Jharsuguda Engineering School, Jharsuguda

Lecture Notes
on
Control Systems & Component
6th Sem, ETC

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Chapter - 1

Fundamentals of Control System

Syllabus: Classification of control system, open loop system and closed loop system and its comparision. Effects of Feedback, standard test signals (Step, Ramp, Parabolic, Impulse functions), Servomechanism and Regulators (Regulating systems).

Control systems are an integral part of modern society. They play a vital role in our day to day life. Control systems finds applications in manufacturing process industries, satellites, guided missiles, navigation, biomedical engineering.

Basic Definations:

System: System is a group of physical components which gives proper output for a given input.

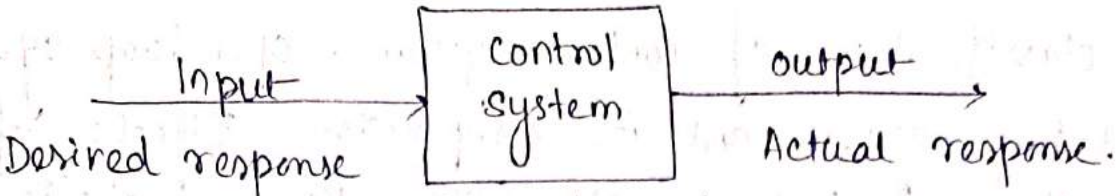
Control system: Control system is a group of physical components which gives controlled output for a given input.

Input: The excitation applied to a control system from an external energy source is usually known as input.

Output: The actual response that is obtained from a control system due to the application of the input is called output.

Plant or Process: It is defined as the portion of a system which is to be controlled or regulated. It is also called process.

Controller: It is an element within the system itself, or external to the system and it controls the plant or the process.



The input variable is generally known as reference input and output is generally called controlled output.

Disturbances: Disturbance is a signal which tends to adversely affect the value of the output of the system.

If such a disturbance is generated within the system itself, it is called an internal disturbance.

The system generated outside the system acting as a extra input to the system in addition to its normal input, affecting the output adversely is called the external disturbance.

Automatic control system: An automatic control system is one that maintains the actual output at the desired value in the presence of disturbance.

Process control system: An automatic control system in which the output is a variable such as, temperature, pressure, flow, liquid level is called a process control system.

Transducer: A transducer is a device which converts a signal from one form to another usually, the transducers applied in control systems convert a signal from any form to electrical form. This is because the electrical signal can be handled, amplified and transmitted easily.

Classification of control systems :

1. Linear and non-linear control systems :

For linear systems, the principle of superposition applies. Those systems for which this principle doesnot apply are non-linear systems. Most reallife control system have non-linear characteristics.

2. Time invariant and Time varying control systems :

A time invariant control system is one whose parameters do not vary with time. A time varying control system is a system in which one or more parameters vary with time. The response depends on the time at which an input is applied.

3. Continuous time and Discrete time control systems :

In a continuous time control system, all system variables are functions of a continuous time 't'. A discrete time control system involves one or more variables that are known only at discrete instance of time.

4. Single input Single output and Multiple input Multiple output control systems :

A system may have one input and one output, such a system is called a single input, single output control system. some systems may have multiple inputs and multiple outputs.

5. Lumped - parameter and Distributed parameter control systems:

Control systems that can be described by ordinary differential equations are lumped parameter control systems, where as distributed - parameter control systems are those that may be described by partial differential equations.

6. Deterministic and stochastic control systems :

A control system is deterministic if the response to input is predictable and repeatable. If not, the control system is a stochastic control system.

Control system configurations :

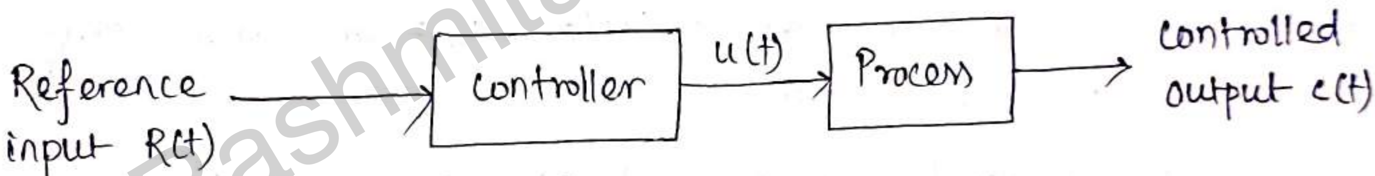
There are two major configurations of control systems,

(1) open loop system

(2) closed loop system.

1. Open loop system :

A system in which the control system action is totally independent of the output of the system is called as open loop system.



u(t) = Actuating signal

Reference input R(t) is applied to the controller which generates the actuating signal u(t) required to control the process which is to be controlled. Process is giving out the necessary desired controlled output c(t).

Advantages of open loop system :

(1) They are simple in construction and design.

(2) They are economical.

(3) Easy for maintenance.

(4) Not much problems of stability.

(5) Convenient to use when output is difficult to measure.

Disadvantages of open loop system :

(1) Inaccurate and unreliable because accuracy is dependent on accuracy of calibration.

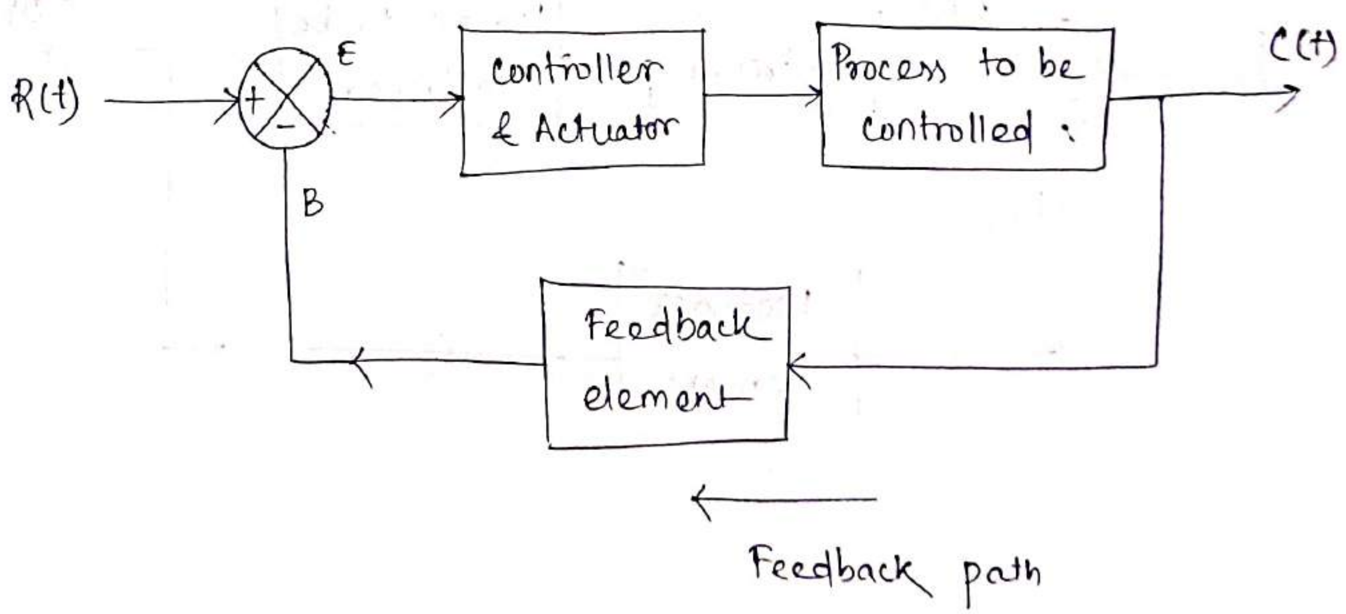
(2) Inaccurate results are obtained with parameter variations, internal disturbances.

(3) To maintain quality and accuracy recalibration of the controller is necessary for time to time.

2. Closed loop control system :

A system in which the controlling action is somehow dependent on the output is called closed loop control system. Such system uses a feedback.

A part of the output is feedback or connected to the input is feedback is that property of a system which permits the output to be compared with the reference input so that appropriate controlling action can be decided.

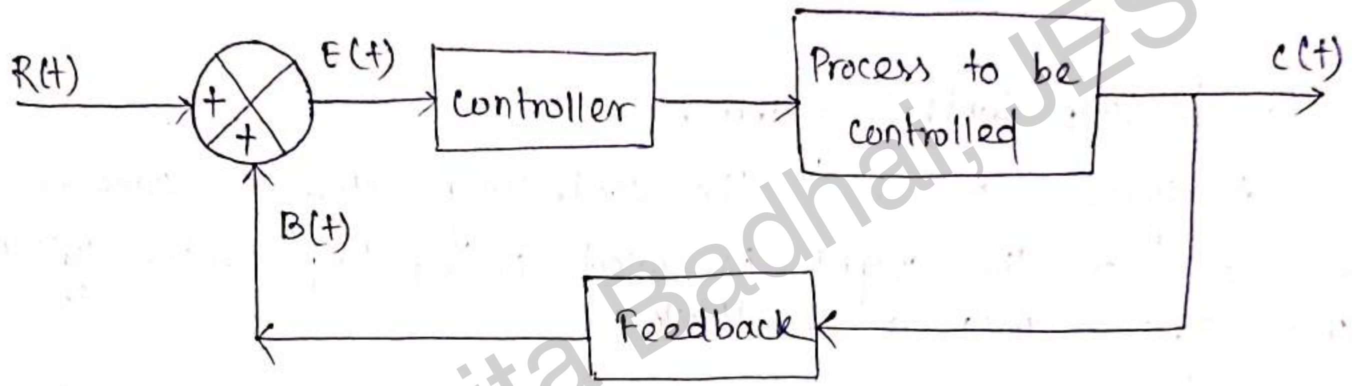


R(t) – Reference input
c(t) – controlled output
B – Feedback signal
E – Error signal.

There are two types of feedbacks

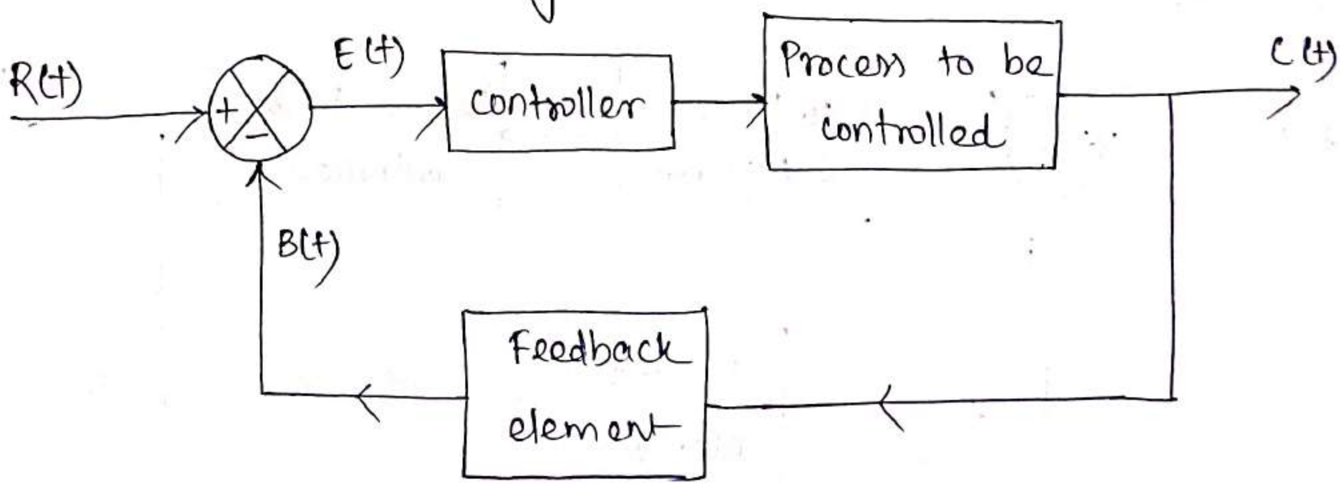
(1) Positive feedback (Regenerative feedback) :

When output is connected to input with '+' sign then it is called as positive feedback.



(2) Negative feedback (Degenerative feedback) :

When output is connected to input with '–' sign, then it is called negative feedback.



Advantages of closed loop system :

(1) Accuracy is very high as any error arising is corrected.

(2) It senses changes in output due to environmental or parametric changes or internal disturbances.

(3) Reduces effect of non-linearity.

(4) Increases Bandwidth.

Disadvantages of closed loop system:

(1) Complicated in design.

(2) Maintenance is costlier.

(3) System may become unstable.

Effect of Feedback :

When feedback is given, the error between system input and output is reduced. However, improvement of error is not only advantage.

The effects of feedback are

(1) Gain is reduced by a factor $\frac{G}{1 \pm GH}$.

(2) Reduction of parameter variation by a factor $1 \pm GH$.

(3) Improvement in sensitivity.

(4) Stability may be affected.

(5) Linearity of system improves.

(6) System bandwidth increases.

Comparision between open loop and closed loop system :

Open loop system	closed loop system.
1. Any change in output has no effect on the input i.e. feedback does not exists.	1. Changes in output, affects the input which is possible by the use of feedback.
2. Output is difficult to measure.	2. output measurement is necessary.
3. Feedback element is absent.	3. Feedback element is present.
4. Error detector is absent.	4. Error detector is necessary.
5. It is inaccurate and unreliable.	5. Highly accurate and reliable.
6. Highly sensitive to the disturbance.	6. less sensitive to the disturbance.
7. Highly sensitive to the environmental changes.	7. less sensitive to the environmental change.
8. Simple in construction and cheap.	8. Complex to design and hence costly.
9. These systems are non-linear.	9. These system are linear.
10. Eg: automatic washing machine, traffic light etc.	10. Eg: AC, control systems for temperature, pressure etc.

Standard Test Signals :

(Step , Ramp , parabolic , impulse function)

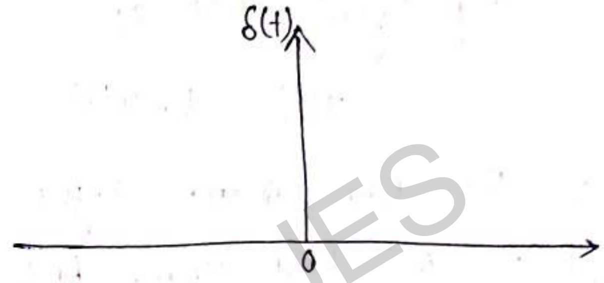
The standard test signals are impulse , step, ramp and parabolic. These signals are used to know the performance of the control systems using time response of the output.

