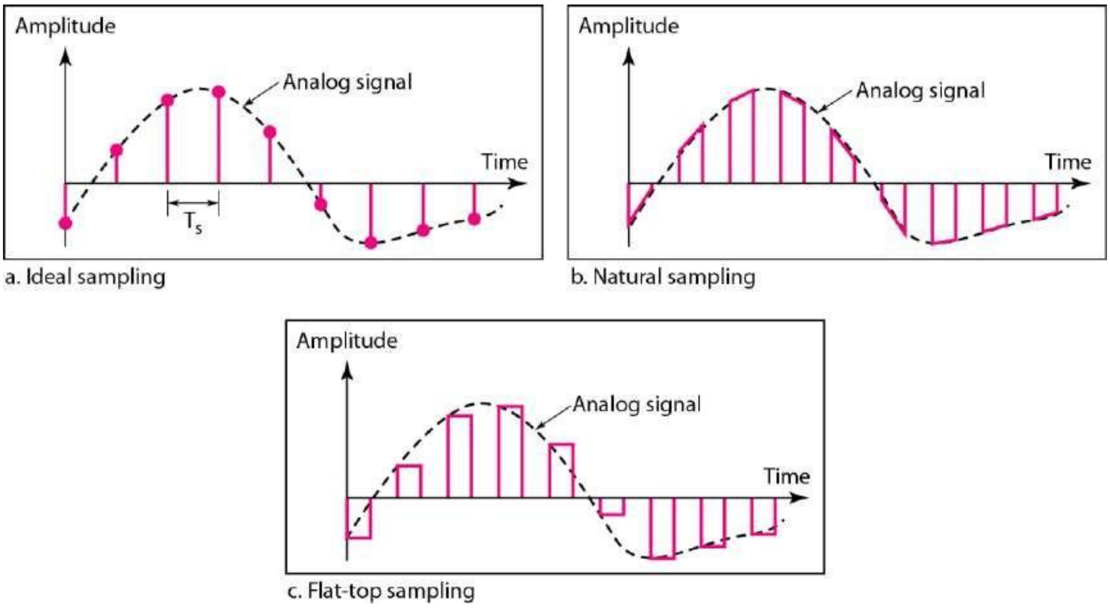


Fig. 4 Types of Sampling

Sampling of band-pass Signals:

- A band-pass signal of bandwidth $2f_m$ can be completely recovered from its samples.

  Min. sampling rate $=2\times Bandwidth$

  $=2\times 2f_m=4f_m$

- Range of minimum sampling frequencies is in the range of $2\times BW\ to\ 4\times BW$

Instantaneous Sampling or Impulse Sampling:

- Sampling function is train of spectrum remains constant impulses throughout frequency range. It is not practical.

Natural sampling:

- The spectrum is weighted by a **sinc** function.
- Amplitude of high frequency components reduces.

Flat top sampling:

- Here top of the samples remains constant.
- In the spectrum high frequency components are attenuated due sinc pulse roll off. This is known as **Aperture effect.**
- If pulse width increases aperture effect is more i.e. more attenuation of high frequency components.

PCM Generator

The pulse code modulator technique samples the input signal $x(t)$ at frequency $f_s \geq 2W$. This sampled 'Variable amplitude' pulse is then digitized by the analog to digital converter. The parallel bits obtained are converted to a serial bit stream. Fig. 8 shows the PCM generator.

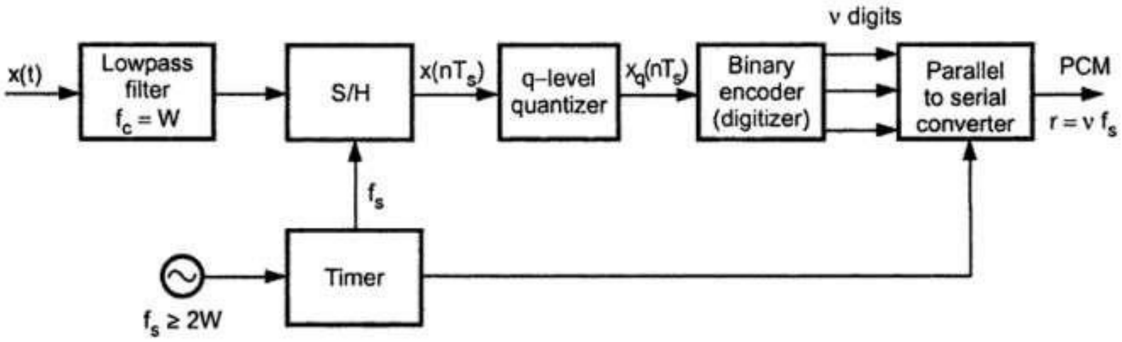


Fig. 8 PCM generator

In the PCM generator of above figure, the signal $x(t)$ is first passed through the lowpass filter of cutoff frequency 'W' Hz. This lowpass filter blocks all the frequency components above 'W' Hz. Thus $x(t)$ is bandlimited to 'W' Hz. The sample and hold circuit then samples this signal at the rate of f_s. Sampling frequency f_s is selected sufficiently above Nyquist rate to avoid aliasing i.e.,

$$f_s \geq 2W$$

In Fig. 8 output of sample and hold is called $x(nT_s)$. This $x(nT_s)$ is discrete in time and continuous in amplitude. A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels. It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called *quantization error*. Thus output of quantizer is a digital level called $x_q(nT_s)$.

Now coming back to our discussion of PCM generation, the quantized signal level $x_q(nT_s)$ is given to binary encoder. This encoder converts input signal to 'v' digits binary word. Thus $x_q(nT_s)$ is converted to 'V' binary bits. The encoder is also called digitizer.

It is not possible to transmit each bit of the binary word separately on transmission line. Therefore 'v' binary digits are converted to serial bit stream to generate single baseband signal. In a parallel to serial converter, normally a shift register does this job. The output of PCM generator is thus a single baseband signal of binary bits.

An oscillator generates the clocks for sample and hold an parallel to serial converter. In the pulse code modulation generator discussed above ; sample and hold, quantizer and encoder combinely form an analog to digital converter.

Transmission BW in PCM

Let the quantizer use 'v' number of binary digits to represent each level. Then the number of levels that can be represented by 'v' digits will be,

$$q = 2^v \quad \ldots \quad 1$$

Here 'q' represents total number of digital levels of q-level quantizer.

For example if $v = 3$ bits, then total number of levels will be,

$$q = 2^3 = 8 \text{ levels}$$

Each sample is converted to 'v' binary bits. i.e. Number of bits per sample = v

We know that, Number of samples per second = f_s

$\therefore$ Number of bits per second is given by,

$$\begin{aligned}(\text{Number of bits per second}) &= (\text{Number of bits per samples}) \\ &\quad \times (\text{Number of samples per second}) \\ &= v \text{ bits per sample} \times f_s \text{ samples per second} \quad \ldots \quad 2\end{aligned}$$

The number of bits per second is also called signaling rate of PCM and is denoted by 'r' i.e.,

$$\boxed{\textbf{Signaling rate in PCM : } r = v f_s} \quad \ldots \quad 3$$

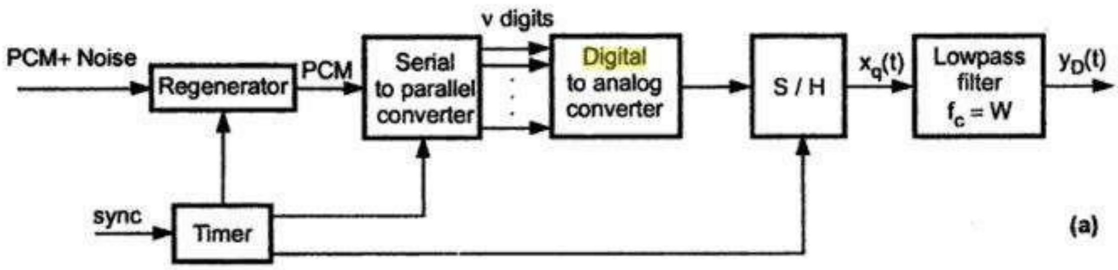
Here $f_s \geq 2W$.

Bandwidth needed for PCM transmission will be given by half of the signaling rate i.e.,

$$\text{Transmission Bandwidth of PCM :} \begin{cases} B_T \geq \frac{1}{2} r & \ldots \quad 4 \\ B_T \geq \frac{1}{2} v f_s \quad \text{Since } f_s \geq 2W & \ldots \quad 5 \\ B_T \geq v W & \ldots \quad 6 \end{cases}$$

PCM Receiver

Fig. 9 (a) shows the block diagram of PCM receiver and Fig. 9 (b) shows the reconstructed signal. The regenerator at the start of PCM receiver reshapes the pulses and removes the noise. This signal is then converted to parallel digital words for each sample.



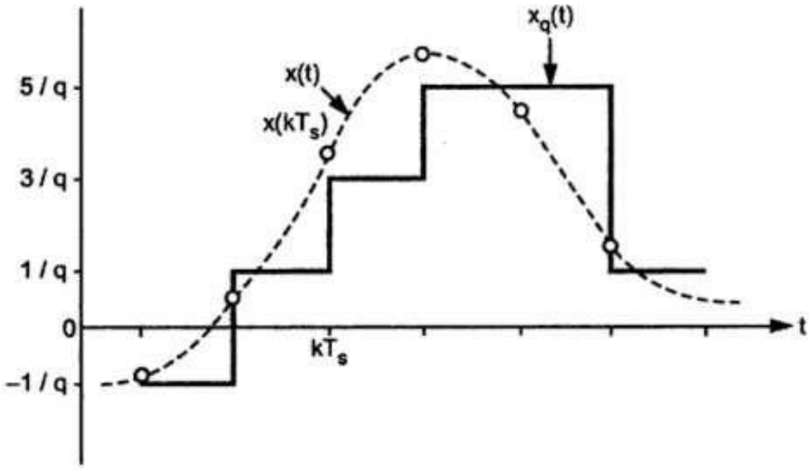


Fig. 9 (a) PCM receiver
(b) Reconstructed waveform

The digital word is converted to its analog value $x_q(t)$ along with sample and hold. This signal, at the output of S/H is passed through lowpass reconstruction filter to get $y_D(t)$. As shown in reconstructed signal of Fig. 9 (b), it is impossible to reconstruct exact original signal $x(t)$ because of permanent quantization error introduced during quantization at the transmitter. This quantization error can be reduced by increasing the binary levels. This is equivalent to increasing binary digits (bits) per sample. But increasing bits 'v' increases the signaling rate as well as transmission bandwidth as we have seen in equation 3 and equation 6. Therefore the choice of these parameters is made, such that noise due to quantization error (called as quantization noise) is in tolerable limits.

<u>Quantization</u>

- The quantizing of an analog signal is done by discretizing the signal with a number of quantization levels.

- **Quantization** is representing the sampled values of the amplitude by a finite set of levels, which means converting a continuous-amplitude sample into a discrete-time signal
- Both sampling and quantization result in the loss of information.
- The quality of a Quantizer output depends upon the number of quantization levels used.
- The discrete amplitudes of the quantized output are called as **representation levels** or **reconstruction levels**.
- The spacing between the two adjacent representation levels is called a **quantum** or **step-size**.
- There are two types of Quantization
    - Uniform Quantization
    - Non-uniform Quantization.
- The type of quantization in which the quantization levels are uniformly spaced is termed as a **Uniform Quantization**.
- The type of quantization in which the quantization levels are unequal and mostly the relation between them is logarithmic, is termed as a **Non-uniform Quantization.**

<u>Uniform Quantization:</u>

- There are two types of uniform quantization.
    - Mid-Rise type
    - Mid-Tread type.
- The following figures represent the two types of uniform quantization.

