Controllers:

A controller is a device introduced in the system to modify the error signal and to produce a control signal. The manner in which the controller produces the control signal is called the control action. The controller modifies the transient response of the system.

(1) Proportional controller (P-controller)

The proportional controller is a device that produces a control signal, $u(t)$ proportional to the input error signal, $e(t)$.

In P-controller, $u(t) \propto e(t)$

$\therefore u(t) = K_p e(t)$

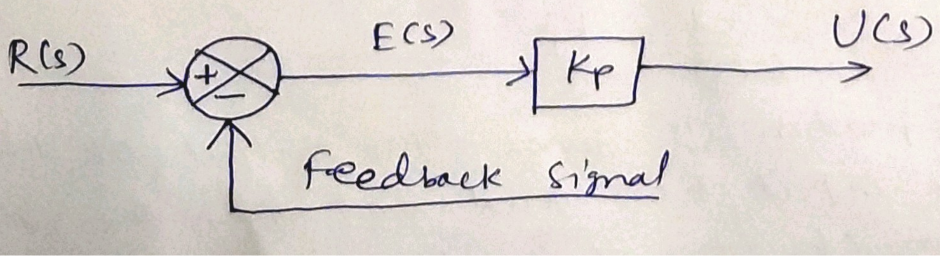
where, K_p = proportional gain or constant.

On taking Laplace transform,

$$U(s) = K_p E(s) \quad \text{——— (1)}$$

$$\therefore \frac{U(s)}{E(s)} = K_p \quad \text{——— (2)}$$

The equation (1) gives the output of the p-controller for the input $E(s)$ & eq^n (2) is the transfer function of the P-controller. The block diagram of the P-controller is shown in the below figure.



→ Proportional controller amplifies the error signal by an amount K_p.

→ Proportional controller
↳ increases the loop gain by an amount K_p.
The increase in loop gain improves the steady state tracking accuracy, disturbance signal rejection & relative stability & also makes the s/s less sensitive to parameter variations.

→ But increasing the gain to very large values may lead to instability of the s/s.

→ The drawback in P-controller is that it leads to a constant steady state error.

② Integral controller (I-controller)

The integral controller is a device that produces a control signal u(t) which is proportional to integral of the input error signal, e(t).

In I-controller,

$$u(t) \propto \int e(t)\,dt \; ; \quad u(t) = K_i \int e(t)\,dt.$$

where, K_i = Integral gain or constant.

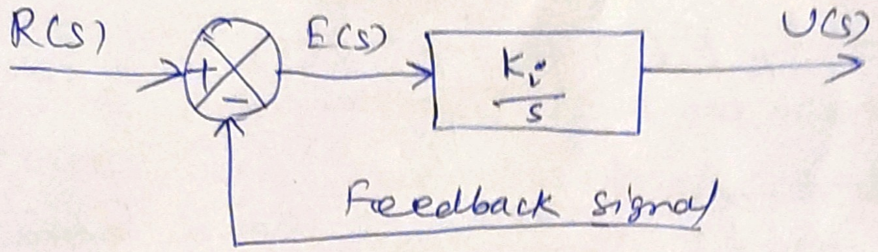
On taking Laplace transform,

$$U(s) = K_i \cdot \frac{E(s)}{s} \quad \text{——— ①}$$

$$\therefore \frac{U(s)}{E(s)} = \frac{K_i}{s} \quad \text{——— ②}$$

The equation ① gives the output of the I-controller for the input E(s) and eqⁿ ② is the transfer function of the I-controller.

The block diagram of I-controller is shown below.



→ The integral controller removes or reduces the steady error without the need for manual reset.

→ Hence the I-controller is sometimes called automatic reset.

→ The drawback in integral controller is that it may lead to oscillatory response of increasing or decreasing amplitude which is undesirable and the s/s may become unstable.

(3) proportional plus integral controller (PI - controllers)

The proportional plus integral controller (PI-controller) produces an output signal consisting of two terms: one proportional to error signal and the other proportional to the integral of error signal.

In PI-controller, $u(t) \propto \left[e(t) + \int e(t)\,dt\right]$

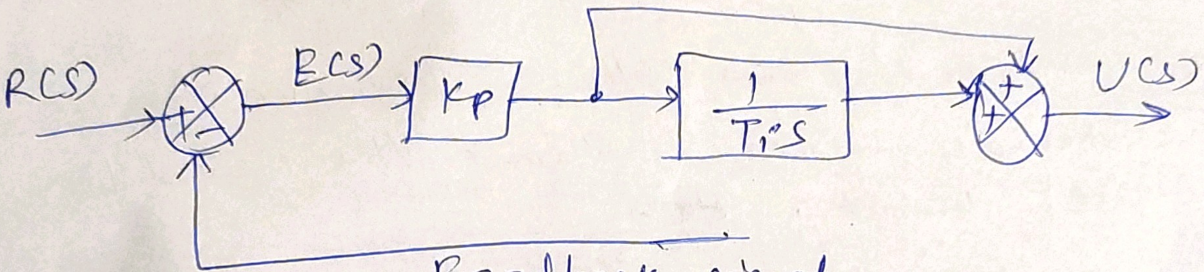
$$\therefore u(t) = K_p\, e(t) + \frac{K_p}{T_i}\int e(t)\,dt \quad \text{——— (1)}$$

where, K_p = proportional gain
T_i = Integral time.