1. Unit impulse signal :

A unit impulse signal , $\delta(t)$ is defined as

$$\delta(t) = 0 \quad \text{for} \quad t \neq 0$$

and $$\int_{0^-}^{0^+} \delta(t)\, dt = 1$$

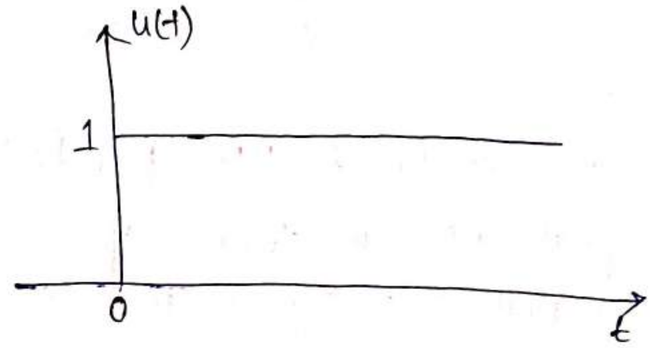


So, unit impulse signal exists only at 't' is equal to zero. The area of this signal under small interval of time around 't' is equal to zero is one. The value of unit impulse signal is zero for all other values of 't'.

2. Unit step signal :

A unit step signal , $u(t)$ is defined as

$$u(t) = 1 \; ; \; t \geq 0$$
$$= 0 \; ; \; t < 0$$

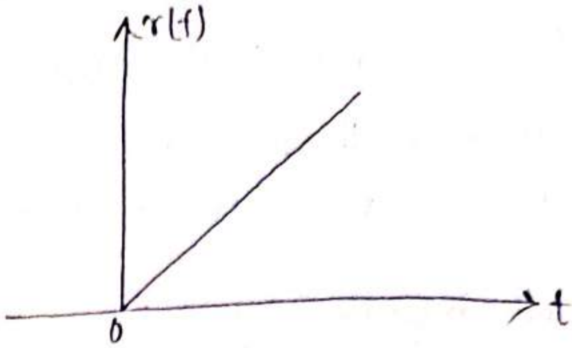


So, unit step signal exists for all values of 't' including zero. And its value is one during this interval. The value of unit step signal is zero for all negative values of 't'.

3. Unit Ramp Signal :

A unit ramp signal, $r(t)$ is defined as

$$r(t) = t \quad ; \; t \geq 0$$
$$= 0 \quad ; \; t < 0$$



We can write unit ramp signal $r(t)$ in terms of unit step signal $u(t)$ as

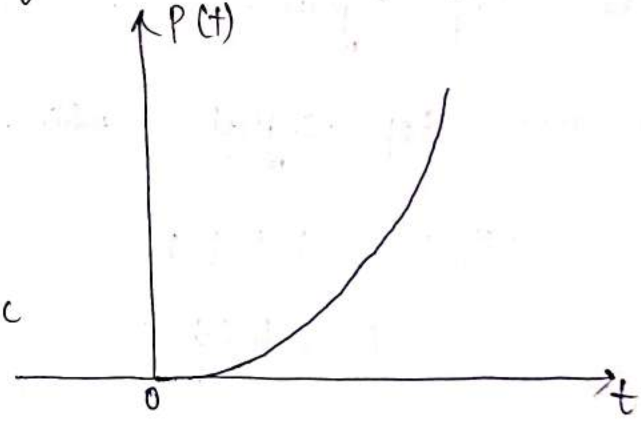
$$r(t) = t \cdot u(t)$$

So, the unit ramp signal exists for all positive values of 't' including zero. And its value increases linearly with respect to 't' during this interval. The value of unit ramp signal is zero for all negative values of 't'.

4. Unit parabolic signal :

A unit parabolic signal, $P(t)$ is defined as

$$P(t) = \frac{t^2}{2} \quad ; \; t \geq 0$$
$$= 0 \quad ; \; t < 0$$



We can write unit parabolic signal $p(t)$ in terms of the unit step signal $u(t)$ as

$$P(t) = \frac{t^2}{2} u(t)$$

So, the unit parabolic signal exists for all the positive values of 't' including zero. And its value increases non-linearly with respect to 't' during this interval.

The value of the unit parabolic signal is zero for all negative values of 't'.

Servomechanism :

A servomechanism is a control system in which the output is a mechanical position, or the rate of change of position or rate of change of velocity, ie. velocity or acceleration.

A robot, automatic positioning of anti-aircraft gun, radar, antenna are some examples of servomechanism.

Regulators (Regulating systems) :

Regulating system is a feedback control system in which for a present value of the reference input, the output is kept constant at its desired value.

A regulating system differ from a servomechanism in that the main function of a regulator is usually to maintain a constant output for a fixed input, while that of a servomechanism is mostly to cause the output of a system to follow the varying input.

A temperature regulator, speed governor are examples of regulating system.

Assignment - 1

1. Define the following terms
   (i) system
   (ii) control system
   (iii) Distance.

2. Define open loop and closed loop control system.

3. Draw the block diagram of a closed loop system and indicate the following on it.
   (i) Reference input
   (ii) controlled output
   (iii) Feedback signal
   (iv) error signal.

4. What is feedback? Explain the effect of feedback.

5. Explain standard test signals.

6. Explain the following terms giving suitable examples
   (i) Servomechanism
   (ii) Regulators

—— 0 ——

Chapter - 2

Transfer Functions

Syllabus :
Transfer function of a system and impulse response, properties, Advantages, disadvantages of transfer function, poles and zeros of transfer function, representation of poles and zeros on the s-plane, simple problems of transfer function of networks.

The transfer function is defined as the ratio of the laplace transform of the output response to the laplace transform of the input, provided that all the initial conditions are zero.

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

If $G(s)$ is the transfer function of the system, we can write mathematically

$$G(s) = \frac{\text{laplace transform of output}}{\text{laplace transform of input}} \Bigg|_{\text{all initial conditions are zero.}}$$

$$G(s) = \frac{C(s)}{R(s)} \Bigg|_{\text{all initial conditions are zero.}}$$

The transfer function can be represented as a block diagram as shown above, with the input on the left and output on the right side and the system transfer function is inside the block.

The input may be regarded as the "cause" and the output as the 'effect'. The block diagram is "unidirectional" since the effect cannot produce the cause.

Procedure for finding transfer functions of electric Networks :

The following procedure is used to find transfer function of an electric network :

(1) Draw the given network in the s-domain with each inductance L replaced by sL and each capacitance by $\frac{1}{sC}$

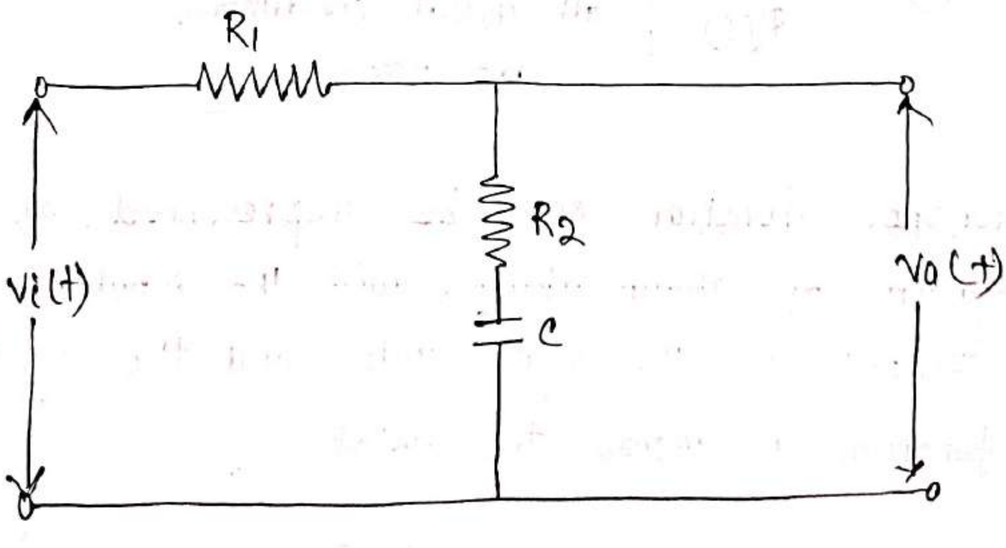
(2) Replace all sources and time variables with their laplace transforms. That is replace $v(t)$ and $i(t)$ by their laplace transforms $V(s)$ and $I(s)$ respectively.

(3) Use KCL, KVL, mesh analysis, node analysis to write network equations.

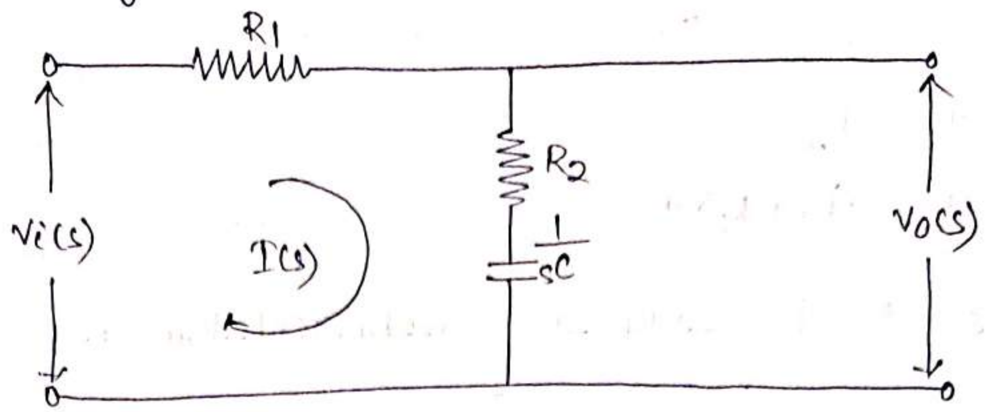
(4) Solve the simultaneous equations for the output.

(5) Form the transfer function.

Eg-1 : Determine the transfer function of the phase lag network.



Ans: The given network in s-domain is



By applying KVL in the left hand mesh

$$V_i(s) = R_1 I(s) + R_2 I(s) + \frac{1}{sC} I(s)$$

$$= \left[R_1 + R_2 + \frac{1}{sC} \right] I(s)$$

$$= \left[\frac{(R_1+R_2)sC + 1}{sC} \right] I(s)$$

By applying KVL in the right hand mesh

$$V_o(s) = R_2 I(s) + \frac{1}{sC} I(s)$$

$$= \left[\frac{R_2 sC + 1}{sC} \right] I(s)$$

Therefore the transfer function is given by

$$\frac{V_o(s)}{V_i(s)} = \frac{\left[\frac{R_2 sC + 1}{\cancel{sC}} \right] \cancel{I(s)}}{\left[\frac{(R_1+R_2)sC + 1}{\cancel{sC}} \right] \cancel{I(s)}}$$

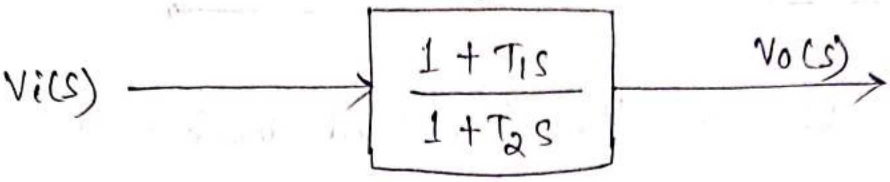
$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{sR_2C + 1}{(R_1+R_2)sC + 1}$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1 + T_1 s}{1 + T_2 s}$$

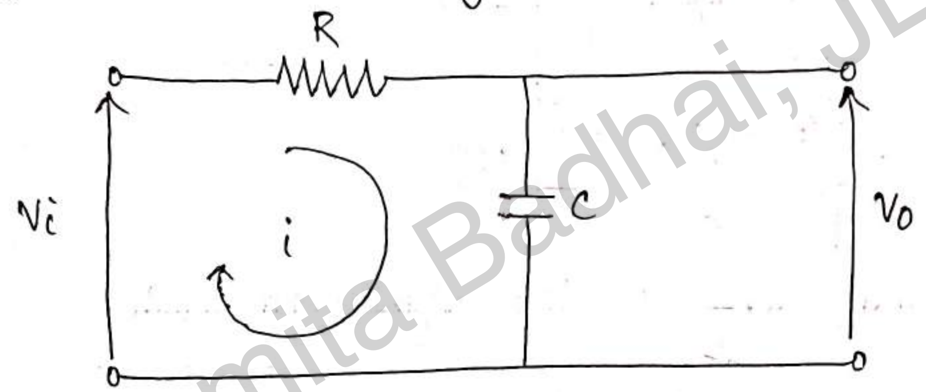
Where $T_1 = R_2 C$

$T_2 = (R_1 + R_2) C$

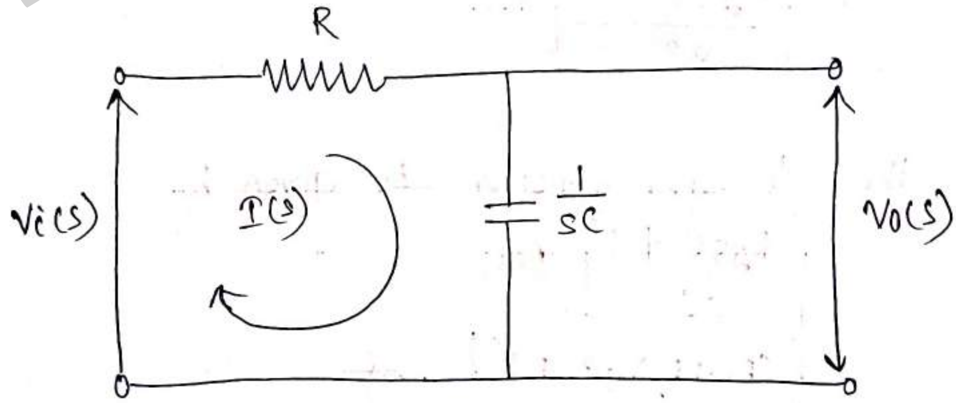
Then the block diagram representation is



Eg: 2 Determine the transfer function for the simple lag network assuming no external load.