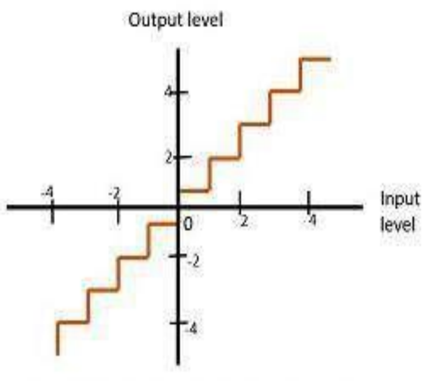


Fig 1 : Mid-Rise type Uniform Quantization

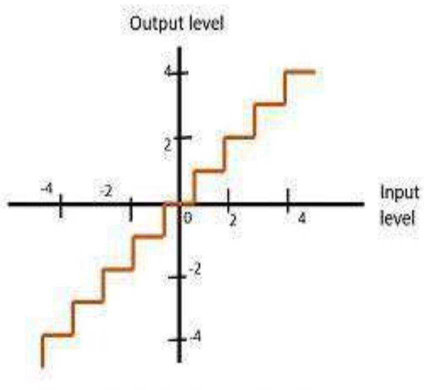


Fig 2 : Mid-Tread type Uniform Quantization

- The **Mid-Rise** type is so called because the origin lies in the middle of a raising part of the stair-case like graph. The quantization levels in this type are even in number.
- The **Mid-tread** type is so called because the origin lies in the middle of a tread of the stair-case like graph. The quantization levels in this type are odd in number.
- Both the mid-rise and mid-tread type of uniform quantizer is symmetric about the origin.

Quantization Noise and Signal to Noise ratio in PCM System

Derivation of Quantization Error/Noise or Noise Power for Uniform (Linear) Quantization

Step 1 : Quantization Error

Because of quantization, inherent errors are introduced in the signal. This error is called *quantization error*. We have defined quantization error as,

$$\varepsilon = x_q(nT_s) - x(nT_s) \qquad \text{.........................} (1)$$

Step 2 : Step size

Let an input $x(nT_s)$ be of continuous amplitude in the range $-x_{max}$ to $+x_{max}$.

Therefore the total amplitude range becomes,

$$\text{Total amplitude range} = x_{max} - (-x_{max}) = 2x_{max} \qquad \text{.........................} (2)$$

If this amplitude range is divided into 'q' levels of quantizer, then the step size 'δ' is given as,

$$\delta = \frac{x_{max} - (-x_{max})}{q}$$

$$= \frac{2x_{max}}{q} \qquad \text{.....................} (3)$$

If signal $x(t)$ is normalized to minimum and maximum values equal to 1, then

$$x_{max} = 1$$
$$-x_{max} = -1 \qquad \text{..............................} (4)$$

Therefore step size will be,

$$\delta = \frac{2}{q} \quad \text{(for normalized signal)} \qquad \text{..............................} (5)$$

Step 3 : Pdf of Quantization error

If step size 'δ' is sufficiently small, then it is reasonable to assume that the quantization error 'ε' will be uniformly distributed random variable. The maximum quantization error is given by

$$\varepsilon_{max} = \left|\frac{\delta}{2}\right| \qquad \text{..............................} (6)$$

i.e. $$-\frac{\delta}{2} \geq \varepsilon_{max} \geq \frac{\delta}{2} \qquad \text{..............................} (7)$$

Thus over the interval $\left(-\frac{\delta}{2}, \frac{\delta}{2}\right)$ quantization error is uniformly distributed random variable.

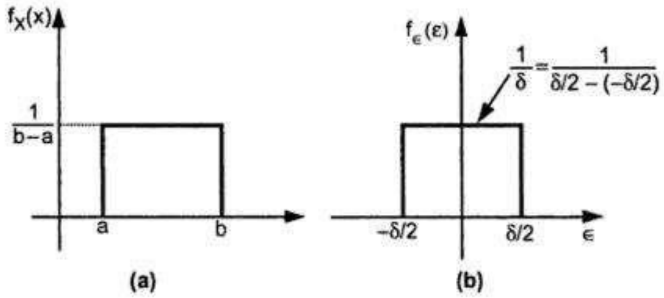


Fig. 10 (a) Uniform distribution
(b) Uniform distribution for quantization error

In above figure, a random variable is said to be uniformly distributed over an interval (a, b). Then PDF of 'X' is given by, (from equation of Uniform PDF).

$$f_X(x) = \begin{cases} 0 & \text{for} \quad x \le a \\ \dfrac{1}{b-a} & \text{for} \quad a < x \le b \\ 0 & \text{for} \quad x > b \end{cases} \qquad \text{..........................(8)}$$

Thus with the help of above equation we can define the probability density function for quantization error 'ε' as,

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & \text{for} \quad \varepsilon \le \dfrac{\delta}{2} \\ \dfrac{1}{\delta} & \text{for} \quad -\dfrac{\delta}{2} < \varepsilon \le \dfrac{\delta}{2} \\ 0 & \text{for} \quad \varepsilon > \dfrac{\delta}{2} \end{cases} \qquad \text{....................................(9)}$$

Step 4 : Noise Power

quantization error 'ε' has zero average value. That is mean 'm_ε' of the quantization error is zero.

The signal to quantization noise ratio of the quantizer is defined as,

$$\frac{S}{N} = \frac{\text{Signal power (normalized)}}{\text{Noise power (normalized)}} \qquad \cdots 10$$

If type of signal at input i.e., $x(t)$ is known, then it is possible to calculate signal power.

The noise power is given as,

$$\text{Noise power} = \frac{V_{noise}^2}{R} \qquad \cdots (11)$$

Here V_{noise}^2 is the mean square value of noise voltage. Since noise is defined by random variable 'ε' and PDF $f_\varepsilon(\varepsilon)$, its mean square value is given as,

$$\text{mean square value} = E[\varepsilon^2] = \bar{\varepsilon}^2 \qquad \cdots (12)$$

The mean square value of a random variable 'X' is given as,

$$\overline{X}^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx \quad \text{By definition} \qquad \cdots (13)$$

Here $$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon \qquad \cdots (14)$$

From equation 9 we can write above equation as,

$$E[\varepsilon^2] = \int_{-\delta/2}^{\delta/2} \varepsilon^2 \times \frac{1}{\delta}\, d\varepsilon$$

$$= \frac{1}{\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\delta/2}^{\delta/2} = \frac{1}{\delta}\left[\frac{(\delta/2)^3}{3} + \frac{(\delta/2)^3}{3}\right]$$

$$= \frac{1}{3\delta}\left[\frac{\delta^3}{8} + \frac{\delta^3}{8}\right] = \frac{\delta^2}{12} \qquad \cdots (15)$$

∴ From equation 1.8.25, the mean square value of noise voltage is,

$$V_{noise}^2 = \text{mean square value} = \frac{\delta^2}{12}$$

When load resistance, $R = 1$ ohm, then the noise power is normalized i.e.,

$$\text{Noise power (normalized)} = \frac{V_{noise}^2}{1} \qquad \text{[with } R = 1 \text{ in equation 11]}$$

$$= \frac{\delta^2 / 12}{1} = \frac{\delta^2}{12}$$

Thus we have,

Normalized noise power

or Quantization noise power $= \dfrac{\delta^2}{12}$ **; For linear quantization.**

or Quantization error (in terms of power) ... (16)

Derivation of Maximum Signal to Quantization Noise Ratio for Linear Quantization:

signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

$$= \frac{\text{Normalized signal power}}{(\delta^2 / 12)} \qquad \text{... (17)}$$

The number of bits 'v' and quantization levels 'q' are related as,

$$q = 2^v \qquad \text{... (18)}$$

Putting this value in equation (3) we have,

$$\delta = \frac{2\, x_{\max}}{2^v} \qquad \text{... (19)}$$

Putting this value in equation 1.8.30 we get,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\left(\dfrac{2\, x_{\max}}{2^v}\right)^2 + 12}$$

Let normalized signal power be denoted as 'P'.

$$\frac{S}{N} = \frac{P}{\dfrac{4\, x_{\max}^2}{2^{2v}} \times \dfrac{1}{12}} = \frac{3P}{x_{\max}^2} \cdot 2^{2v}$$

This is the required relation for maximum signal to quantization noise ratio. Thus,

$$\textbf{Maximum signal to quantization noise ratio : } \frac{S}{N} = \frac{3P}{x_{max}^2} \cdot 2^{2v} \quad \cdots (20)$$

This equation shows that signal to noise power ratio of quantizer increases exponentially with increasing bits per sample.

If we assume that input $x(t)$ is normalized, i.e.,

$$x_{max} = 1 \quad \cdots (21)$$

Then signal to quantization noise ratio will be,

$$\frac{S}{N} = 3 \times 2^{2v} \times P \quad \cdots (22)$$

If the destination signal power 'P' is normalized, i.e.,

$$P \leq 1 \quad \cdots (23)$$

Then the signal to noise ratio is given as,

$$\frac{S}{N} \leq 3 \times 2^{2v} \quad \cdots (24)$$

Since $x_{max} = 1$ and $P \leq 1$, the signal to noise ratio given by above equation is normalized.

Expressing the signal to noise ratio in decibels,

$$\left(\frac{S}{N}\right) dB = 10 \log_{10} \left(\frac{S}{N}\right) dB \quad \text{since power ratio.}$$

$$\leq 10 \log_{10} [3 \times 2^{2v}]$$

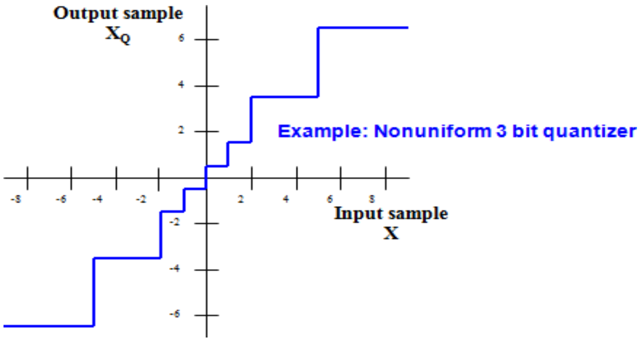
$$\leq (4.8 + 6v)\, dB$$

Thus,

Signal to Quantization noise ratio for normalized values of power 'P' and amplitude of input $x(t)$: $\left(\frac{S}{N}\right) dB \leq (4.8 + 6v)\, dB$ ··· (25)

Non-Uniform Quantization:

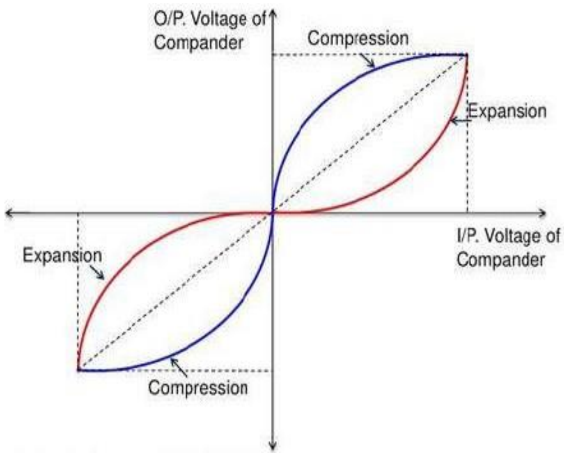
In non-uniform quantization, the step size is not fixed. It varies according to certain law or as per input signal amplitude. The following fig shows the characteristics of Non uniform quantizer.