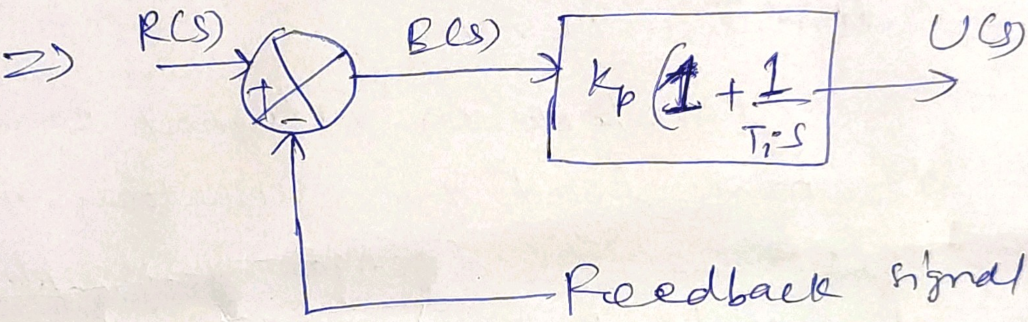
On taking laplace transform, with zero ~~initig~~ initial conditions,

$$U(s) = K_p E(s) + \frac{K_p}{T_i} \frac{E(s)}{s}$$

The block diagrame is shown below.



Feedback signal.



→ The advantages of both P-controller & I-controller are combined in PI-controller.

→ The proportional action increases the loop gain and makes the s/s less sensitive to variations of s/s parameters.

→ The integral action eliminates or reduces the steady state error.

→ The integral control action is adjusted by varying the integral time. The change in value of 'K_p' affects both the proportional and integral parts of control action. The inverse of the integral time 'T_i' is called the reset rate.

PROPORTIONAL PLUS DERIVATIVE CONTROLLER (PD-CONTROLLER):

The proportional plus derivative controller produces an output signal consisting of two terms; one proportional to error signal & the other proportional to the derivative of error signal.

In PD - controller,

$$u(t) \propto \left[e(t) + \frac{d}{dt} e(t) \right]$$

$$\therefore u(t) = K_p e(t) + K_p T_d \frac{d}{dt} e(t) \quad \text{--- (1)}$$

where, K_p = proportional gain

T_d = Derivative time

On taking Laplace transformation, with zero initial conditions, we get,

$$U(s) = K_p E(s) + K_p T_d s E(s)$$

$$\therefore \frac{U(s)}{E(s)} = K_p \left[1 + T_d s \right] \quad \text{--- (2)}$$

This equation (1), give the output of the PD - controller for the input E(s) & the eq^n (2) is the transfer function of PD - controller.

The block diagram of PD - controller is shown below.

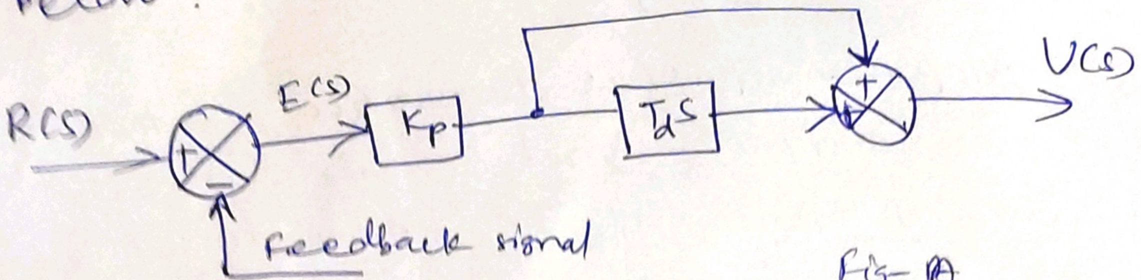


Fig-A

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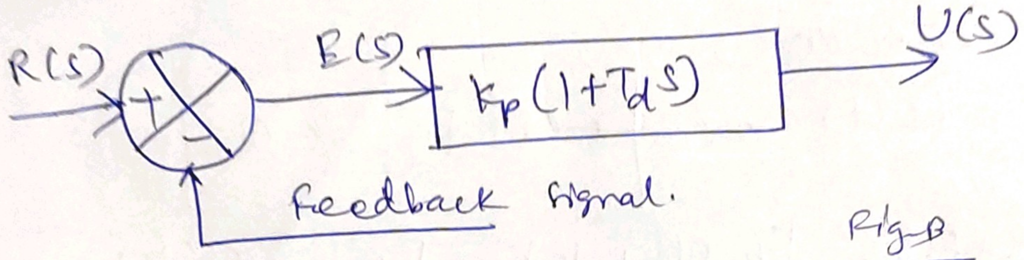


Fig-B

→ The derivative control acts on rate of change of error & not on the actual error signal. The derivative control action is effective only during transient periods & so it does not produce corrective measures for any constant error,

→ Hence derivative controller is never used alone, but it is employed in association with proportional & integral controllers.