Ans: The s-domain network is



Applying KVL in the left-hand mesh of

$$V_i(s) = R\,I(s) + \frac{1}{sC} I(s)$$

By applying KVL in the right-hand mesh

$$V_o(s) = \frac{1}{sC} I(s)$$

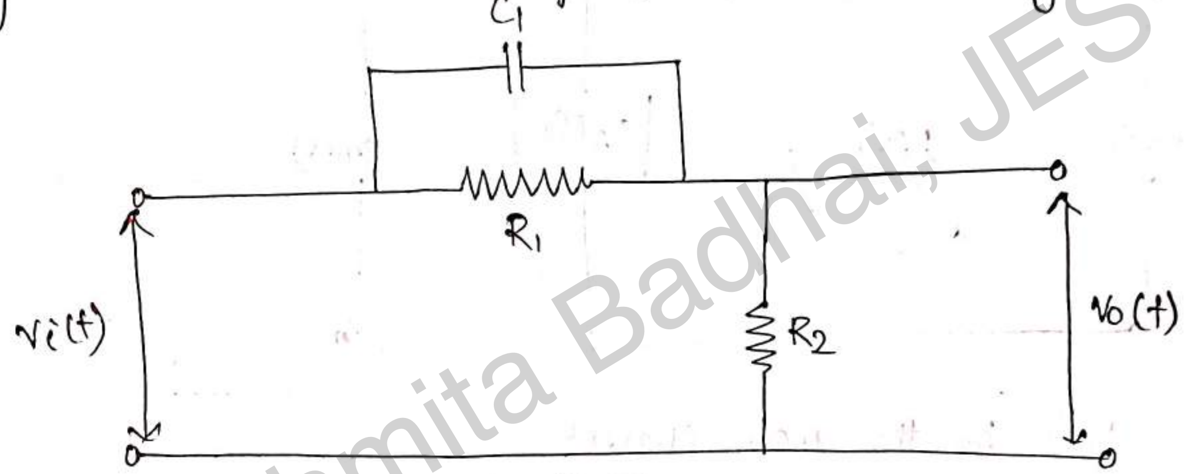
Therefore,

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}\cancel{I(s)}}{\left(R+\frac{1}{sC}\right)\cancel{I(s)}}$$

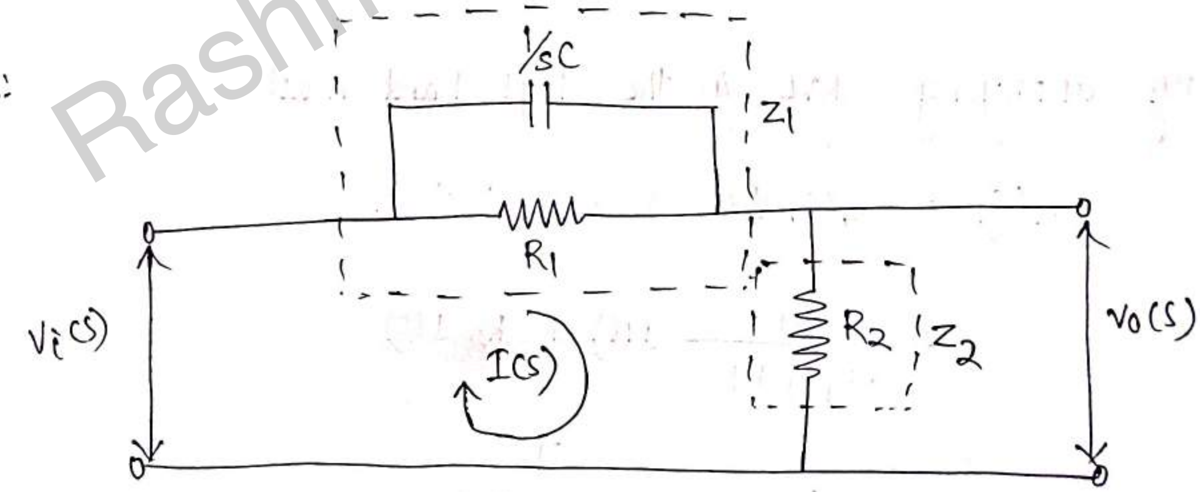
$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1/\cancel{sC}}{\frac{RsC+1}{\cancel{sC}}}$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{RsC+1}$$

Eg: 3 Determine the transfer function of the given network



Ans:



(Transformed Network)

In the transformed network

$$Z_1(s) = R_1 \parallel \frac{1}{sC_1}$$

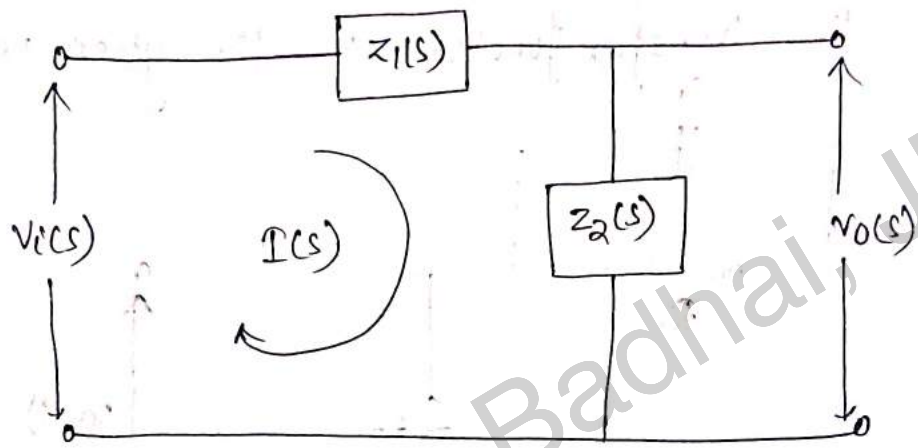
$$= \frac{R_1 \cdot \frac{1}{sC_1}}{R_1 + \frac{1}{sC}}$$

$$= \frac{R_1/\cancel{sC_1}}{\frac{sR_1C_1+1}{\cancel{sC_1}}}$$

$$= \frac{R_1}{sR_1C_1+1}$$

$$Z_2(s) = R_2$$

Then the equivalent circuit is



Let $I(s)$ be the mesh current

By applying KVL in the left hand mesh

$$V_i(s) = Z_1(s)\,I(s) + Z_2(s)\,I(s)$$

$$= \frac{R_1}{sR_1C_1+1}\,I(s) + R_2\,I(s)$$

$$= \left[\frac{R_1}{sR_1C_1+1} + R_2\right] I(s)$$

By applying KVL in the right hand mesh

$$V_o(s) = R_2\,I(s)$$

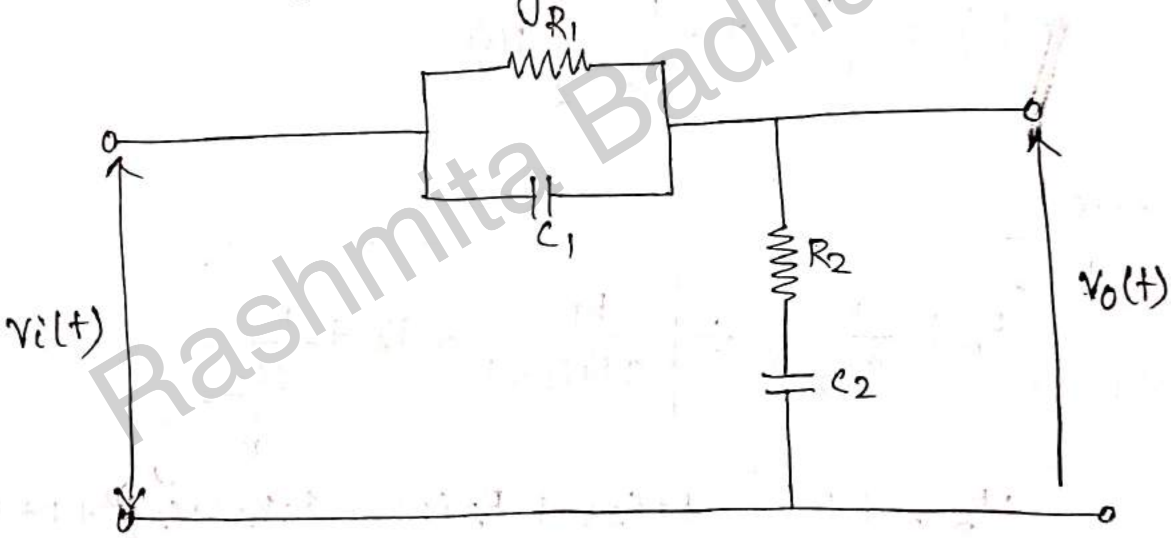
Transfer Function,

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 \cancel{I(s)}}{\left[\frac{R_1}{sR_1C_1+1} + R_2\right]\cancel{I(s)}}$$

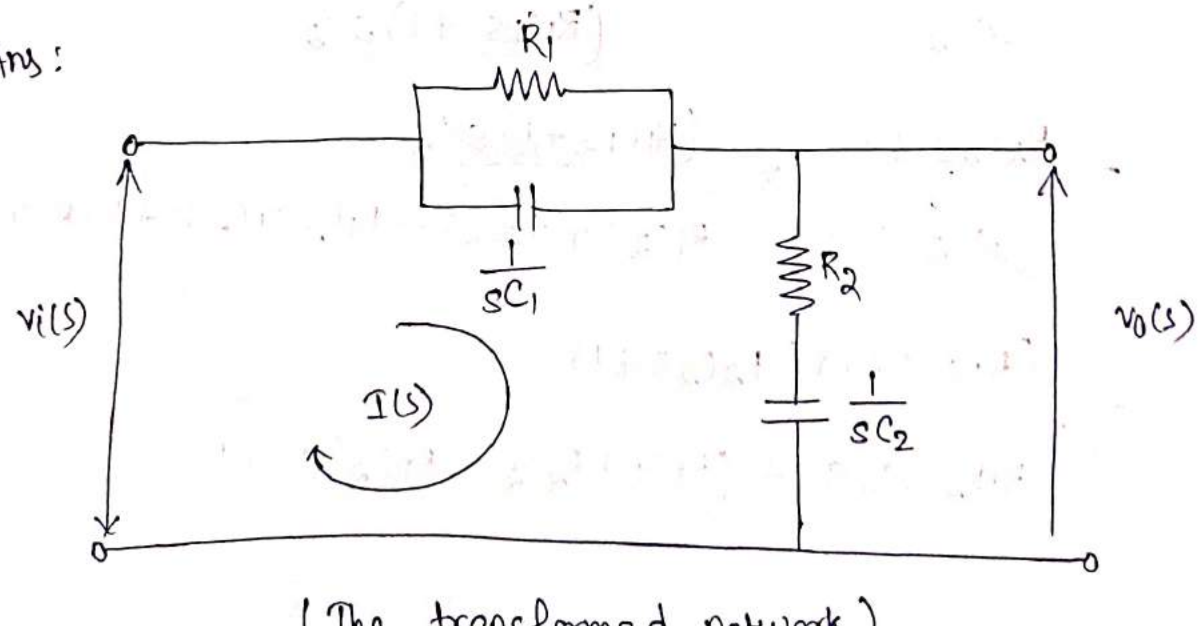
$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2}{\frac{R_1 + R_2(sR_1C_1+1)}{sR_1C_1+1}}$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2(1+sR_1C_1)}{R_1+R_2+sR_1R_2C_1}$$

Eg: 4 Determine the transfer function $\frac{V_o(s)}{V_i(s)}$ of the RC network of the Lag-lead compensator shown below.



Ans:



(The transformed network)

By applying KVL in the left hand mesh

$$V_i(s) = \left[\left(R_1 \parallel \frac{1}{sC_1}\right) + \left(R_2 + \frac{1}{sC_2}\right)\right] I(s)$$

$$= \left[\frac{\frac{R_1}{sC_1}}{R_1 + \frac{1}{sC_1}} + R_2 + \frac{1}{sC_2}\right] I(s)$$

$$= \left[\frac{R_1}{sR_1C_1 + 1} + R_2 + \frac{1}{sC_2}\right] I(s)$$

By applying KVL in the right hand mesh

$$V_o(s) = \left[R_2 + \frac{1}{s\cdot C_2}\right] I(s)$$

Therefore,

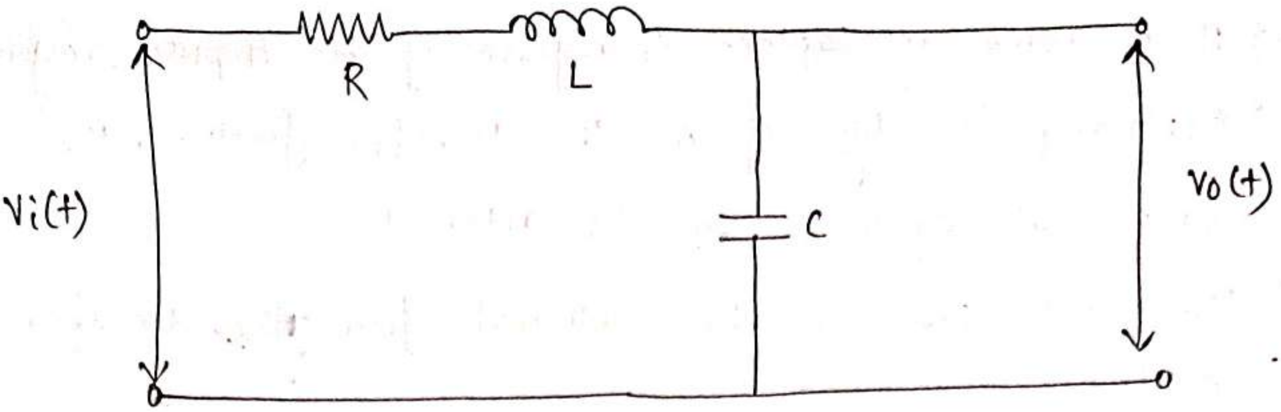
$$\frac{V_o(s)}{V_i(s)} = \left(R_2 + \frac{1}{sC_2}\right) \div \left[\frac{R_1}{sR_1C_1 + 1} + R_2 + \frac{1}{sC_2}\right]$$

$$= \frac{sR_2C_2 + 1}{sC_2} \div \frac{R_1C_2s + R_2C_2s + R_1R_2C_1C_2s^2 + 1 + R_1C_1s}{(R_1C_1s + 1)sC_2}$$