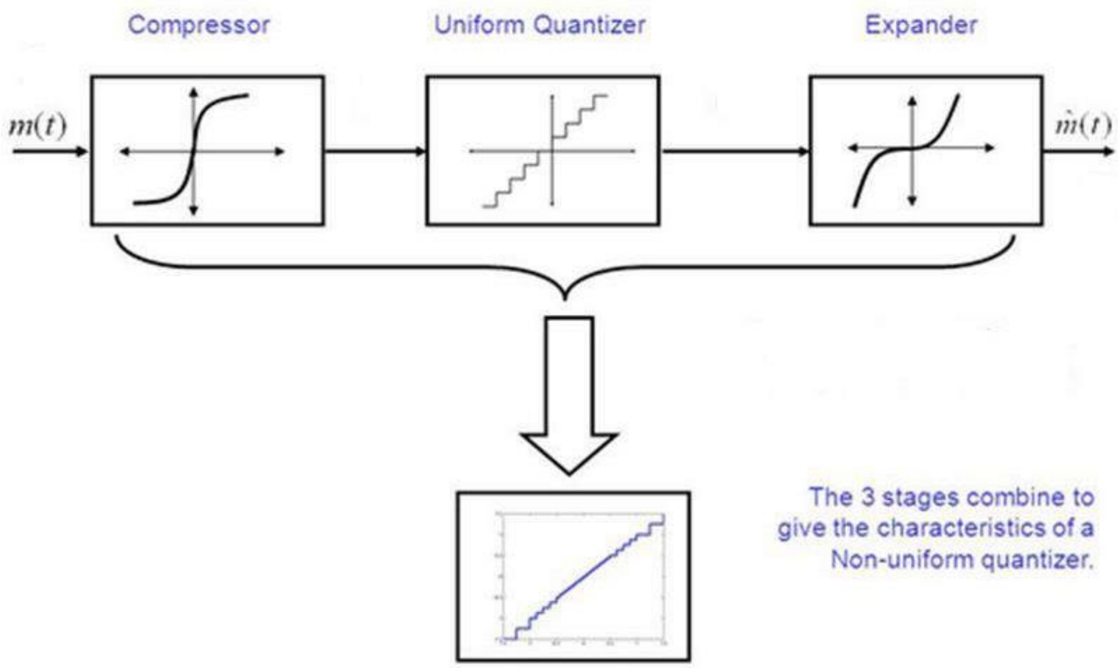


In this figure observe that step size is small at low input signal levels. Hence quantization error is also small at these inputs. Therefore signal to quantization noise power ratio is improved at low signal levels. Stepsize is higher at high input levels. Hence signal to noise power ratio remains almost same throughout the dynamic range of quantizer.

Companding PCM System

- Non-uniform quantizers are difficult to make and expensive.
- An alternative is to first pass the speech signal through nonlinearity before quantizing with a uniform quantizer.
- The nonlinearity causes the signal amplitude to be ***compressed***.
  - The input to the quantizer will have a more uniform distribution.
- At the receiver, the signal is ***expanded*** by an inverse to the nonlinearity.
- The process of compressing and expanding is called ***Companding***.





μ - Law Companding for Speech Signals

Normally for speech and music signals a μ - law compression is used. This compression is defined by the following equation,

$$Z(x) = (\text{Sgn } x)\frac{ln(1+\mu|x|)}{ln(1+\mu)} \quad |x| \leq 1 \qquad \cdots (1)$$

Below Fig shows the variation of signal to noise ratio with respect to signal level without companding and with companding.

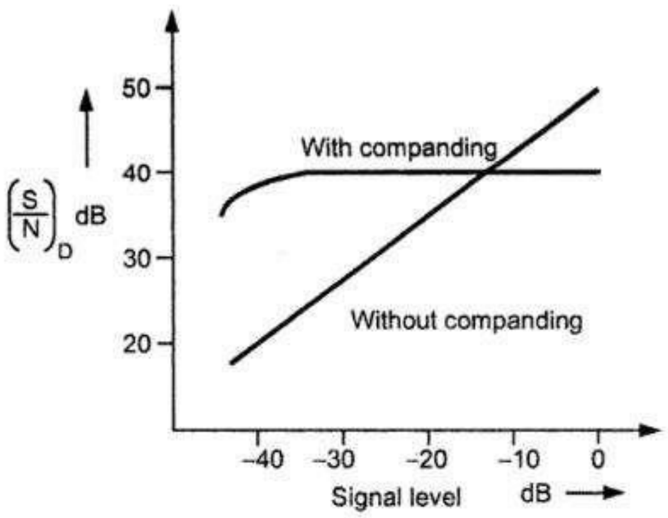


Fig. 11 PCM performance with μ - law companding

It can be observed from above figure that signal to noise ratio of PCM remains almost constant with companding.

A-Law for Companding

The A law provides piecewise compressor characteristic. It has linear segment for low level inputs and logarithmic segment for high level inputs. It is defined as,

$$Z(x) = \begin{cases} \dfrac{A|x|}{1+\ln A} & \text{for } 0 \le |x| \le \dfrac{1}{A} \\ \dfrac{1+\ln(A|x|)}{1+\ln A} & \text{for } \dfrac{1}{A} \le |x| \le 1 \end{cases} \quad \cdots (2)$$

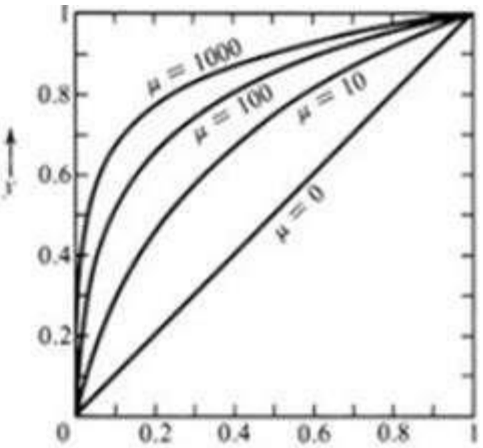
When A = 1, we get uniform quantization. The practical value for A is 87.56. Both A-law and μ-law companding is used for PCM telephone systems.

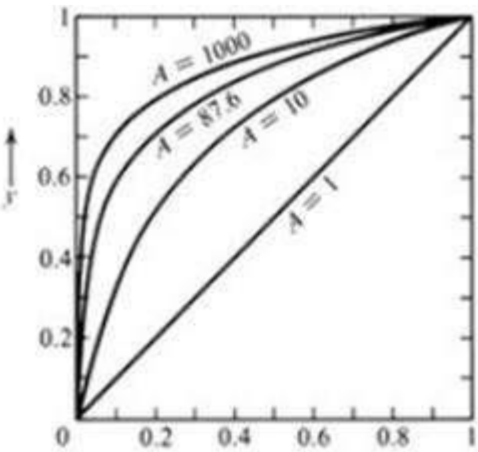
Signal to Noise Ratio of Companded PCM

The signal to noise ratio of companded PCM is given as,

$$\frac{S}{N} = \frac{3q^2}{[\ln(1+\mu)]^2} \quad \cdots (3)$$

Here $q = 2^v$ is number of quantization levels.





Differential Pulse Code Modulation

(DPCM)Redundant Information in PCM

The samples of a signal are highly corrected with each other. This is because any signal does not change fast. That is its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with little difference. When these samples are encoded by standard PCM system, the resulting encoded signal contains redundant information.

Fig. shows a continuous time signal $x(t)$ by dotted line. This signal is sampled by flat top sampling at intervals $T_s, 2T_s, 3T_s \ldots nT_s$. The sampling frequency is selected to be higher than nyquist rate. The samples are encoded by using 3 bit (7 levels) PCM. The sample is quantized to the nearest digital level as shown by small

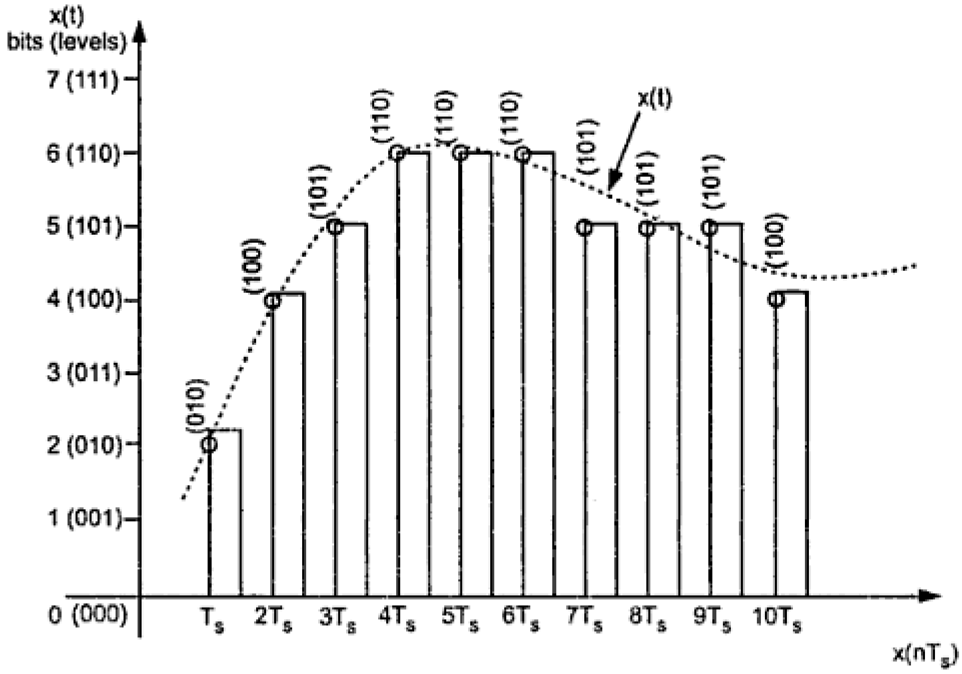


Fig. Redundant information in PCM

circles in the diagram. The encoded binary value of each sample is written on the top of the samples. We can see from Fig. that the samples taken at $4T_s, 5T_s$ and $6T_s$ are encoded to same value of (110). This information can be carried only by one sample. But three samples are carrying the same information means it is redundant. Consider another example of samples taken at $9T_s$ and $10T_s$. The difference between these samples is only due to last bit and first two bits are redundant, since they do not change.

Principle of DPCM

If this redundancy is reduced, then overall bit rate will decrease and number of bits required to transmit one sample will also be reduced. This type of digital pulse modulation scheme is called Differential Pulse Code Modulation.