→ The derivative controller does not affect the steady state error directly but anticipates the error, initiates an early corrective action & tends to increase the stability of the s/s.

→ Derivative control action has an advantage of being anticipatory it has the disadvantage of that it amplifies noise signals and may cause a saturation effect in the actuator.

The derivative control action is adjusted by varying the derivative time. The change in the value of 'K_p' affects both the proportional and derivative parts of control action. The derivative control is also called <u>rate control</u>.

Proportional plus Integral plus derivative controller (PID - controller)

The PID - controller produces an output signal consisting of three terms: one proportional to error signal, another one proportional to integral of error signal & the third one proportional to derivative of error signal.

In PID controller,

$$u(t) \propto \left[e(t) + \int e(t)\,dt + \frac{d}{dt} e(t) \right]$$

$$\therefore u(t) = K_p e(t) + \frac{K_p}{T_i} \int e(t)\,dt + K_p T_d \frac{d}{dt} e(t) \quad \text{--- (1)}$$

where, K_p = Proportional gain

T_i = Integral time

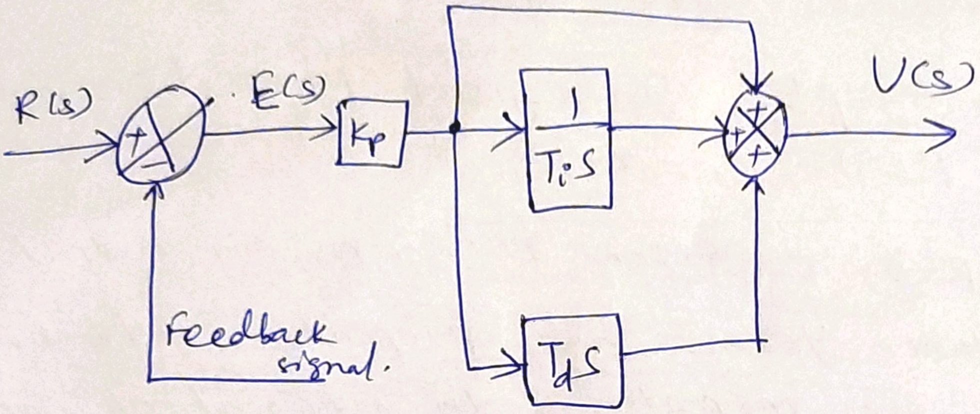
T_d = Derivative time.

On taking Laplace transform of eq^n (1) with zero initial cond^n, we get,

$$U(s) = K_p E(s) + \frac{K_p}{T_i} \frac{E(s)}{s} + K_p T_d\, s\, E(s)$$

$$\therefore \frac{U(s)}{E(s)} = K_p \left[1 + \frac{1}{T_i s} + T_d s \right] \quad \text{--- (2)}$$

The eqn (1) gives the output of the PID controller for the input E(s) & eqn (2) is the transfer function of the PID-controller. The block diagram of PID-controller is shown below.



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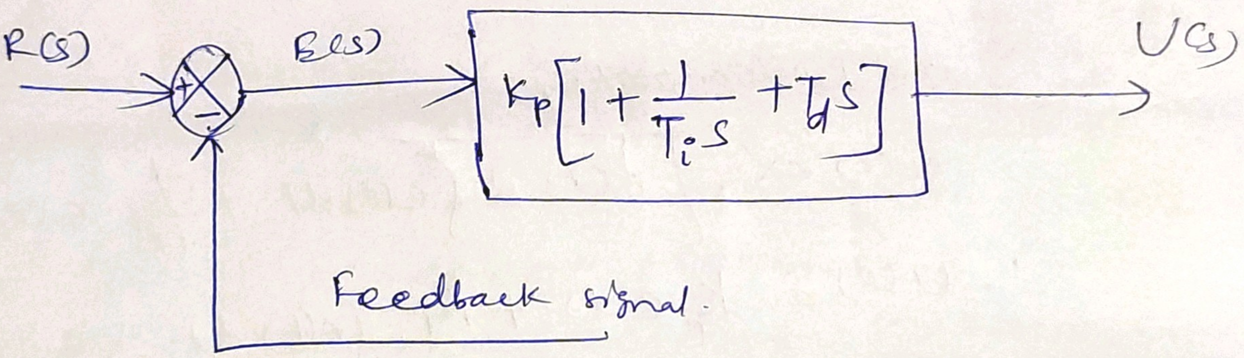

→ The combination of proportional control action, integral control action & derivative control action is called PID - control action.

→ The proportional controller stabilizes the gain but produces a steady state error.

→ The integral controller reduces or eliminates the steady state error.

→ The derivative controller reduces the rate of change of error.