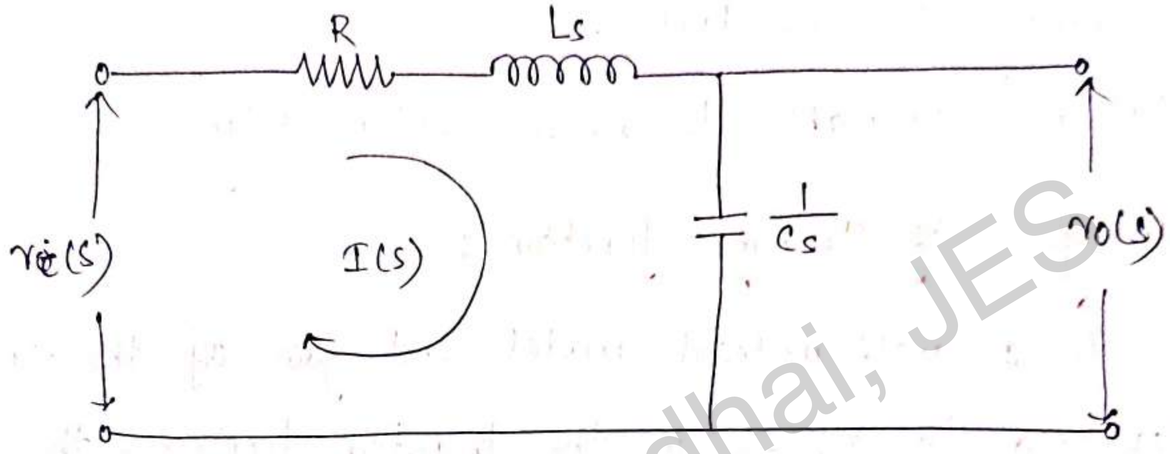
$$= \frac{R_2C_2s + 1}{\cancel{sC_2}} \times \frac{(R_1C_1s + 1)\cancel{sC_2}}{R_1C_2s + R_2C_2s + R_1R_2C_1C_2s^2 + 1 + R_1C_1s}$$

$$= \frac{(R_1C_1s + 1)(R_2C_2s + 1)}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_2C_2 + R_1C_2)s + 1}$$

Eg: 5 Determine the transfer function of the system



Ans :



(The transformed network)

By applying KVL in the left hand mesh

$$V_i(s) = \left[R + Ls + \frac{1}{Cs}\right] I(s)$$

By applying KVL in the right hand mesh

$$V_o(s) = \left[\frac{1}{Cs}\right] I(s)$$

Then the transfer function is

$$\frac{V_o(s)}{V_i(s)} = \frac{[1/Cs]\,\cancel{I(s)}}{\left[R + Ls + \frac{1}{Cs}\right]\cancel{I(s)}}$$

$$= \frac{1/\cancel{Cs}}{\dfrac{RCs + LCs^2 + 1}{\cancel{Cs}}} = \frac{1}{LCs^2 + RCs + 1}$$

Properties of Transfer function :

(1) Zero initial condition.

(2) It is same as laplace transform of its impulse response

(3) Replacing 's' by $\frac{d}{dt}$ in the transfer function, the differential equation can be obtained.

(4) Poles and zeros can be obtained from the transfer function.

(5) Stability can be known.

(6) Can be applicable to linear system only.

Advantages of Transfer function :

(1) It is a mathematical model and gain of the system.

(2) Replacing 's' by $\frac{d}{dt}$ in the transfer function, the differential equation can be obtained.

(3) Poles and zeros can be obtained from the transfer function.

(4) Stability can be known.

(5) Impulse response can be found.

Disadvantages of Transfer function :

(1) Applicable only to linear system.

(2) Not applicable if initial condition cannot be neglected.

(3) It gives no information about the actual structure of a physical system.

Impulse Response :

In a control system, when there is a single i/p of unit impulse function, then there will be some response of the linear system.

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow C(s)$$

$$G(s) = \frac{C(s)}{R(s)}$$

$$R(s) = \mathcal{L}[\delta(t)] = 1$$

The laplace Transform of the i/p will be $R(s) = 1$

$$C(s) = G(s) \cdot R(s)$$

$$\Rightarrow C(s) = G(s) \cdot 1$$

$$\Rightarrow C(s) = G(s)$$

The Laplace transform of the system output will be simply the "Transfer function" of the system.

Taking laplace inverse

$$\mathcal{L}^{-1} \rightarrow c(t) = G(t)$$

Here $G(t)$ will be impulse of the linear system.

This is called Weighing Function. Hence, Laplace Transfer of the impulse response is the Transfer function of the system itself.

Poles and zeroes of Transfer Function :

$$G(s) = \frac{K(s-z_1)(s-z_2) \cdots \cdots (s-z_m)}{(s-p_1)(s-p_2) \cdots \cdots (s-p_n)}$$

Where, K is known as gain factor of the transfer function.

$z_1, z_2, \ldots z_m$ are called zeros.

$p_1, p_2, \ldots p_n$ are called poles.

Number of poles 'n' will always be greater than the number of zeros 'm'.

Eg: 1 Obtain the pole-zero map of the following transfer function.

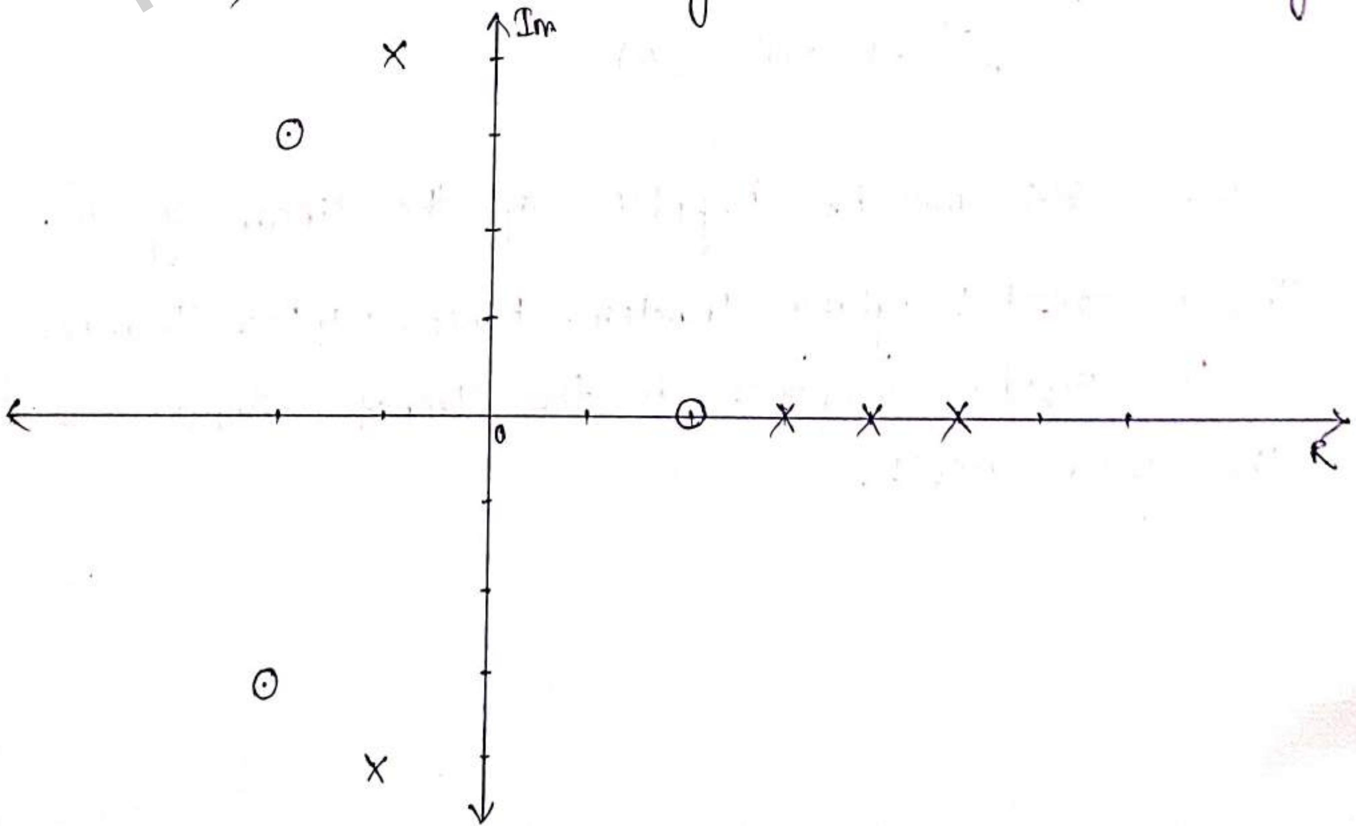
$$G(s) = \frac{(s-2)(s+2+j4)(s+2-j4)}{(s-3)(s-4)(s-5)(s+1+5j)(s+1-j5)}$$

Ans: Poles and zeros of the transfer function are

zeros : $s=2$, $s=-2-j4$, $s=-2+j4$

Poles : $s=3$, $s=4$, $s=5$, $s=-1-j5$, $s=-1+j5$

where as, Pole is denoted by 'X' and zero is denoted by 'O'.



Eg: 2 Obtain the pole-zero map of the following transfer function.

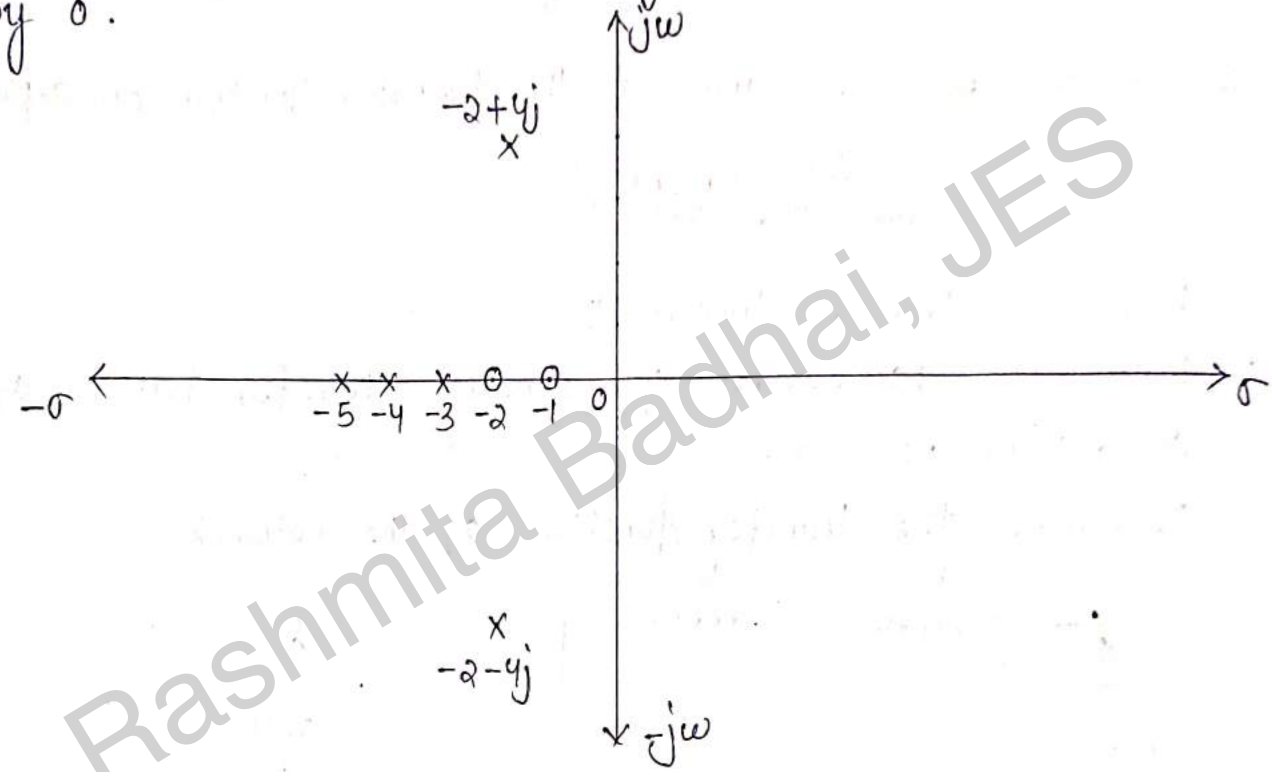
$$G(s) = \frac{(s+1)(s+2)}{(s+3)(s+4)(s+5)(s+2-4j)(s+2+4j)}$$

Ans: Poles and zeros of a transfer function are

zeros : $-1, -2$

poles : $-3, -4, -5, -2+4j, -2-4j$

Where as, poles is denoted by 'x' and zeros are denoted by o.



Eg: 3 Obtain the pole-zero map of the following transfer function.