DPCM Transmitter

The differential pulse code modulation works on the principle of prediction. The value of the present sample is predicted from the past samples. The prediction may not be exact but it is very close to the actual sample value. Fig. shows the transmitter of Differential Pulse Code Modulation (DPCM) system. The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$. The comparator finds out the difference between the actual sample value $x(nT_s)$ and predicted sample value $\hat{x}(nT_s)$. This is called error and it is denoted by $e(nT_s)$. It can be defined as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \qquad \text{..................(1)}$$

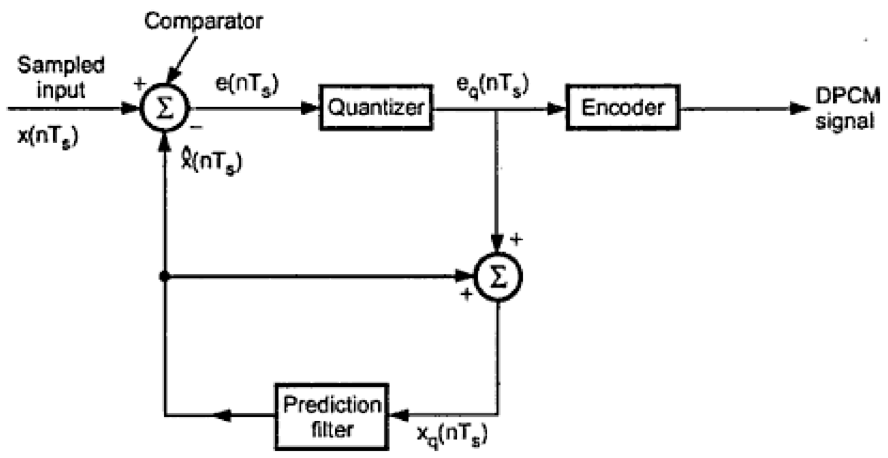


Fig. Differential pulse code modulation transmitter

Thus error is the difference between unquantized input sample $x(nT_s)$ and prediction of it $\hat{x}(nT_s)$. The predicted value is produced by using a prediction filter. The quantizer output signal $e_q(nT_s)$ and previous prediction is added and given as input to the prediction filter. This signal is called $x_q(nT_s)$. This makes the prediction more and more close to the actual sampled signal. We can see that the quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small number of bits. Thus number of bits per sample are reduced in DPCM.

The quantizer output can be written as,

$$e_q(nT_s) = e(nT_s) + q(nT_s) \quad \text{.................(2)}$$

Here $q(nT_s)$ is the quantization error. As shown in Fig. the prediction filter input $x_q(nT_s)$ is obtained by sum $\hat{x}(nT_s)$ and quantizer output i.e.,

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \quad \text{..................(3)}$$

Putting the value of $e_q(nT_s)$ from equation 2 in the above equation we get,

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \quad \text{................(4)}$$

Equation 1 is written as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$\therefore \quad e(nT_s) + \hat{x}(nT_s) = x(nT_s) \quad \text{...............(5)}$$

$\therefore$ Putting the value of $e(nT_s) + \hat{x}(nT_s)$ from above equation into equation 4 we get,

$$x_q(nT_s) = x(nT_s) + q(nT_s) \quad \text{..................(6)}$$

Thus the quantized version of the signal $x_q(nT_s)$ is the sum of original sample value and quantization error $q(nT_s)$. The quantization error can be positive or negative. Thus equation 6 does not depend on the prediction filter characteristics.

Reconstruction of DPCM Signal

Fig. shows the block diagram of DPCM receiver.

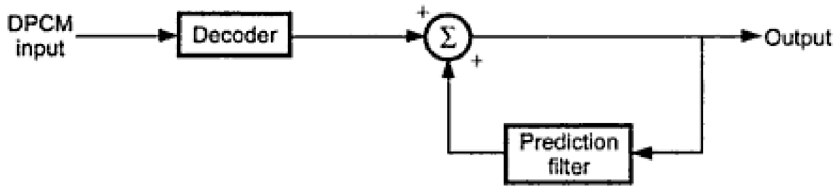


Fig. DPCM receiver

The decoder first reconstructs the quantized error signal from incoming binary signal. The prediction filter output and quantized error signals are summed up to give the quantized version of the original signal. Thus the signal at the receiver differs from actual signal by quantization error $q(nT_s)$, which is introduced permanently in the reconstructed signal.

Introduction to Delta Modulation

PCM transmits all the bits which are used to code the sample. Hence signaling rate and transmission channel bandwidth are large in PCM. To overcome this problem Delta Modulation is used.

Delta Modulation

Operating Principle of DM

Delta modulation transmits only one bit per sample. That is the present sample value is compared with the previous sample value and the indication,whether the amplitude is increased or decreased is sent. Input signal $x(t)$ is approximated to step signal by the delta modulator. This step size is fixed . The difference between the input signal $x(t)$ and staircase approximated signal confined to two levels, i.e. $+\delta$ and $-\delta$. If the difference is positive, then approximated signal is increased by one step i.e. 'δ'. If the difference is negative, then approximated signal is reduced by 'δ'. When the step is reduced, '0' is transmitted and if the step is increased, '1' is transmitted. Thus for each sample, only one binary bit is transmitted. Fig. shows the analog signal $x(t)$ and its staircase approximated signal by the delta modulator.

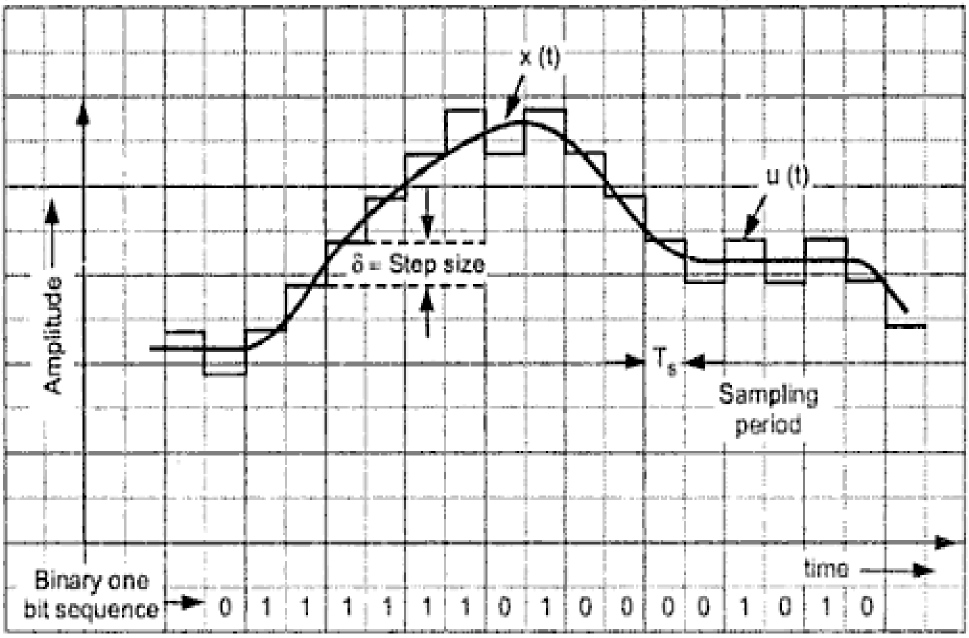


Fig. Delta modulation waveform

The principle of delta modulation can be explained by the following set of equations. The error between the sampled value of $x(t)$ and last approximated sample is given as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \qquad \ldots (\quad 1)$$

Here, $e(nT_s)$ = Error at present sample

$x(nT_s)$ = Sampled signal of $x(t)$

$\hat{x}(nT_s)$ = Last sample approximation of the staircase waveform.

We can call $u(nT_s)$ as the present sample approximation of staircase output.

Then, $u[(n-1)T_s] = \hat{x}(nT_s) \qquad \ldots (\quad 2)$

= Last sample approximation of staircase waveform.

Let the quantity $b(nT_s)$ be defined as,

$$b(nT_s) = \delta \, sgn[e(nT_s)] \qquad \ldots (\quad 3)$$

That is depending on the sign of error $e(nT_s)$ the sign of step size δ will be decided. In other words,

$$\begin{aligned} b(nT_s) &= +\delta \quad \text{if} \quad x(nT_s) \geq \hat{x}(nT_s) \\ &= -\delta \quad \text{if} \quad x(nT_s) < \hat{x}(nT_s) \end{aligned} \qquad \ldots (\quad 4)$$

If $b(nT_s) = +\delta$; binary '1' is transmitted

and if $b(nT_s) = -\delta$; binary '0' is transmitted.

T_s = Sampling interval.

DM Transmitter

Fig. (a) shows the transmitter based on equations 3 to 5.

The summer in the accumulator adds quantizer output ($\pm\delta$) with the previous sample approximation. This gives present sample approximation. i.e.,

$$\begin{aligned} u(nT_s) &= u(nT_s - T_s) + [\pm\delta] \quad \text{or} \\ &= u[(n-1)T_s] + b(nT_s) \end{aligned} \qquad \ldots (\quad 5)$$

The previous sample approximation $u[(n-1)T_s]$ is restored by delaying one sample period T_s. The sampled input signal $x(nT_s)$ and staircase approximated signal $\hat{x}(nT_s)$ are subtracted to get error signal $e(nT_s)$.

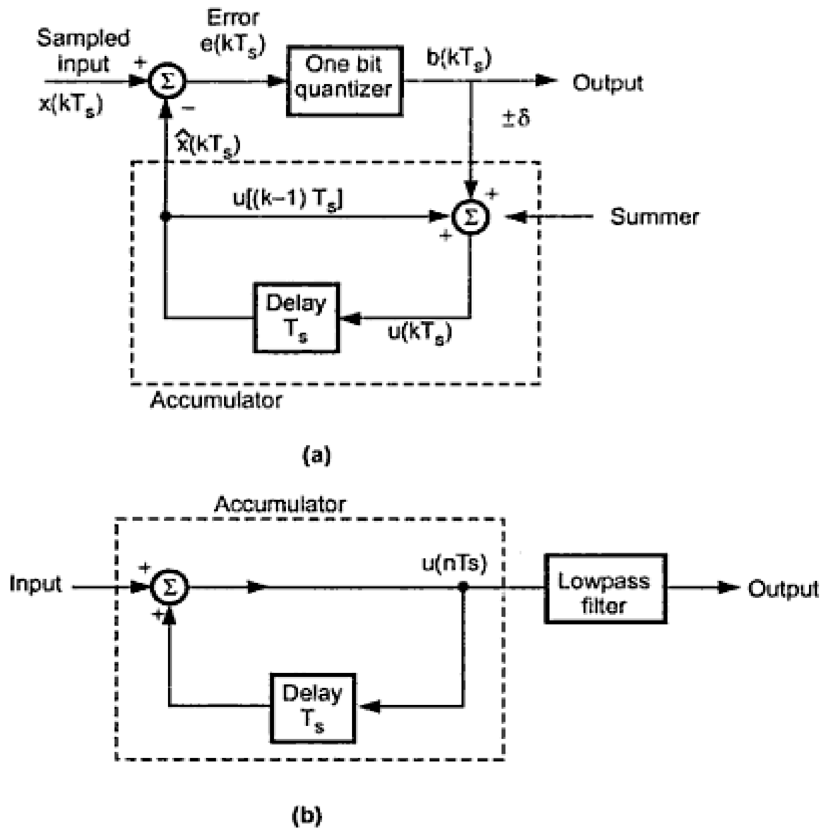


Fig. (a) Delta modulation transmitter and (b) Delta modulation receiver

Depending on the sign of $e(nT_s)$ one bit quantizer produces an output step of $+\delta$ or $-\delta$. If the step size is $+\delta$, then binary '1' is transmitted and if it is $-\delta$, then binary '0' is transmitted.