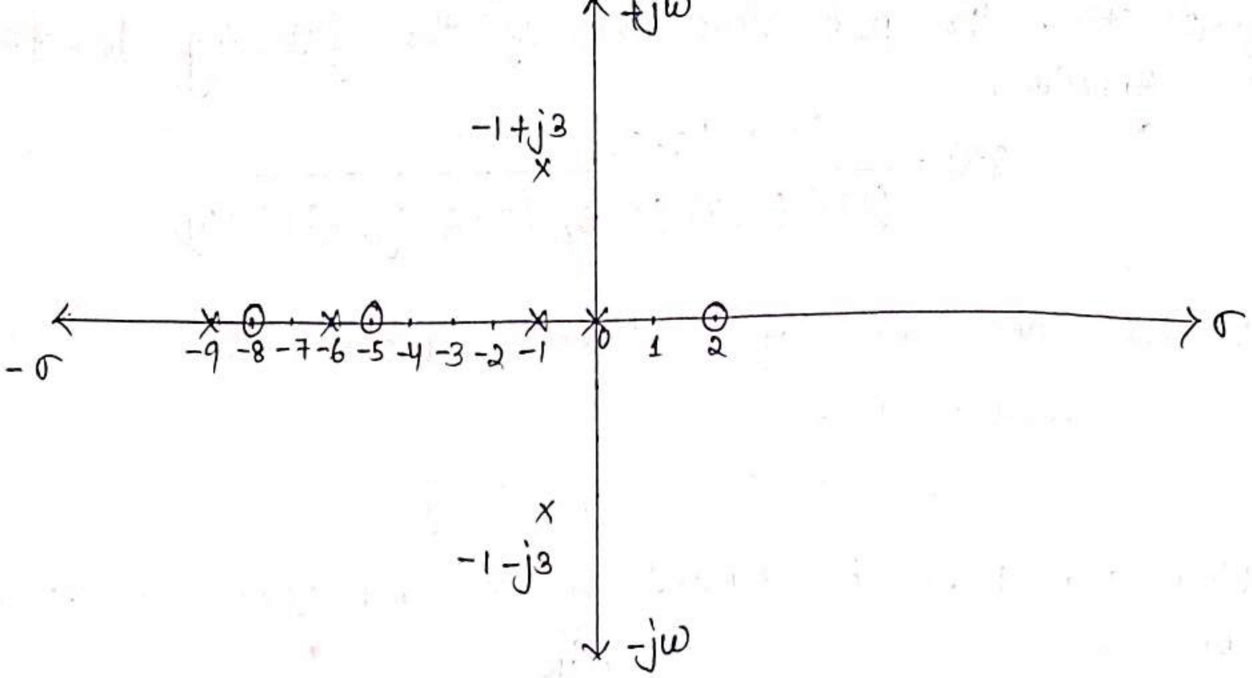
$$G(s) = \frac{(s-2)(s+5)(s+8)}{s\,(s+1)(s+6)(s+9)\,(s+1-j3)(s+1+j3)}$$

Ans: Poles and zeros of the transfer function are

zeros : $2, -5, -8$

poles : $0, -1, -6, -9, -1+j3, -1-j3$

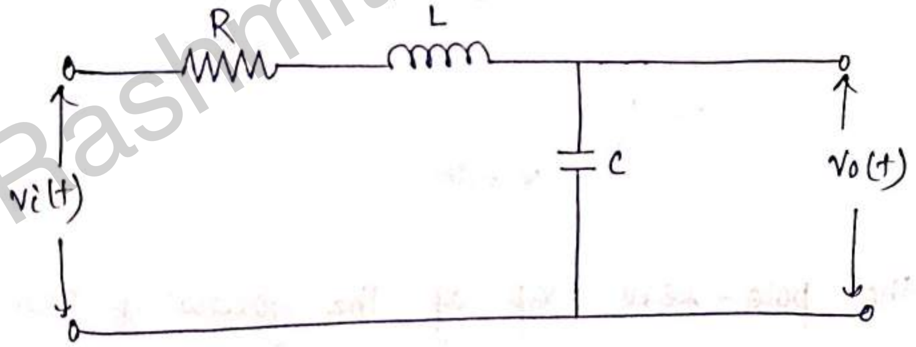
Where, poles is denoted by 'x' and zeros is denoted by 'o'.



This is the pole-zero map of the transfer function on s-plane.

Assignment - 2

1. What is Transfer function?
2. Write the procedure for finding transfer function of an electric network.
3. Determine the transfer function of the network



4. Write Properties, advantages and disadvantages of transfer function.
5. Obtain the pole-zero map of the following transfer function

$$G(s) = \frac{(s-2)(s+5)(s+8)}{s(s+1)(s+6)(s+9)(s+1-j3)(s+1+j3)}$$

Chapter-3

Control System Components & mathematical modelling of physical system

Syllabus:

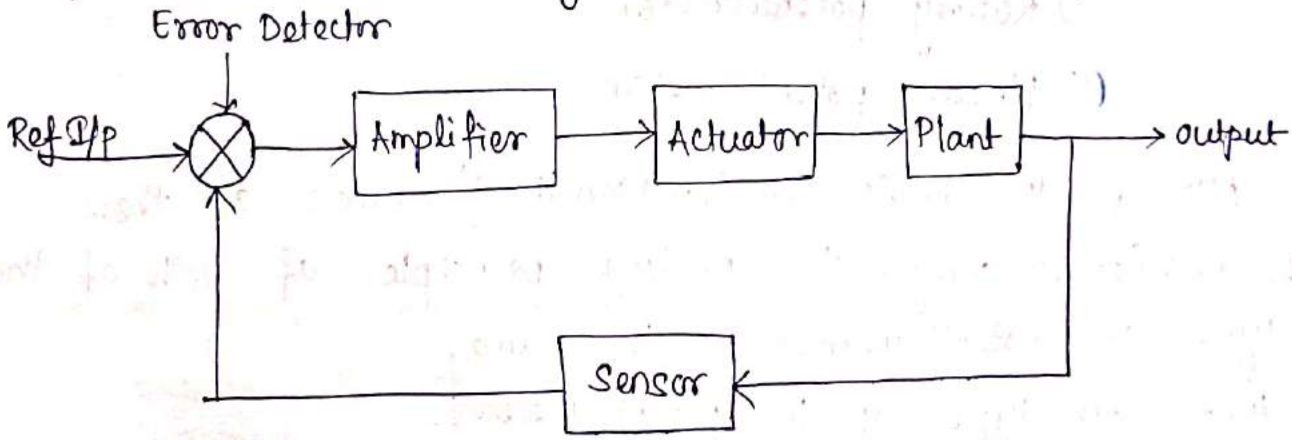
Components of control system, Potentiometer, synchros, Diode modulator & demodulator, DC motors, AC servomotors, Modelling of Electrical systems (R, L, C, Analogous systems)

A physical system is a collection of physical objects connected together to serve an objective. An idealized physical system is called a physical model. Once a physical model is obtained, the next step is to obtain mathematical model. When a mathematical model is solved for various input conditions, the result represents the dynamic behaviour of the system.

Components of control system:

The components of automatic control systems are

(1) Error detector
(2) Amplifier and controller
(3) Actuator
(4) Plant
(5) Sensor of feedback system.

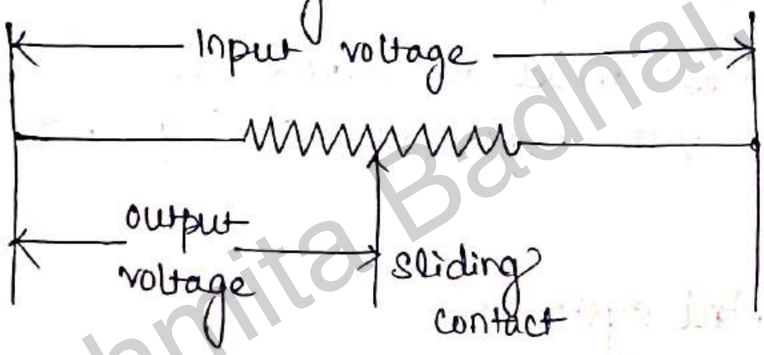


Potentiometer :

A potentiometer is defined as 3-terminal variable resistor in which the resistance is manually varied to control the flow of electric current. A potentiometer acts as an adjustable voltage divider.

A potentiometer is a passive electronic component. Potentiometers work by varying the position of a sliding contact across a uniform resistance.

In a potentiometer, the entire input voltage is applied across the whole length of the resistor, and the output voltage is the voltage drop between the fixed and sliding contact.



A potentiometer has the two terminals of the input source fixed to the end of the resistor. To adjust the output voltage the sliding contact gets moved along the resistor on the output side.

There are two main types of potentiometers:

(1) Rotary potentiometer

(2) Linear potentiometer.

Although the basic constructional features of these potentiometers vary, the working principle of both of these types of potentiometers is the same.

→ These are types of DC potentiometers.

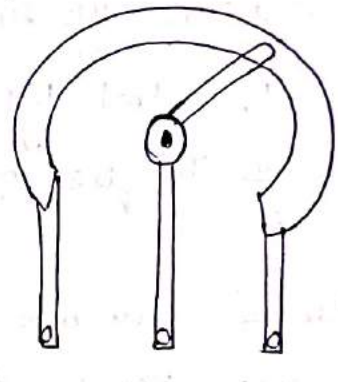
(1) Rotary Potentiometers :

The rotary type potentiometers are used for obtaining adjustable supply voltage to a part of electronic circuits and electrical circuits.

The volume controller of a radio transistor is an example of a rotary potentiometer where the rotary knob of the potentiometer controls the supply to the amplifier.

This type of potentiometer has two terminal contacts between which a uniform resistance is placed in a semi-circular pattern. The device also has a middle terminal which is connected to the resistance through a sliding contact attached with a rotary knob. By rotating the knob one can move the sliding contact on the semicircular resistance. The voltage is taken between a resistance end contact and the sliding contact.

There are many more uses of rotary type potentiometer where smooth voltage control is required.



2. Linear Potentiometers :

The linear potentiometer is basically the same but the only difference is that here instead of rotary movement the sliding contact gets moved on the resistor linearly.

Here two ends of a straight resistor are connected across the source voltage. A sliding contact can be slide on the resistor through a track attached along with the resistor. The terminal connected to the sliding is connected to one end of the output circuit and one of the terminals of the resistor is connected to the other end of the output circuit.

This is commonly used in the equalizer of music and sound mixing systems.

Synchros:

The synchros are the type of transducer which transforms the angular position of the shaft into an electric signal. It is used as an error detector and as a rotary position sensor.

The error occurs in the system because of the misalignment of the shaft. The transmitter and the control transformer are the main parts of the synchro.

The synchro system is of two types. They are

1. Control type synchro
2. Torque Transmission Type synchro

The Torque transmission type synchros has small output torque, and hence they are used for running the very light load like a pointer.

The control type synchro is used for driving the large loads. The control synchros is used for error detection in positional control system.

Their system consists of two units. They are

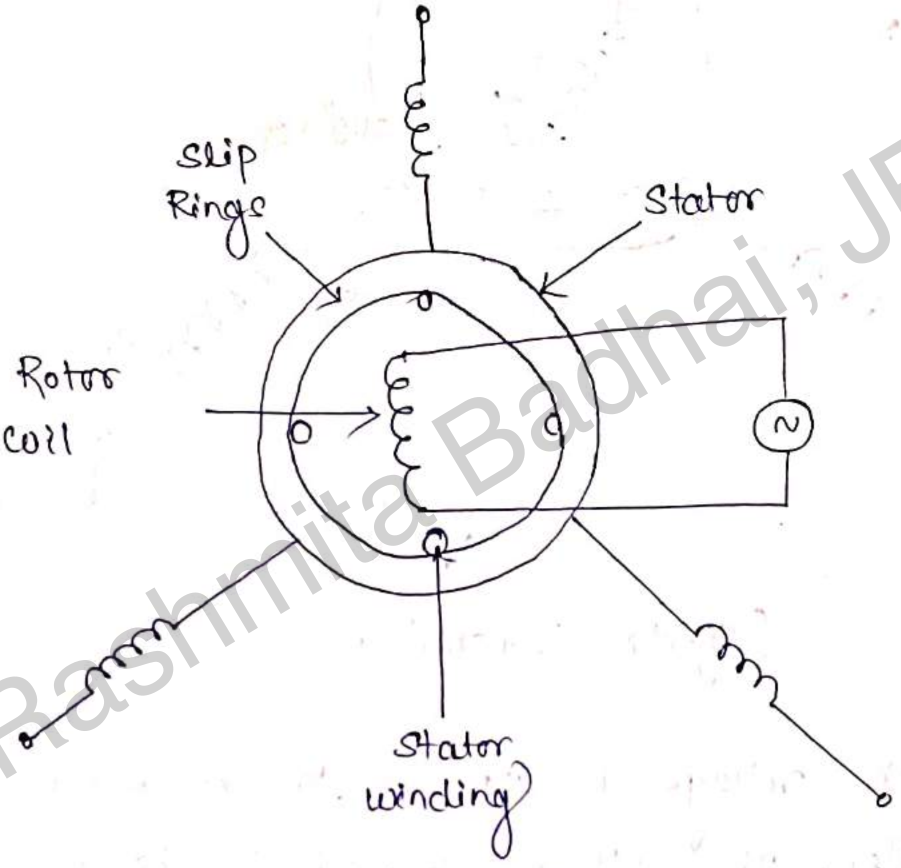
1. Synchro Transmitter
2. Synchro Receiver.

The synchro always work with these two parts.

1. Synchros Transmitter :—

The construction of synchro-transmitter is similar to three phase alternator. The stator of the synchros is made of steel for reducing the iron losses.

The starter is slotted for housing the three phase windings. The axis of the stator winding is kept 120° apart from each other.



(Constructional Feature of Synchros Transmitter)

The AC voltage is applied to the rotor of the transmitter and it is expressed as

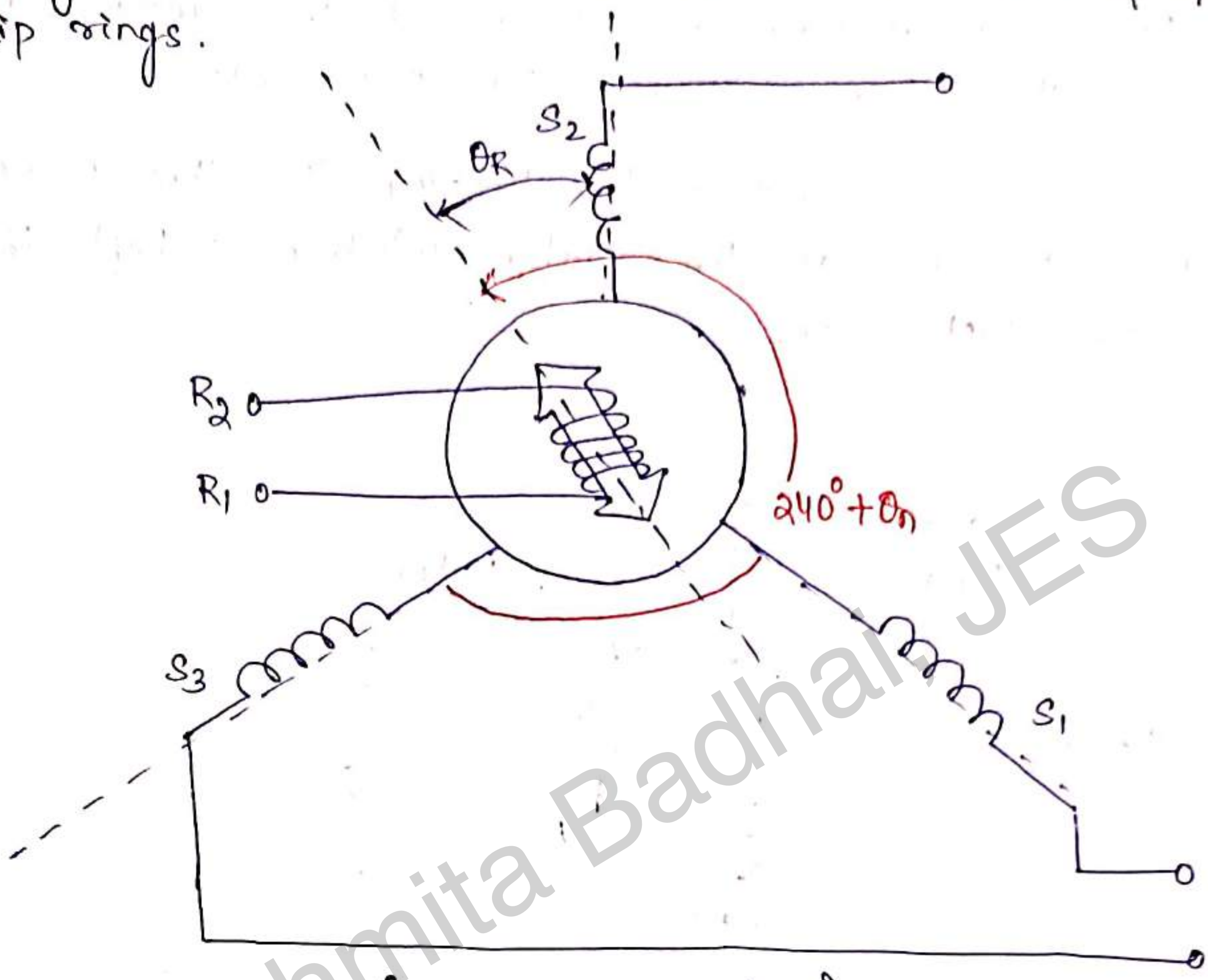
$$V_r = \sqrt{2} \sin w_c t$$

where;

$V_r \rightarrow$ r.m.s value of rotor voltage

$w_c \rightarrow$ carrier frequency.

The coils of the stator windings are connected in star. The rotor of the synchros is a dumbbell in shape, and a concentric coil is wound on it. The AC voltage is applied to the rotor with the help of slip rings.



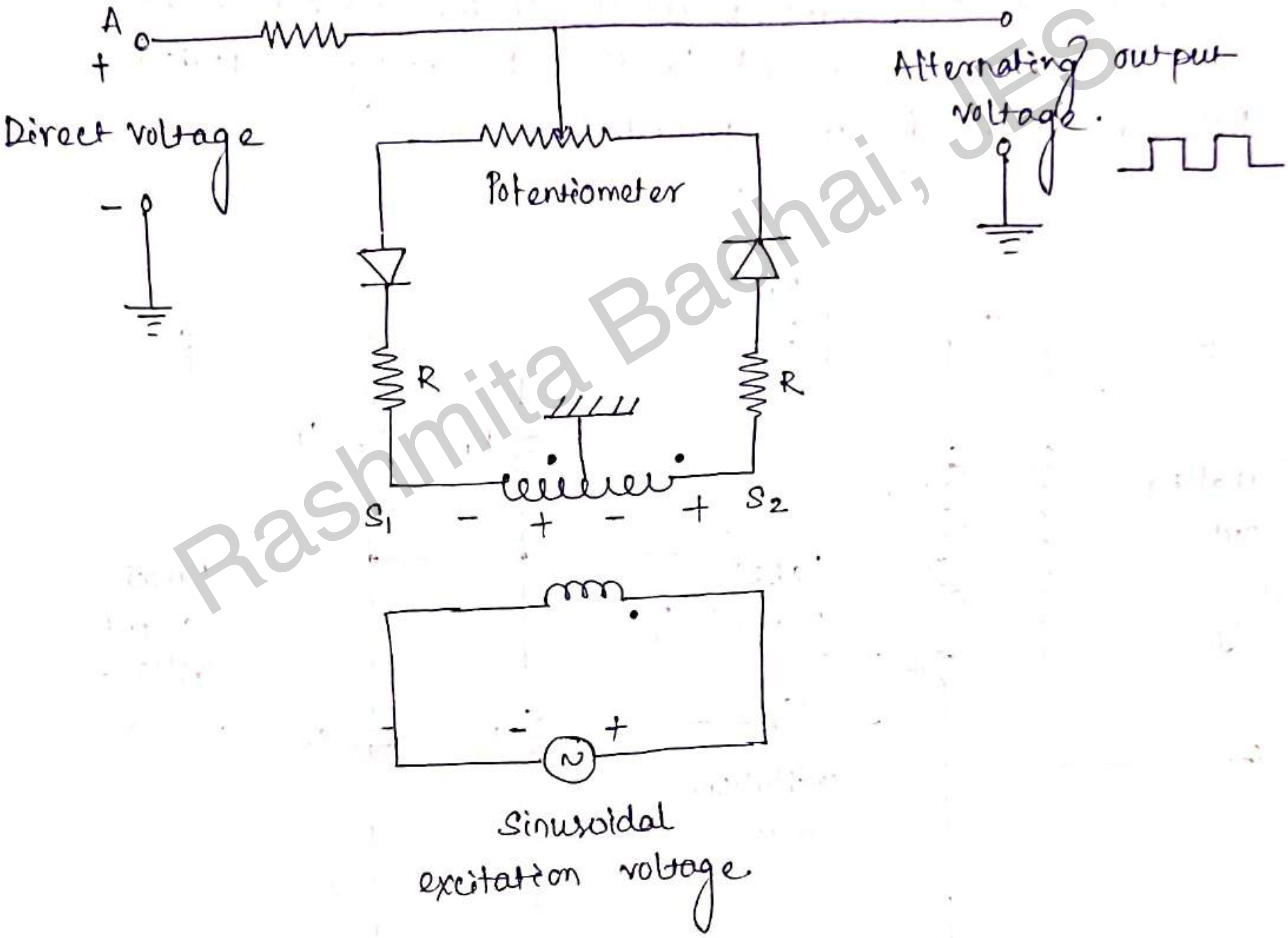
(Synchro transmitter)

Consider the voltage is applied to the rotor of the transmitter. The voltage applied to the rotor induces the magnetizing current and an alternating flux along its axis. The voltage is induced in the stator winding because of the mutual induction between the rotor and stator flux. The flux linked in the stator winding is equal to the cosine of the angle between the rotor and stator. The voltage is induced in the stator winding.

Diode Modulator and Demodulator :—

Modulation is defined as the process of superimposing a low frequency signal on a high frequency carrier signal.

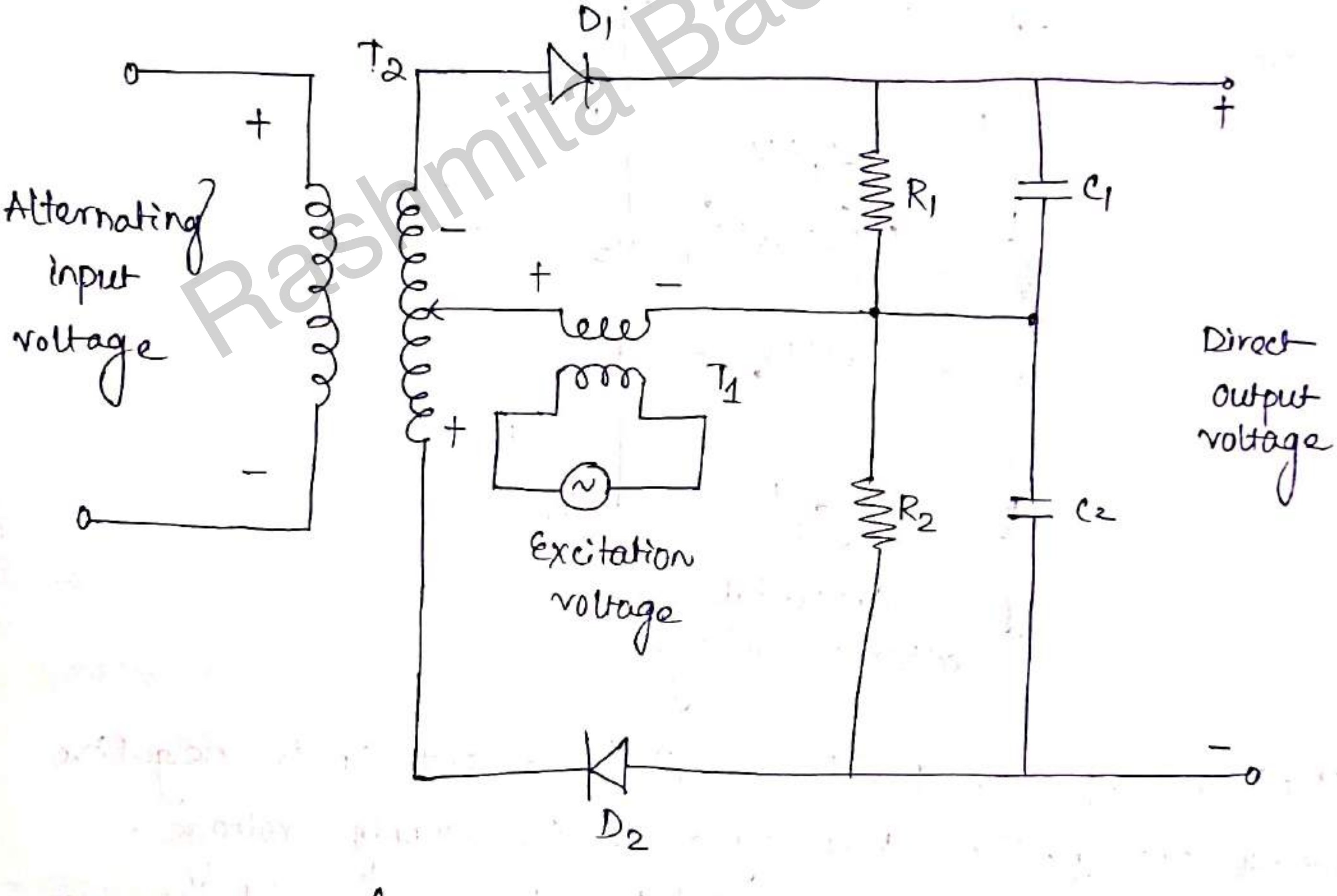
A diode modulation in which two diodes have been used. A sinusoidal supply of frequency ω_c was used here to excite the primary of the centre-tapped transformer. The centre point of the secondary winding of the transformer is grounded and hence the phase shift of the emfs included in the two halves of secondary will be 180°.



The end S_2 is positive while the end S_1 is negative during the positive half cycle of the supply voltage. which leads the diode D_1 and D_2 to be forward biased. Hence, a current circulates in the secondary of transformer.

The voltage between point B and ground becomes zero when the moving contact of potentiometer is located exactly at the midpoint. This is valid strictly under the assumption that the diodes are identical and the resistances in series with the diode are identical. Therefore, the output voltage becomes zero.

The end S_2 becomes negative while the end S_1 becomes positive during the negative half cycle of the supply voltage. The diodes D_1 and D_2 becomes reverse biased. The point B is disconnected from the transformer and it is at the same potential of A. Therefore, the output voltage is equal to the DC voltage applied between point A and ground.



(Diode Demodulator)

A diode demodulator in which the excitation voltage is alternating having frequency same as that of input signal. The input signal has been applied to the primary of the transformer (T_1). The input voltage is assumed to be in phase with the excitation voltage. During positive half cycle, the diode D_1 and D_2 will conduct due to the nature of the polarities of the induced emfs in the secondary of T_2.

The available voltage across D_1 is more than that of D_2 because there exists an additive secondary voltage in the closed path of D_1 and the subtractive voltage in the closed path of D_2. Hence the diode D_1 conducts more current than D_2, resulting in great charge storage in C_1 compared to that of C_2.

During the negative half cycle, the diode D_1 and D_2 will be reverse biased because the polarity of the secondary emfs is exactly opposite to the polarity. The capacitor C_1 and C_2 in this will function as filter and hence the direct output voltage will be maintained constant. The excitation voltage in the secondary of T_1 must be greater than the voltage induced in one half of secondary of T_1 to make the operation successful.

The diode D_2 conducts more current than the diode D_1 if the alternating voltage applied at the input is in phase opposition with the excitation voltage. The capacitor maintain the same direct voltage levels during the reactive half cycles.

DC Motors :—

A DC motor is an electrical machine that transforms electrical energy into mechanical energy, by creating a magnetic field that is powered by direct current.

When a DC Motor is powered, a magnetic field is created in its stator. The field attracts and repels magnects on the rotor; this causes the rotor to rotate. To keep the rotor continually rotating, the commutator that is attached to brushes connected to the power source supply current to the motors wire windings.

One of the reasons DC motors are prefered over other types of motors is their ability to precision control their speed, which is a necessity for industrial machinery.

DC motors are able to immediately start, stop, and reverse, which is an essential factor for controlling the operation of production equipment.

There are two types of DC Motors.

(1) Armature control (2) Field control.

Comparision between Armature control and Field control DC Motors

Armature Control	Field Control
→ In armature control large current is required as the source has to meet the full power requirement of the motor.	→ In field control the requirement of current is small.
→ Time constant is less.	→ Time constant will be more.
→ Efficiency is very high.	→ Efficiency is low

→ It is easy to provide constant field current.	→ It is difficult to provide constant field current.
→ Speed of response of the motor to change in current is fast.	→ Speed of response to change in current is low.

Advantages of DC Motor :—

(i) It has linear characteristics.

(ii) It is used for large power applications.

(iii) It is easier to control.