DM Receiver

At the receiver shown in Fig. (b), the accumulator and low-pass filter are used. The accumulator generates the staircase approximated signal output and is delayed by one sampling period T_s. It is then added to the input signal. If input is binary '1' then it adds $+\delta$ step to the previous output (which is delayed). If input is binary '0' then one step 'δ' is subtracted from the delayed signal. The low-pass filter has the cutoff frequency equal to highest frequency in $x(t)$. This filter smoothen the staircase signal to reconstruct $x(t)$.

Advantages and Disadvantages of Delta Modulation

Advantages of Delta Modulation

The delta modulation has following advantages over PCM,

1. Delta modulation transmits only one bit for one sample. Thus the signaling rate and transmission channel bandwidth is quite small for delta modulation.
2. The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter involved in delta modulation.

Disadvantages of Delta Modulation

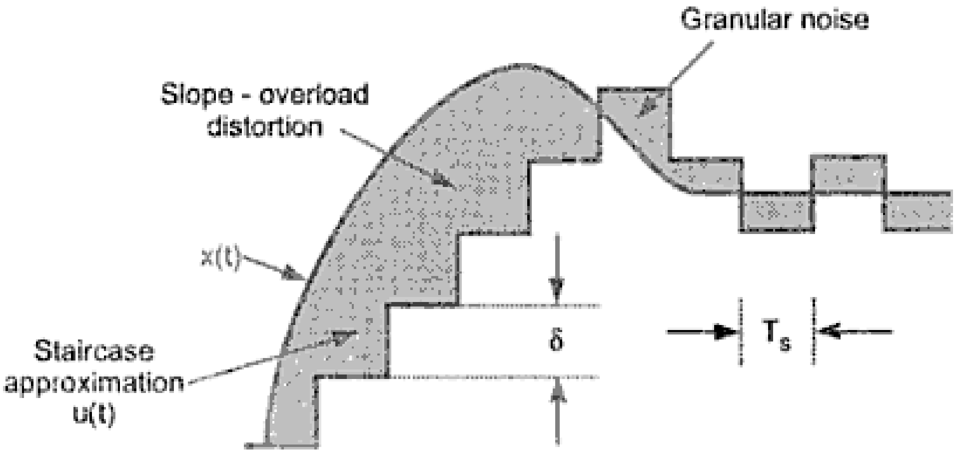


Fig. Quantization errors in delta modulation

The delta modulation has two drawbacks -

Slope Overload Distortion (Startup Error)

This distortion arises because of the large dynamic range of the input signal.

As can be seen from Fig. the rate of rise of input signal $x(t)$ is so high that the staircase signal cannot approximate it, the step size 'δ' becomes too small for staircase signal $u(t)$ to follow the steep segment of $x(t)$. Thus there is a large error between the staircase approximated signal and the original input signal $x(t)$. This error is called *slope overload distortion*. To reduce this error, the step size should be increased when slope of signal of $x(t)$ is high.

Since the step size of delta modulator remains fixed, its maximum or minimum slopes occur along straight lines. Therefore this modulator is also called Linear Delta Modulator (LDM).

Granular Noise (Hunting)

Granular noise occurs when the step size is too large compared to small variations in the input signal. That is for very small variations in the input signal, the staircase signal is changed by large amount (δ) because of large step size. Fig shows that when the input signal is almost flat, the staircase signal $u(t)$ keeps on oscillating by $\pm\delta$ around the signal. The error between the input and approximated signal is called *granular noise.* The solution to this problem is to make step size small.

Thus large step size is required to accommodate wide dynamic range of the input signal (to reduce slope overload distortion) and small steps are required to reduce granular noise. Adaptive delta modulation is the modification to overcome these errors.

Adaptive Delta Modulation

Operating Principle

To overcome the quantization errors due to slope overload and granular noise, the step size (δ) is made adaptive to variations in the input signal $x(t)$. Particularly in the steep segment of the signal $x(t)$, the step size is increased. When the input is varying slowly, the step size is reduced. Then the method is called *Adaptive Delta Modulation (ADM).*

The adaptive delta modulators can take continuous changes in step size or discrete changes in step size.

Transmitter and Receiver

Fig. (a) shows the transmitter and (b) shows receiver of adaptive delta modulator. The logic for step size control is added in the diagram. The step size increases or decreases according to certain rule depending on one bit quantizer output.

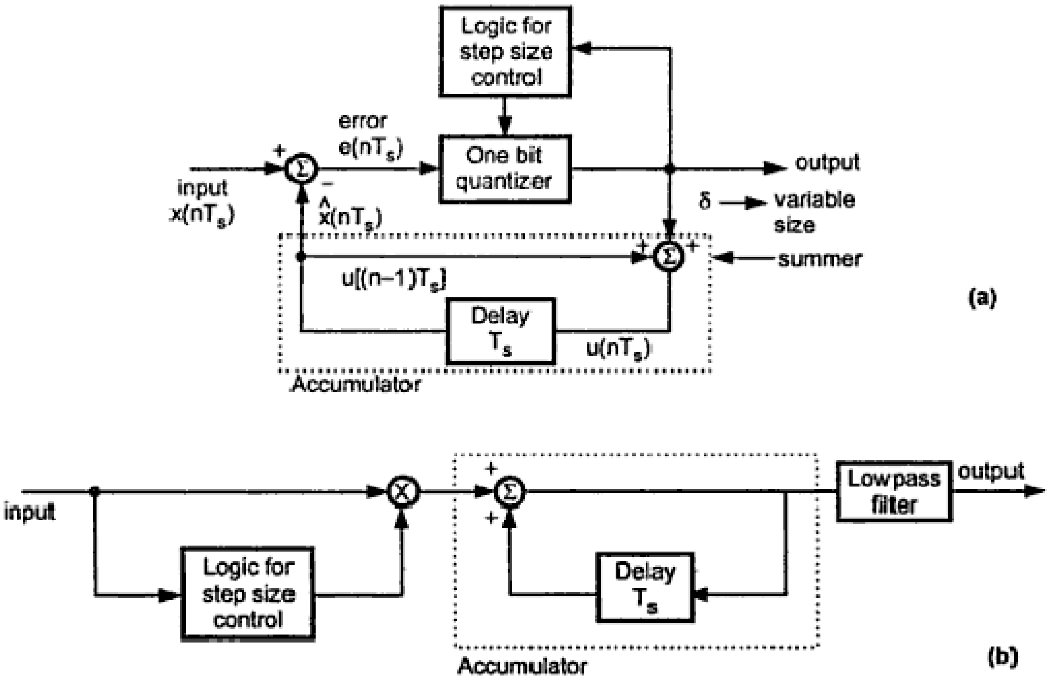


Fig. Adaptive delta modulator (a) Transmitter (b) Receiver

For example if one bit quantizer output is high (1), then step size may be doubled for next sample. If one bit quantizer output is low, then step size may be reduced by one step. Fig. shows the waveforms of adaptive delta modulator and sequence of bits transmitted.

In the receiver of adaptive delta modulator shown in Fig. (b) the first part generates the step size from each incoming bit. Exactly the same process is followed as that in transmitter. The previous input and present input decides the step size. It is then given to an accumulator which builds up staircase waveform. The low-pass filter then smoothens out the staircase waveform to reconstruct the smooth signal.

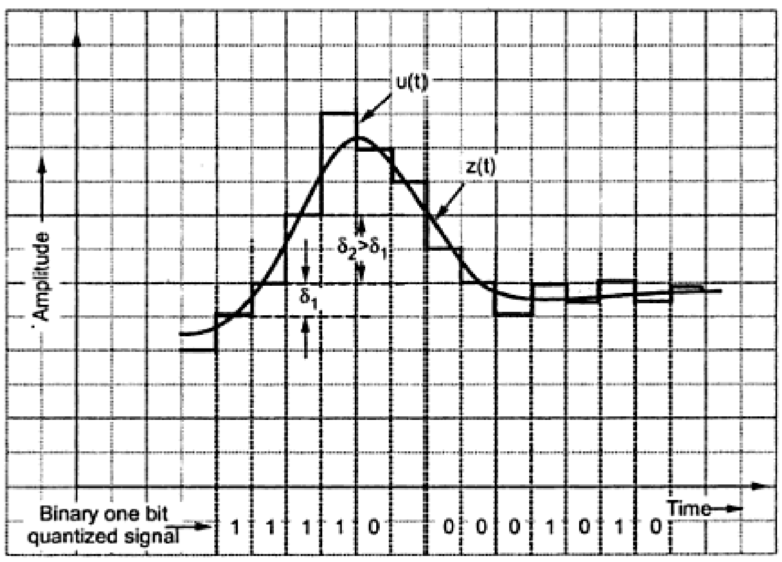


Fig. Waveforms of adaptive delta modulation

Advantages of Adaptive Delta Modulation

Adaptive delta modulation has certain advantages over delta modulation. i.e.,

1. The signal to noise ratio is better than ordinary delta modulation because of the reduction in slope overload distortion and granular noise.
2. Because of the variable step size, the dynamic range of ADM is wide.
3. Utilization of bandwidth is better than delta modulation.

Plus other advantages of delta modulation are, only one bit per sample is required and simplicity of implementation of transmitter and receiver.

Condition for Slope overload distortion occurrence

Slope overload distortion will occur if

$$A_m > \frac{\delta}{2\pi f_m T_s}$$

where T_s is the sampling period.

Let the sine wave be represented as,

$$x(t) = A_m \sin(2\pi f_m t)$$

Slope of $x(t)$ will be maximum when derivative of $x(t)$ with respect to 't' will be maximum. The maximum slope of delta modulator is given

$$\text{Max. slope} = \frac{\text{Step size}}{\text{Sampling period}}$$

$$= \frac{\delta}{T_s} \qquad \text{.............(1)}$$

Slope overload distortion will take place if slope of sine wave is greater than slope of delta modulator i.e.

$$\max\left|\frac{d}{dt}x(t)\right| > \frac{\delta}{T_s}$$

$$\max\left|\frac{d}{dt}A_m \sin(2\pi f_m t)\right| > \frac{\delta}{T_s}$$

$$\max\left|A_m 2\pi f_m \cos(2\pi f_m t)\right| > \frac{\delta}{T_s}$$

$$A_m 2\pi f_m > \frac{\delta}{T_s}$$

or $$\boxed{A_m > \frac{\delta}{2\pi f_m T_s}} \qquad \text{...............(2)}$$

Expression for Signal to Quantization Noise power ratio for Delta Modulation

To obtain signal power :

slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

Here A_m is peak amplitude of sinusoided signal

δ is the step size

f_m is the signal frequency and

T_s is the sampling period.