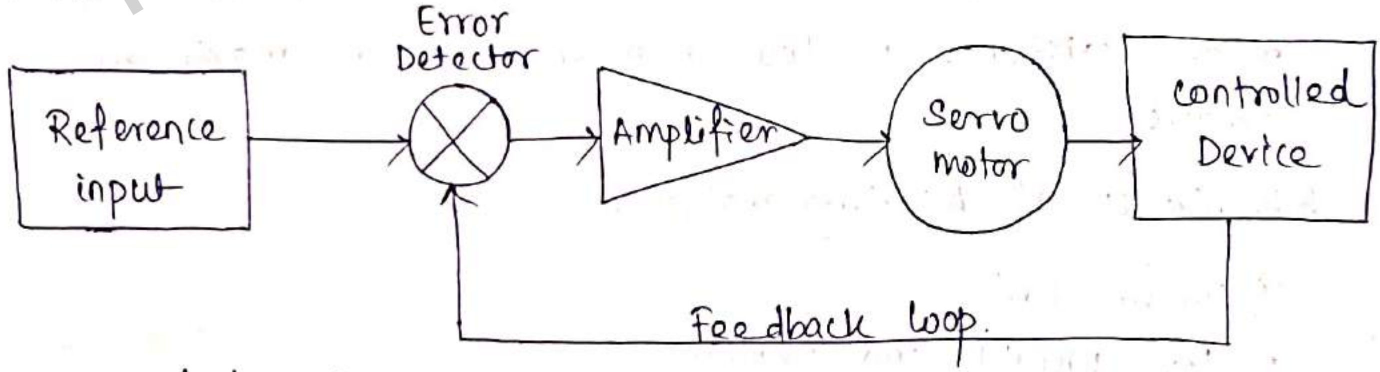
Disadvantages of DC Motor :—

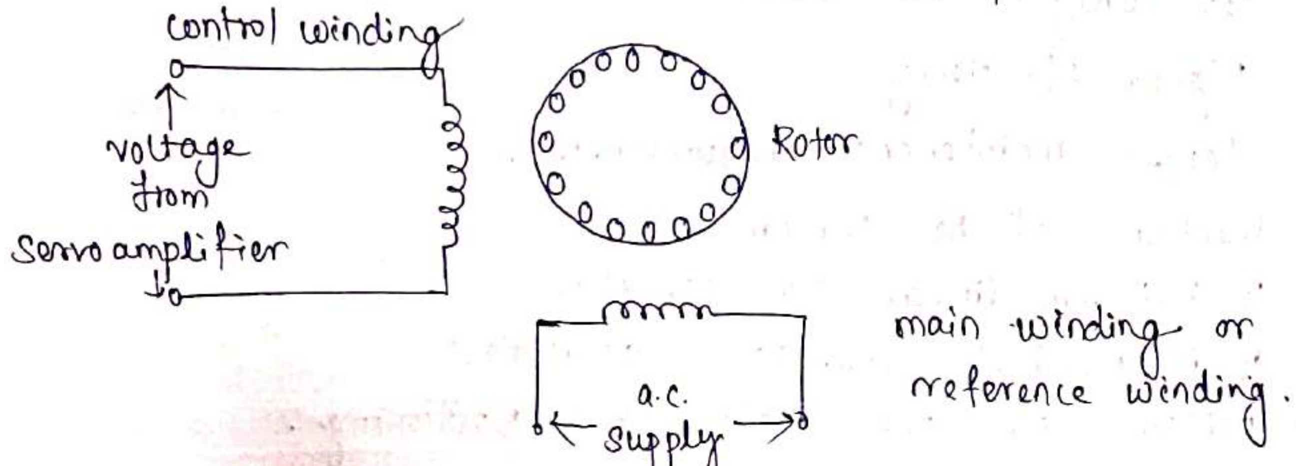
(i) It has low torque to volume ratio.

(ii) It has low torque to inertia ratio.

AC Servomotors :—

A two phase servomotor is commonly used in feedback control systems. In servo applications, an induction motor is required to produce rapid acceleration from stand still.





Constructional Features :

→ Squirrel cage rotor with Cu or Al conductor.
→ It has high rotor resistance.
→ Small diameter to length ratio to minimize inertia.
→ Two stator windings in space quadrature (One is called reference winding and the other is control winding)
→ The two voltages to stator windings must derived from same source.

Principle of operation :

(i) The two applied AC voltage to stators with a phase difference produce a rotating flux.

(ii) As this moving flux sweeps over the rotor conductors, small emf is induced in Rotor. Rotor being short circuited. current will flow and this current interacts with rotating flux to produce a torque in the rotor. This torque causes the rotor to turn so that it chases the rotating magnetic flux.

For induction motor in high power applications, rotor resistance is low in order to obtain maximum torque.

Advantages of AC servomotors :—

(i) Lower cost
(ii) less weight and inertia
(iii) Higher efficiency.
(iv) Fewer maintenance requirements.

Disadvantages of AC servomotors :—

(i) It has non-linear characteristics.
(ii) It is used for low power applications
(iii) Difficult for speed control and positioning.

Comparision between A.C. and D.C. Servomotor :

A.C. Servomotor	D.C. Servomotor
(1) Low power output.	(1) High power output.
(2) Efficiency is less.	(2) Efficiency is high.
(3) No brushes and slip rings hence maintenance free.	(3) Frequent maintenance required.
(4) No radio frequency noise.	(4) Brushes produce radio frequency noise.
(5) Smooth operation	(5) Noisy operation.

Application of servomotors :

(1) Radars.
(2) Electromechanical actuators
(3) Computers.
(4) Machine tools
(5) Tracking and guided system
(6) Process controllers.
(7) Robots.

Modelling of Electrical Systems (R, L, C, Analogous systems) :—

Analogous Systems :—

The circuit which represents the systems for which differential equations are similar in form are called analogous circuits.

The corresponding variables and parameters in two circuit represented by equations of the same form are called analogous.

Mechanical systems can be represented by electric networks or vice-versa either by force voltage analogy or force - current analogy.

Components of an electrical system:

There are three basic elements in an electrical system.

(i) resistor (R)

(ii) Inductor (L)

(iii) Capacitor (C)

Electrical systems are of two types.

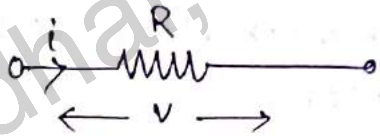
(i) voltage source electrical system

(ii) current source electrical system.

(i) voltage source electrical system:

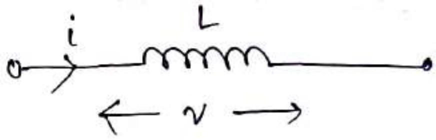
→ If 'i' is the current through a resistor and 'v' is the voltage drop in it, then,

$$v = Ri$$



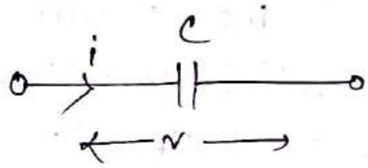
→ If 'i' is the current through an inductor and 'v' is the voltage ~~drop~~ developed in it, then

$$v = L\frac{di}{dt}$$



→ If (i) is the current through a capacitor and 'v' is the voltage developed in it, then

$$v = \frac{1}{C}\int i\,dt$$



(ii) current source electrical system:

→ If 'i' is the current through a resistor and 'v' is the voltage drop in it, then

$$i = \frac{v}{R}$$

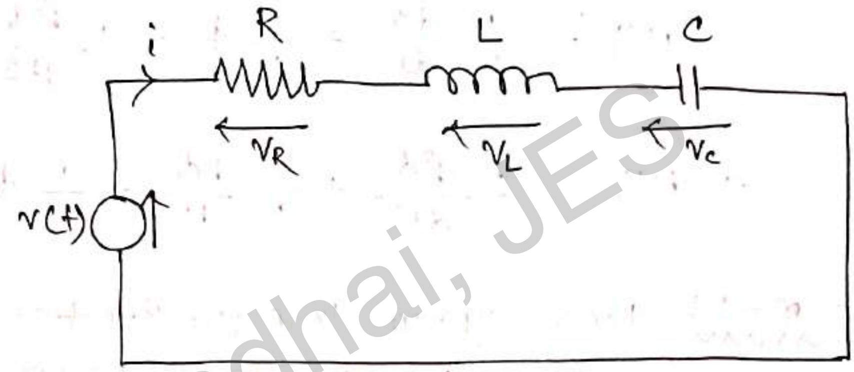
→ If 'i' is the current through an inductor and 'v' is the voltage developed in it, then

$$i = \frac{1}{L}\int v\,dt$$

→ If 'i' is the current through a capacitor and 'v' is the the voltage developed in it, then

$$i = C\frac{dv}{dt}$$

Series RLC Circuit :



This is a series RLC circuit.

By applying KVL,

$$v(t) = V_R + V_L + V_C$$

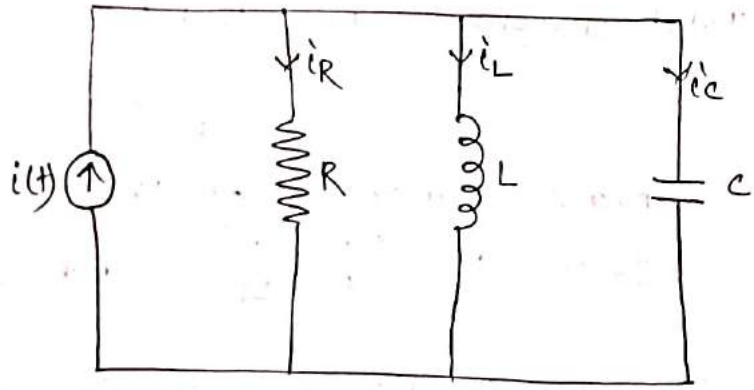
$$= Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$= Ri + L\frac{di}{dt} + \frac{1}{C}\int \frac{dq}{dt}\,dt \qquad \left\{\because i = \frac{dq}{dt}\right\}$$

$$= Ri + L\frac{di}{dt} + \frac{1}{C}\times q$$

$$\Rightarrow v(t) = Ri + L\frac{di}{dt} + \frac{q}{C}$$

Parallel RLC Circuit :



This is a parallel RLC circuit.

By applying KCL

$$i(t) = i_R + i_L + i_C$$

$$= \frac{v}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt}$$

If ϕ is the flux, then

$$v = \frac{d\phi}{dt}$$

and $$\phi = \int v\,dt$$

and $$\frac{dv}{dt} = \frac{d}{dt} \times \frac{d\phi}{dt}$$

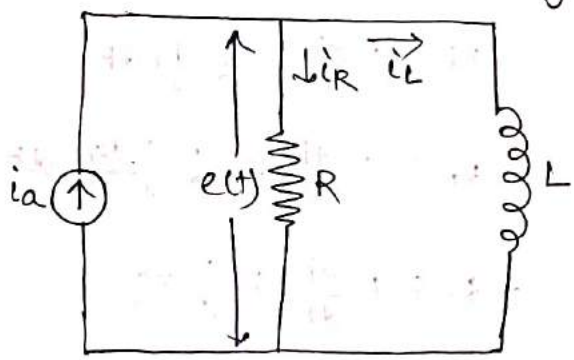
$$= \frac{d^2\phi}{dt^2}$$

By substituting the values, we get

$$i(t) = \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2}$$

$$\Rightarrow i(t) = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi$$

Problem-1: Find system transfer function between the inductance current to the source current in the following RL circuit.



Ans: Voltage across the resistance,

$$e(t) = i_R R$$

$$\Rightarrow i_R = \frac{e(t)}{R}$$

Voltage across the inductance

$$e(t) = L\frac{di_L}{dt}$$

$$\Rightarrow i_L = \frac{1}{L}\int e(t)\,dt$$

Total current $$i_a = i_R + i_L$$

$$= \frac{e(t)}{R} + \frac{1}{L}\int e(t)\,dt$$

Laplace transform of the current source

$$I_a(s) = E(s)\left(\frac{1}{R} + \frac{1}{Ls}\right) \quad \text{and} \quad I_L(s) = \frac{E(s)}{Ls}$$

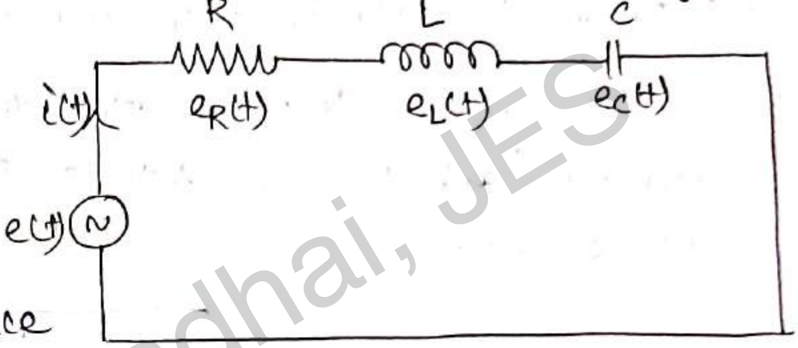
Transfer function between the inductance current to the source current is $$\frac{I_L(s)}{I_a(s)}$$

$$= \frac{E(s)/Ls}{E(s)\left(\frac{1}{R}+\frac{1}{Ls}\right)} = \frac{\cancel{E(s)}}{Ls} \times \frac{1}{\cancel{E(s)}\left(\frac{1}{R}+\frac{1}{Ls}\right)}$$

$$= \frac{1}{\frac{Ls}{R}+\frac{\cancel{Ls}}{\cancel{Ls}}}$$

$$= \frac{1}{\frac{L}{R}s+1} = \frac{1}{\tau s+1} \quad \left(\because \tau = \frac{L}{R} \text{ is the time constant.}\right)$$

Problem-2: Find system transfer function between ~~function~~ capacitance voltage to the source voltage in the following RLC circuit as shown in the figure.



Ans: voltage across the resistance,

$$e_R(t) = iR$$

voltage across the inductance

$$e_L(t) = L\frac{di}{dt}$$

voltage across the capacitance,

$$e_C(t) = \frac{1}{C}\int i\,dt$$

Total voltage $e(t) = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$

laplace transform of the voltage source

$$E(s) = I(s)\left(R + Ls + \frac{1}{Cs}\right)$$

Transfer function between capacitance voltage and source voltage is

$$\frac{E_C(s)}{E(s)} = \frac{\cancel{I(s)}/Cs}{\cancel{I(s)}\left(R+Ls+\frac{1}{Cs}\right)} = \frac{1}{Cs\left(R+Ls+\frac{1}{Cs}\right)}$$

$$= \frac{1}{RCs + LCs^2 + 1}$$

——— o ———

Assignment - 3

1. What are the components of control system?

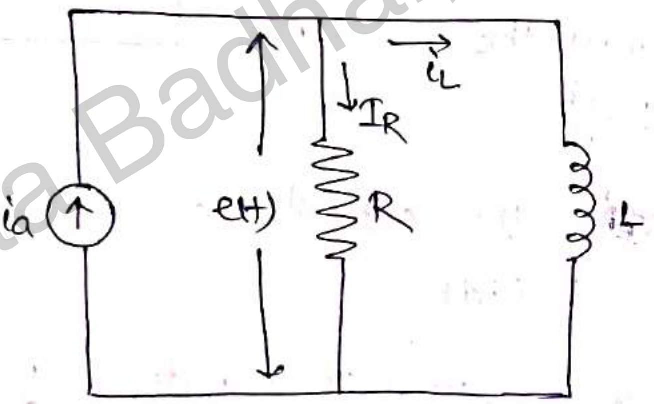
2. Define potentiometer. Explain its types.