From above equation, the maximum signal amplitude will be,

$$A_m = \frac{\delta}{2\pi f_m T_s} \qquad \text{..................(1)}$$

Signal power is given as,

$$P = \frac{V^2}{R}$$

Here V is the rms value of the signal. Here $V = \frac{A_m}{\sqrt{2}}$. Hence above equation becomes,

$$P = \left(\frac{A_m}{\sqrt{2}}\right)^2 / R$$

Normalized signal power is obtained by taking R = 1. Hence,

$$P = \frac{A_m^2}{2}$$

Putting for A_m from equation 1

$$P = \frac{\delta^2}{8\pi^2 f_m^2 T_s^2} \qquad \text{.................(2)}$$

This is an expression for signal power in delta modulation.

(ii) To obtain noise power

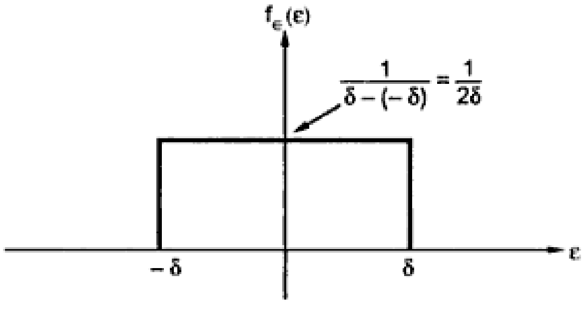


Fig. Uniform distribution of quantization error

We know that the maximum quantization error in delta modulation is equal to step size 'δ'. Let the quantization error be uniformly distributed over an interval $[-\delta, \delta]$ This is shown in Fig. From this figure the PDF of quantization error can be expressed as,

$$f_{\epsilon}(\varepsilon) = \begin{cases} 0 & \textit{for} \quad \varepsilon < \delta \\ \frac{1}{2\delta} & \textit{for} \quad -\delta < \varepsilon < \delta \\ 0 & \textit{for} \quad \varepsilon > \delta \end{cases} \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

The noise power is given as,

$$\text{Noise power} = \frac{V_{\text{noise}}^2}{R}$$

Here V_{noise}^2 is the mean square value of noise voltage. Since noise is defined by random variable 'ε' and PDF $f_{\epsilon}(\varepsilon)$, its mean square value is given as,

$$\text{mean square value} = E\left[\varepsilon^2\right] = \overline{\varepsilon^2}$$

mean square value is given as,

$$E\left[\varepsilon^2\right] = \int_{-\infty}^{\infty} \varepsilon^2 f_{\epsilon}(\varepsilon)\, d\varepsilon$$

From equation 3

$$E\left[\varepsilon^2\right] = \int_{-\delta}^{\delta} \varepsilon^2 \cdot \frac{1}{2\delta}\, d\varepsilon$$

$$= \frac{1}{2\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\delta}^{\delta}$$

$$= \frac{1}{2\delta}\left[\frac{\delta^3}{3} + \frac{\delta^3}{3}\right] = \frac{\delta^2}{3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Hence noise power will be,

$$\text{noise power} = \left(\frac{\delta^2}{3}\right) / R$$

Normalized noise power can be obtained with R = 1. Hence,

$$\text{noise power} = \frac{\delta^2}{3} \qquad \ldots\ldots\ldots\ldots\ldots(5)$$

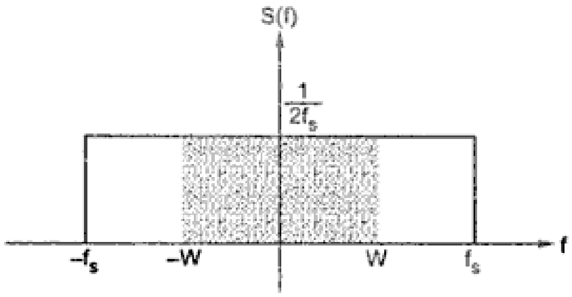


Fig. PSD of noise

This noise power is uniformly distributed over $-f_s$ to f_s range. This is illustrated in Fig. At the output of delta modulator receiver there is lowpass reconstruction filter whose cutoff frequency is 'W'. This cutoff frequency is equal to highest signal frequency. The reconstruction filter passes part of the noise power at the output as Fig. From the geometry of Fig. output noise power will be,

$$\text{Output noise power} = \frac{W}{f_s} \times \text{noise power} = \frac{W}{f_s} \times \frac{\delta^2}{3}$$

We know that $f_s = \frac{1}{T_s}$, hence above equation becomes,

$$\text{Output noise power} = \frac{WT_s\delta^2}{3} \quad \text{.................(6)}$$

(iii) To obtain signal to noise power ratio

Signal to noise power ratio at the output of delta modulation receiver is given as,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

From equation 2. and equation 6

$$\frac{S}{N} = \frac{\dfrac{\delta^2}{8\pi^2 f_m^2 T_s^2}}{\dfrac{WT_s\delta^2}{3}}$$

$$\boxed{\frac{S}{N} = \frac{3}{8\pi^2 W f_m^2 T_s^3}} \quad \text{..............(7)}$$

This is an expression for signal to noise power ratio in delta modulation.

Unit-6

Digital Modulation Techniques

Digital modulation : Digital Modulation is defined as changing the amplitude of the carrier signal with respect to the binary information or digital signal.

Bit rate is the number of bits transmitted during one second between the transmitter and receiver.

Baud rate is the rate of change of signal on transmission medium after encoding and modulation have occurred.

Bandwidth efficiency : Bandwidth efficiency is the ratio of the transmission bit rate to the minimum bandwidth required for a particular modulation

Advantages of Digital communications

- It has a better noise immunity
- Repeaters can be used between transmitters and receivers
- It becomes simpler and cheaper as compared to the analog communication

Disadvantages of Digital communications

- It requires a larger channel bandwidth
- Delta modulation needs synchronization incase of synchronous modulation

Minimum Shift Keying (MSK) : The minimum frequency space that allows the 2 fsk representing symbols 0s and 1s. Thus CP (Continuous Phase) FSK signal with a deviation ratio ifone half is defined as MSK.

Frequency Shift Keying (FSK) : Frequency Shift Keying is the as changing amplitude of the carrier signal with respect to the binary information or digital signal.

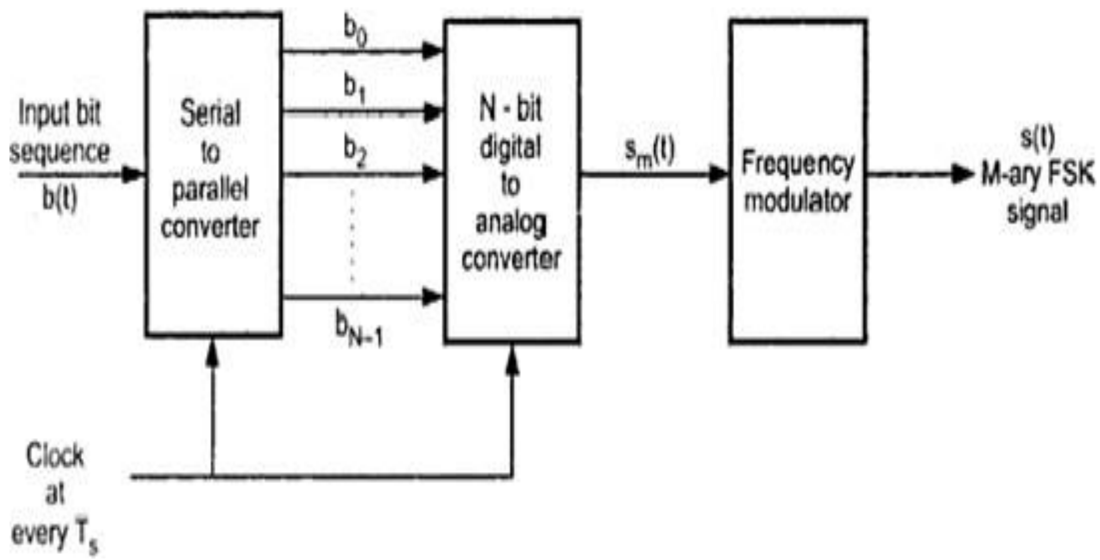
<u>The advantages of Minimum Shift Keying</u> :

MSK baseband waveform are smoother compared with QPSK MSK signals havecontinuous phase It does not have any amplitude variation

Frequency Shift Keying (FSK)

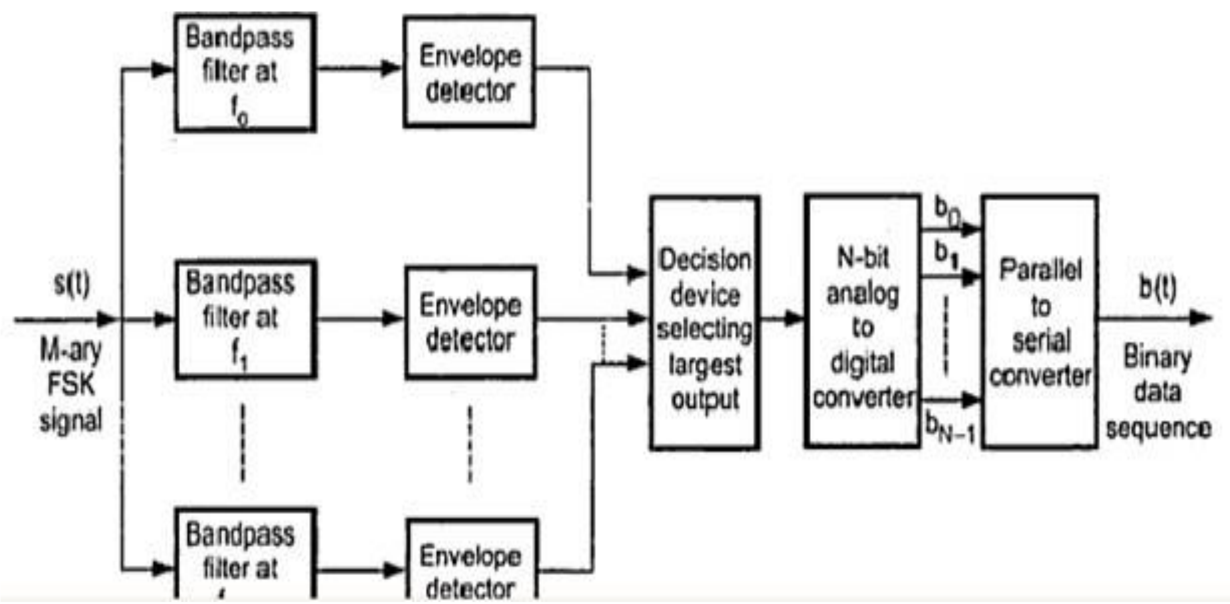
Transmitter

the M-ary FSK transmitter. The 'N' successive bits are presented in parallel to digital to analog converter. These 'N' bits forms a symbol at the output of digital to analog converter. There will be total $2^N = M$ possible symbols. The symbol is presented every $T_s = NT_b$ period. The output of digital to analog converter is given to a frequency modulator. Thus depending upon the value of symbol, the frequency modulator generates the output frequency. For every symbol, the frequency modulator produces different frequency output. This particular frequency signal remains at the output for one symbol duration. Thus for 'M' symbols, there are 'M' frequency signals at the output of modulator. Thus the transmitted frequencies are $f_0, f_1, f_2, ... f_{M-1}$ depending upon the input symbol to the modulator.



Receiver

The M-ary FSM signal is given to the set of 'M' bandpass filters. The center frequencies of those filters are $f_0, f_1, f_2, \ldots f_{M-1}$. These filters pass their particular frequency and alternate others. The envelope detectors outputs are applied to a decision device. The decision device produces its output depending upon the highest input. Depending upon the particular symbol, only one envelope detector will have higher output. The outputs of other detectors will be very low. The output of the decision device is given to 'N' bit analog to digital converter. The analog to digital converter output is the 'N' bit symbol in parallel. These bits are then converted to serial bit stream by parallel to serial converter. In some cases the bits appear in parallel. Then there is no need to use serial to parallel and parallel to serial converters.



2.2 AMPLITUDE SHIFT KEYING (ASK) PHASE SHIFT KEYING (PSK):

Amplitude Shift Keying (ASK) : Amplitude Shift Keying is the as changing amplitude of the carrier signal with respect to the binary information or digital signal.

Define Phase Shift Keying (PSK): Phase Shift Keying is the changing amplitude of the carrier signal with respect to the binary information or digital signal

Concept Of Amplitude Shift Keying In Detail:

The amplitude shift keying is also called on-off keying (OOK). This is the simplest digital modulation technique. The binary input data is converted to unipolar NRZ signal. A product modulator takes this NRZ signal and carrier signal. The output of the product modulator is the ASK signal, which can be expressed mathematically as,

$$v(t) = d \sin(2\pi f_c t) \quad \text{... (2.3.1)}$$

Here f_c is the carrier frequency

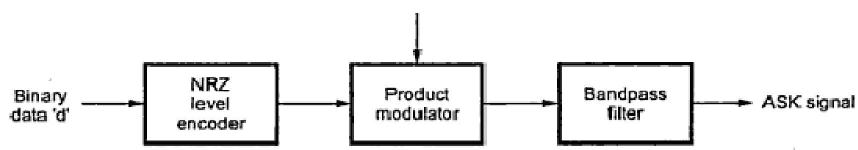
and d is the data bit, which is either 1 or 0.

Fig. 2.3.1 (a) shows the block diagram of the ASK modulator. The binary data sequence 'd' is given to the NRZ level encoder. This NRZ level encoder converts the input binary sequence to the signal suitable for product modulator. The product modulator also accepts a sinusoidal carrier of frequency f_c. The output of the product modulator is passed through a bandpass filter for bandwidth limiting. The output of the bandpass filter is the ASK signal. This signal and other waveforms are shown in Fig. 2.3.1 (b). Observe that the ASK signal has on-off nature. In equation 2.3.1 when $d=0$, $v(t)=0$; i.e. no ASK signal. And when $d=1$, $d=\sin(2\pi f_c t)$. The ASK is very sensitive to noise. It is used for very low bit rates less than around 100 bps. The only advantage of ASK is that it is very simple to implement.

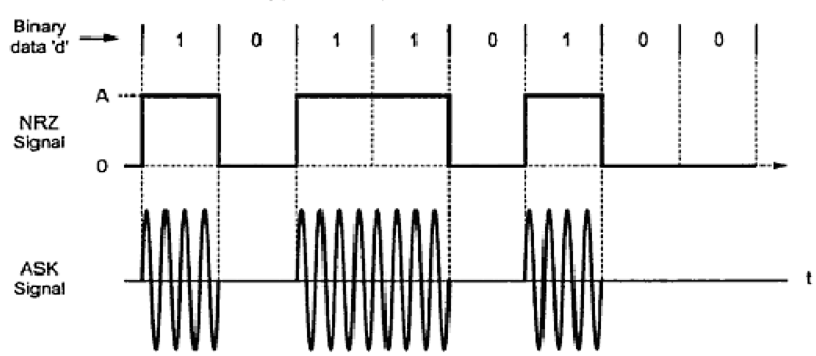
Baud rate

For ASK, the ASK waveform is changed at the bit rate. Hence Baud rate is given as,

$$\text{Baud rate} = f_b \quad \text{... (2.3.2)}$$



(a) Block diagram of ASK modulator



(b) Waveforms

Fig. 2.3.1 Amplitude shift keying (ASK)

2.3 CONCEPT OF BINARY PHASE SHIFT KEYING TO BPSK, 8 PSK, 16 PSK,

<u>CONCEPT OF BINARY PHASE SHIFT KEYING:</u>

Binary Phase Shift Keying (BPSK)

Principle of BPSK

- In binary phase shift keying (BPSK), binary symbol '1' and '0' modulate the phase of the carrier. Let the carrier be,

$$s(t) = A\cos(2\pi f_0 t) \quad \text{... (4.2.1)}$$

'A' represents peak value of sinusoidal carrier. In the standard 1Ω load register, the power dissipated will be,

$$P = \frac{1}{2}A^2$$

$$\therefore \quad A = \sqrt{2P} \quad \text{... (4.2.2)}$$

- When the symbol is changed, then the phase of the carrier is changed by 180 degrees (π radians).
- Consider for example,

$$\text{Symbol '1'} \Rightarrow s_1(t) = \sqrt{2P}\cos(2\pi f_0 t) \quad \text{... (4.2.3)}$$

---

if next symbol is '0' then,

$$\text{Symbol '0'} \Rightarrow s_2(t) = \sqrt{2P}\cos(2\pi f_0 t + \pi) \quad \text{... (4.2.4)}$$

Since $\cos(\theta + \pi) = -\cos\theta$, we can write above equation as,

$$s_2(t) = -\sqrt{2P}\cos(2\pi f_0 t) \quad \text{... (4.2.5)}$$

With the above equation we can define BPSK signal combinely as,

$$\boxed{s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)} \quad \text{... (4.2.6)}$$

Here $b(t)$ = +1 when binary '1' is to be transmitted

= −1 when binary '0' is to be transmitted

Graphical Representation of BPSK Signal

Fig. 4.2.1 shows binary signal and its equivalent signal $b(t)$.

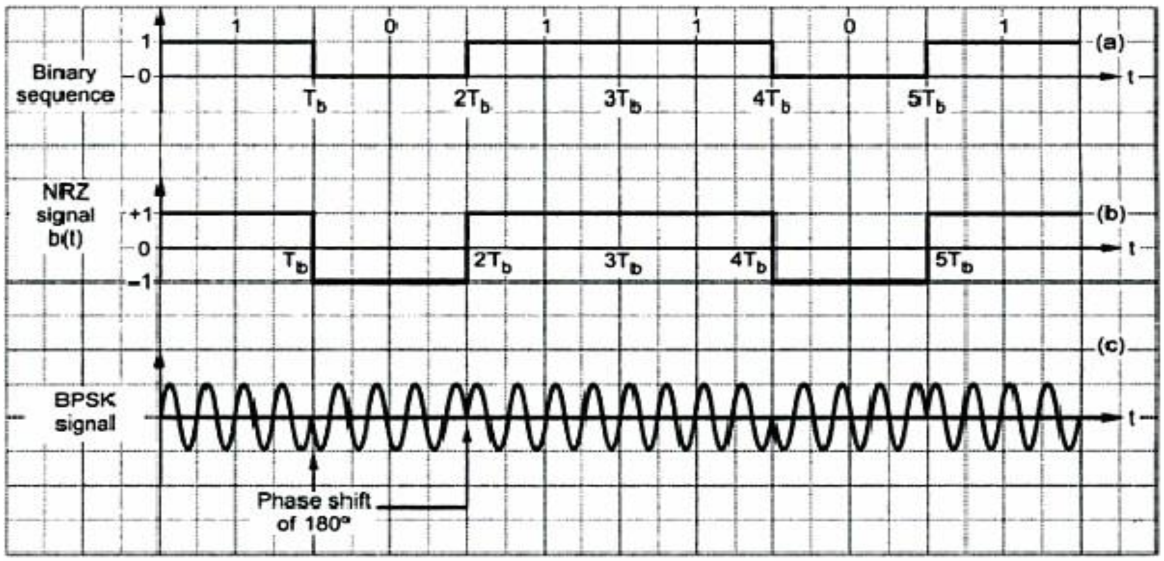


Fig. 4.2.1 (a) Binary sequence
(b) Its equivalent bipolar signal $b(t)$
(c) BPSK signal

As can be seen from Fig. 4.2.1 (b), the signal $b(t)$ is NRZ bipolar signal. This signal directly modulates carrier $cos\,(2\pi f_0\,t)$.

Generation and Reception of BPSK Signal

Generation of BPSK Signal

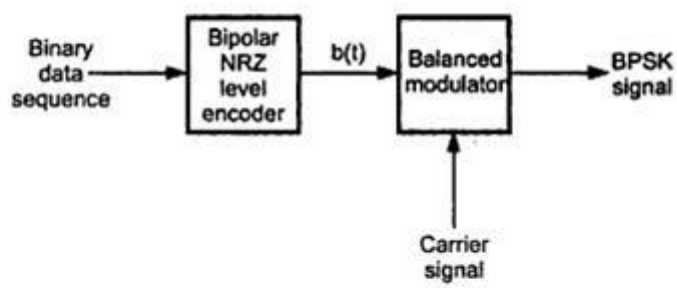


Fig. 4.2.2 BPSK generation scheme

- The BPSK signal can be generated by applying carrier signal to the balanced modulator.
- The baseband signal $b(t)$ is applied as a modulating signal to the balanced modulator. Fig. 4.2.2 shows the block diagram of BPSK signal generator.
- The NRZ level encoder converts the binary data sequence into bipolar NRZ signal.

Reception of BPSK Signal

Fig. 4.2.3 shows the block diagram of the scheme to recover baseband signal from BPSK signal. The transmitted BPSK signal is,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

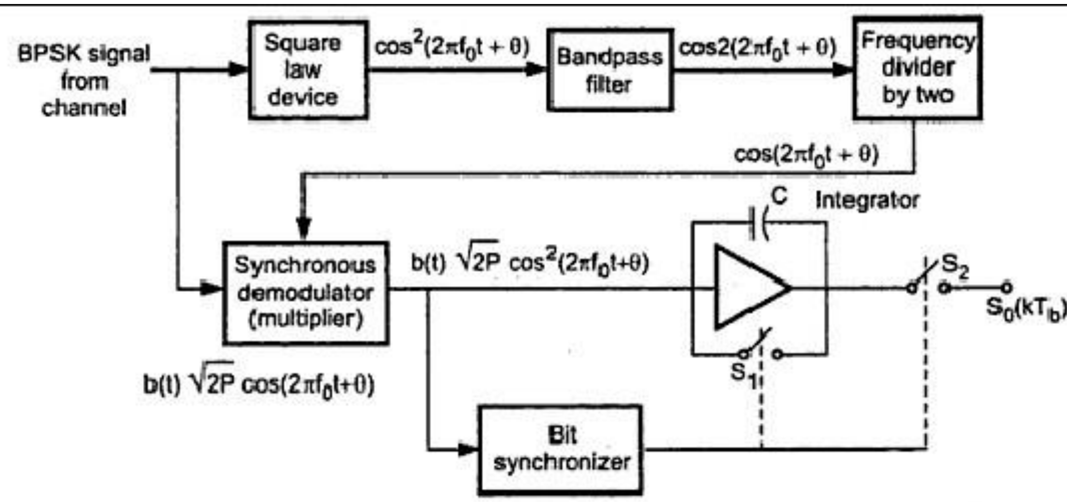


Fig. 4.2.3 Reception BPSK scheme

Operation of the receiver

1) **Phase shift in received signal** : This signal undergoes the phase change depending upon the time delay from transmitter to receiver. This phase change is normally fixed phase shift in the transmitted signal. Let the phase shift be θ. Therefore the signal at the input of the receiver is,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta) \quad \text{... (4.2.7)}$$

2) **Square law device** : Now from this received signal, a carrier is separated since this is coherent detection. As shown in the figure, the received signal is passed through a square law device. At the output of the square law device the signal will be,

$$\cos^2(2\pi f_0 t + \theta)$$

Note here that we have neglected the amplitude, because we are only interested in the carrier of the signal.

We know that,

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\therefore \quad \cos^2(2\pi f_0 t + \theta) = \frac{1+\cos 2(2\pi f_0 t + \theta)}{2}$$

Bandwidth of BPSK Signal

The spectrum of the BPSK signal is centered around the carrier frequency f_0.

If $f_b = \frac{1}{T_b}$, then for BPSK the maximum frequency in the baseband signal will be

f_b see Fig. 4.2.6. In this figure the main lobe is centered around carrier frequency f_0 and extends from $f_0 - f_b$ to $f_0 + f_b$. Therefore Bandwidth of BPSK signal is,

$$BW = \text{Highest frequency} - \text{Lowest frequency in the main lobe}$$

$$= f_0 + f_b - (f_0 - f_b)$$

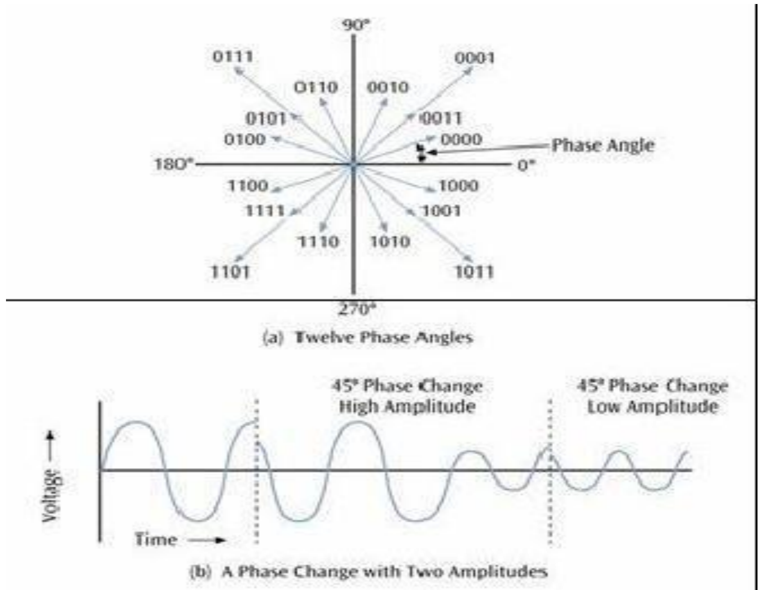
$$\therefore \quad \boxed{BW = 2f_b} \quad \dots (4.2.21)$$

Thus the minimum bandwidth of BPSK signal is equal to twice of the highest frequency contained in baseband signal.

2.4 QUADRATURE AMPLITUDE MODULATION:

Quadrature Amplitude modulation (QAM): QAM is a form of digital modulation similar to PSK except the digital information is contained in both the amplitude and phase of the transmitted carrier.

- QAM is a combination of ASK and PSKTwo different signals sent simultaneously on the same carrier frequency ie,M=4, 16, 32, 64, 128, 256 As an example of QAM, 12 different phases are combined with two different amplitudes.
- Since only 4 phase angles have 2 different amplitudes, there are a total of 16 combinations. With 16 signal combinations, each baud equals 4 bits of information (2 ^ 4 = 16).
- Combine ASK and PSK such that each signal corresponds to multiple bits.
- More phases than amplitudes. Minimum bandwidth requirement same as ASK or PSK.



(a) Twelve Phase Angles

(b) A Phase Change with Two Amplitudes

2.5 <u>QUADRATURE PHASE SHIFT KEYING (QPSK) TECHNIQUES AND ITS BLOCK DIAGRAM:</u>

<u>Advantages of QPSK</u>

- Very good noise immunity
- Effective utilization of available bandwidth
- Low error probability

Very high bit rate data transmission

Quadrature Phase Shift Keying (QPSK)

Principle

- In communication systems we know that there are two main resources, i.e. transmission power and the channel bandwidth. The channel bandwidth depends upon the bit rate or signalling rate f_b. In digital bandpass transmission, a carrier is used for transmission. This carrier is transmitted over a channel.
- If two or more bits are combined in some symbols, then the signalling rate is reduced. Therefore the frequency of the carrier required is also reduced. This reduces the transmission channel bandwidth. Thus because of grouping of bits in symbols, the transmission channel bandwidth is reduced.
- In quadrature phase shift keying, two successive bits in the data sequence are grouped together. This reduces the bits rate of signalling rate (i.e. f_b) and hence reduces the bandwidth of the channel.

- In BPSK we know that when symbol changes the level, the phase of the carrier is changed by 180°. Since there were only two symbols in BPSK, the phase shift occurs in two levels only.
- In QPSK two successive bits are combined. This combination of two bits forms four distinct symbols. When the symbol is changed to next symbol the

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Since $b_o(t)$ and $b_e(t)$ cannot change at the same time, the phase change in QPSK signal will be maximum $\pi / 2$. This is clear from Fig. 4.4.3.

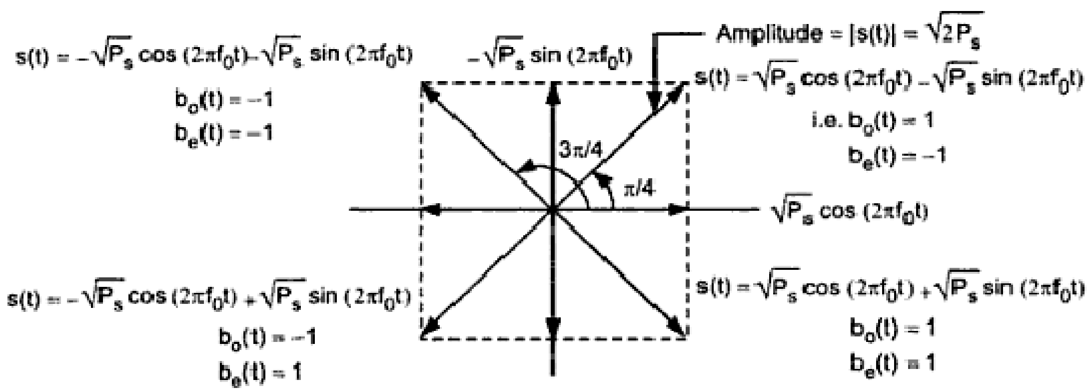
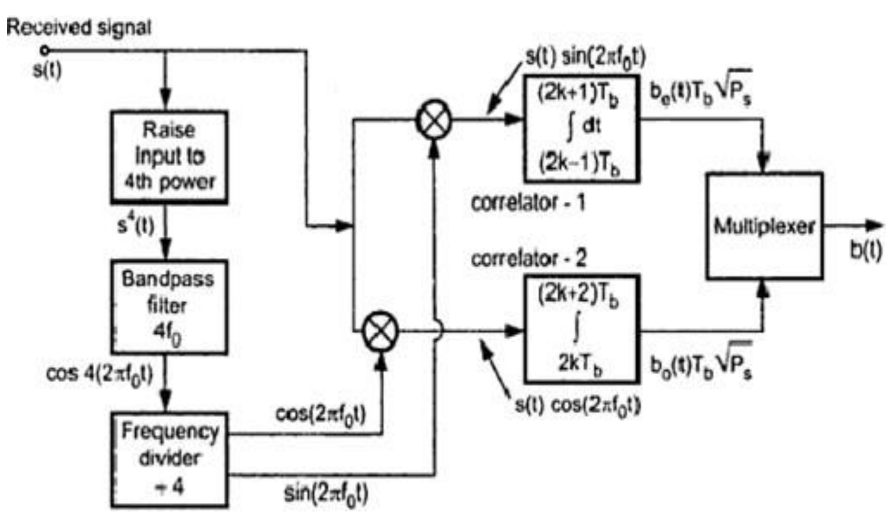


Fig. 4.4.3 Phasor diagram of QPSK signal

The QPSK Receiver



This is Synchronous reception. Therefore coherent carrier is to be recovered from the received signal s(t).

Operation

Step 1 : Isolation of carrier

The received signal $s(t)$ is first raised to its 4^{th} power, i.e. $s^4(t)$. Then it is passed through a bandpass filter centered around $4f_0$. The output of the bandpass filter is a coherent carrier of frequency $4f_0$. This is divided by 4 and it gives two coherent quadrature carriers $cos(2\pi f_0 t)$ and $sin(2\pi f_0 (t)$.

Step 2 : Synchronous detection

These coherent carriers are applied to two synchronous demodulators. These synchronous demodulators consist of multiplier and an integrator.

Step 3 : Integration over two bits interval

The incoming signal is applied to both the multipliers. The integrator integrates the product signal over two bit interval (i.e. $T_s = 2T_b$).

Step 4 : Sampling and multiplexing odd and even bit sequences

At the end of this period, the output of integrator is sampled. The outputs of the two integrators are sampled at the offset of one bit period, T_b. Hence the output of

Advantages of QPSK

QPSK has some definite advantages and disadvantages as compared to BPSK and DPSK.

Advantages :

1) For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK.
2) Because of reduced bandwidth, the information transmission rate of QPSK is higher.
3) Variation in OQPSK amplitude is not much. Hence carrier power almost remains constant.

Bandwidth Efficiency (Information Density)

Definition : It is the ratio of transmission bit rate to minimum required bandwidth.

i.e.,

$$\text{BW efficiency} = \frac{\text{Transmission rate(Bits / sec)}}{\text{Minimum bandwidth (cycles / sec)}}$$

$$= \frac{\text{Transmission rate}}{\text{Minimum bandwidth}} \text{ bits/cycle} \qquad ...(2.9.1)$$

- When bandwidth efficiency is normalized to 1-Hz bandwidth, it gives number of bits that can be propagated per hertz of bandwidth.
- Bandwidth efficiency is used to compare the performance of digital modulation techniques.

Compare binary PSK with QPSK.

BPSK QPSK

1. One bit forms a symbol. Two bits form a symbol.

2. Two possible symbols. Four possible symbols.

3. Minimum bandwidth is twice of fb. Minimum bandwidth is equal to fb.

4. Symbol duration = Tb. Symbol duration = 2Tb.

9. What are the advantages of M-ary signaling scheme?

1. M-ary signaling schemes transmit bits at a time.

2. Bandwidth requirement of M-ary signaling schemes is reduced.

The probability of error in M-Ary FSK as the value of m increases:

As the value of „M" increases, the Euclidean distance between the symbols reduces. Hence the symbols come closer to each other. This increases the probability of error in M-arysystems.

Correlative coding allows the signaling rate of 2B0 in the channel of bandwidth B0. Thisis made physically possible by allowing ISI in the transmitted signal in controlled manner. This ISI is known to the receiver. Hence effects of ISI are eliminated at the receiver. Correlative coding is implemented by duobinary signaling and modified duobinary signaling.