3. Define synchros.

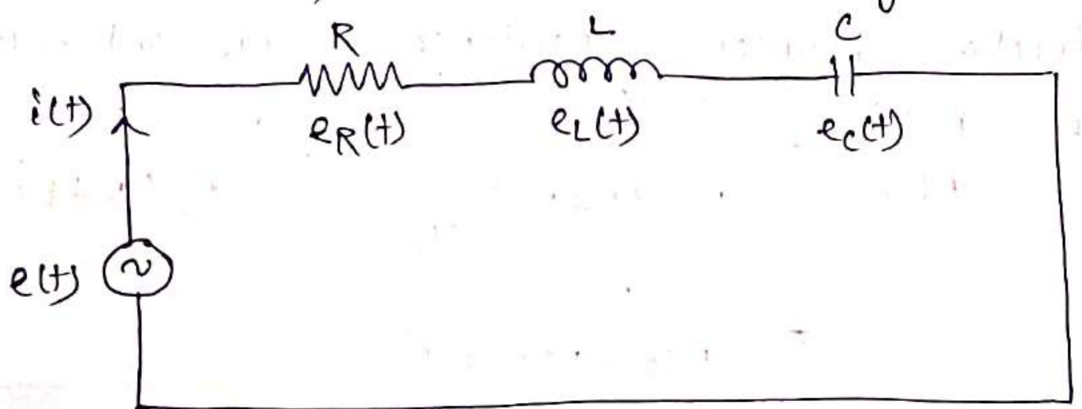
4. Define DC Motor. Differentiate between Armature control and Field control DC Motor.

5. Explain the construction and working of AC servomotors.

6. Find system transfer function between the inductance current ~~and~~ to the source current in the following RL circuit.



7. Find the system transfer function between capacitance voltage to the source voltage in the following RLC circuit as shown in the figure.



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Chapter - 4

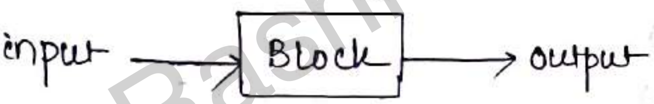
Block Diagram and Signal flow graph (SFG)

Syllabus : Defination of Basic elements of a Block diagram, Canonical form of closed loop systems, Rules for Block diagram reduction, Procedure for reduction of Block diagram, simple problem for equivalent transfer function, Basic defination in SFG and properties, Mason's Gain formula, Steps for solving signal flow graph, simple problems in signal flow graph for network.

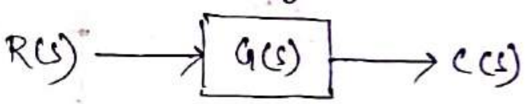
Defination of Basic elements of a Block diagram :-

Block Diagram :

Block diagram is the pictorial representation of each system and it makes easier to understand the system. The short hand pictorial representation the cause and response relationship between input and output of a physical system is known as block diagram.



(Block representation of a system)



(Block representation with gain of a system)

Output : The value of input multiplied by the gain of the system.

$$G(s) = \frac{C(s)}{R(s)}$$

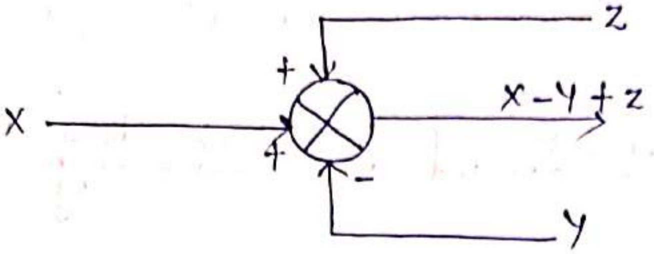
$$C(s) = G(s) \cdot R(s)$$

where, $G(s)$ = Gain
$C(s)$ = output
$R(s)$ = input

Arrows :

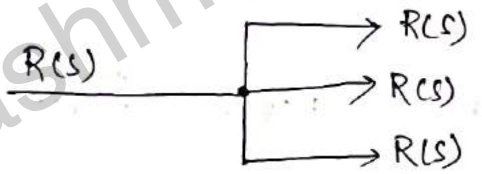
Block diagrams contain arrows which indicate the unidirectional flow of signals in these diagrams.

Summing point : A summing point is a point in a system where two or more signals are added algebraically. A circle with a cross is the symbol that indicates the summing operation. The plus or minus sign at each arrowhead indicates whether the signal is to be added or subtracted.



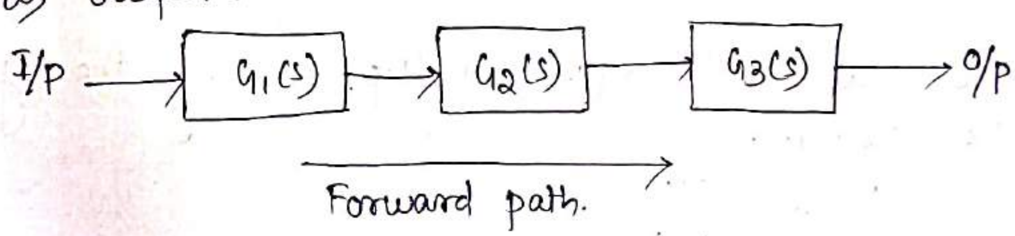
Takeoff points :

A takeoff point is the point in a system from which output signal from a block goes concurrently to other blocks or summing points.



Forward path :

It is the direction of signal flow from input towards output.

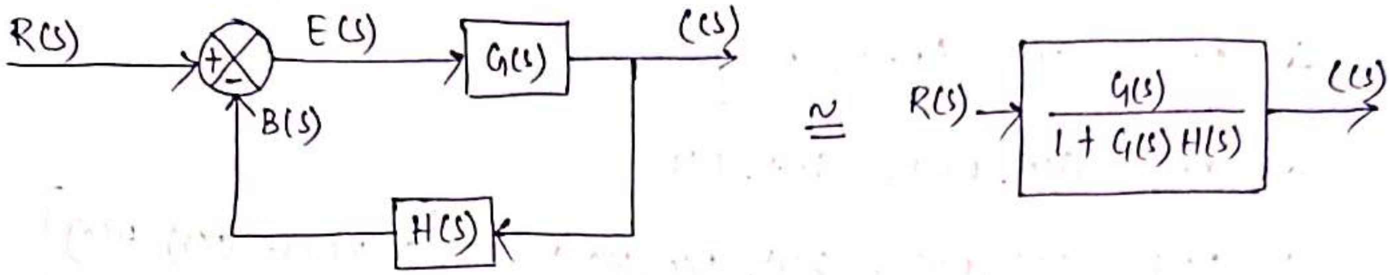


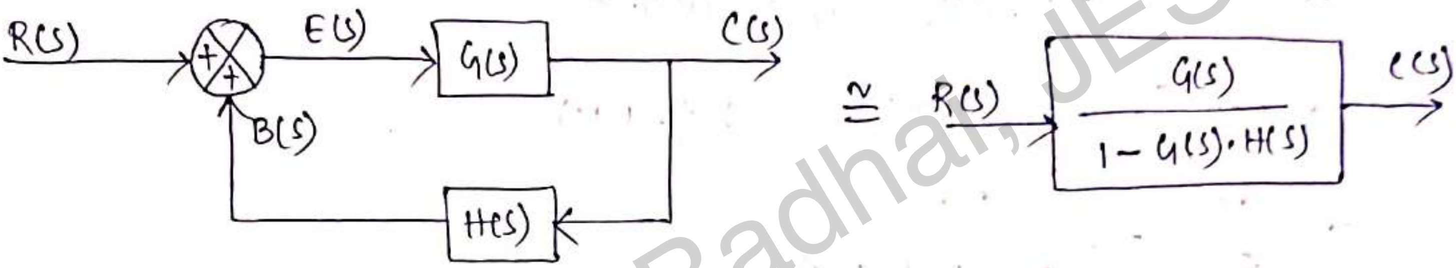
Feedback path :

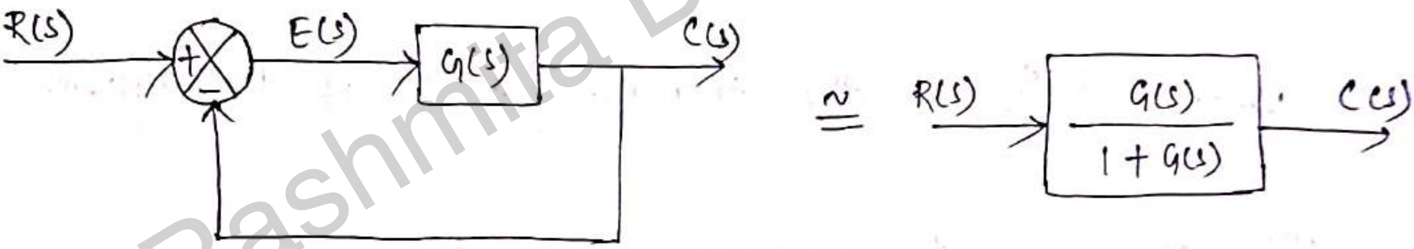
It is the direction of signal flow from output towards input.

Canonical form of closed loop system :

A block diagram in which forward path contains only one block, feedback path having one block and one take off point represents a canonical form of closed loop system.







Where, $R(s)$ = Reference input

$C(s)$ = controlled output

$B(s)$ = Feedback signal

$E(s)$ = actuating signal or error signal.

$G(s) = \frac{C(s)}{E(s)}$ is called forward path transfer function

$H(s) = \frac{B(s)}{C(s)}$ is called feedback transfer function.

$G(s) \cdot H(s)$ is called loop gain.

The ratio $\frac{C(s)}{R(s)}$ is called the closed loop transfer function or system transfer function.

The transfer function of the simple closed loop system can be deduced as

$$C(s) = G(s) \cdot E(s)$$

$$\Rightarrow C(s) = G(s) \cdot [R(s) - B(s)]$$

$$\Rightarrow C(s) = G(s) \cdot R(s) - G(s) \cdot B(s)$$

$$\Rightarrow C(s) = G(s) \cdot R(s) - G(s) \cdot C(s) \cdot H(s) \qquad [\because B(s) = C(s) \cdot H(s)]$$

$$\Rightarrow C(s) + G(s) \cdot C(s) \cdot H(s) = G(s) \cdot R(s)$$

$$\Rightarrow C(s) \cdot [1 + G(s) \cdot H(s)] = G(s) \cdot R(s)$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

where; $\frac{C(s)}{R(s)}$ is the transfer function of the system.

This is valid for negative feedback.

For positive feedback system, the transfer function is

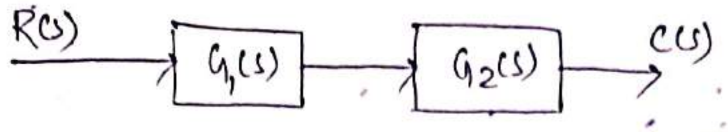
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s) \cdot H(s)}$$

A system with $H(s) = 1$, is said to be unity feedback. The transfer function with unity negative feedback is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

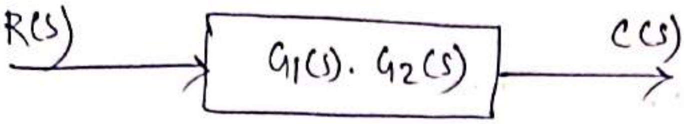
Rules for Block diagram Reduction :

Rule - 1 :

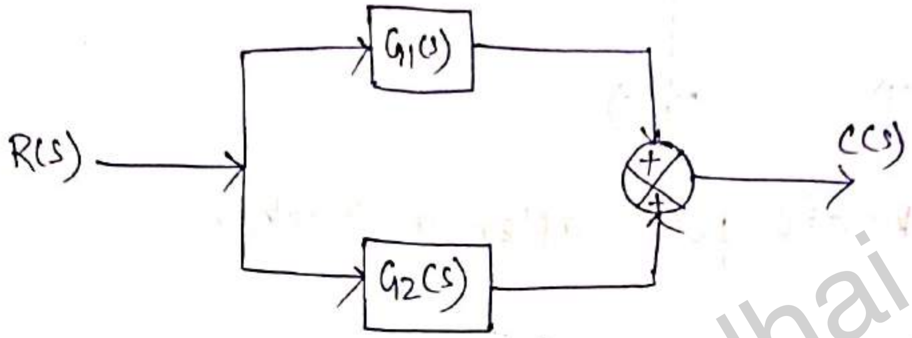


(cascade)

equivalent is .



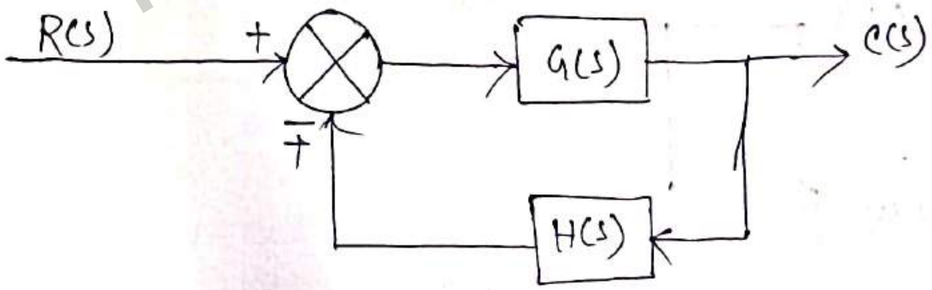
Rule - 2 :



(parallel)

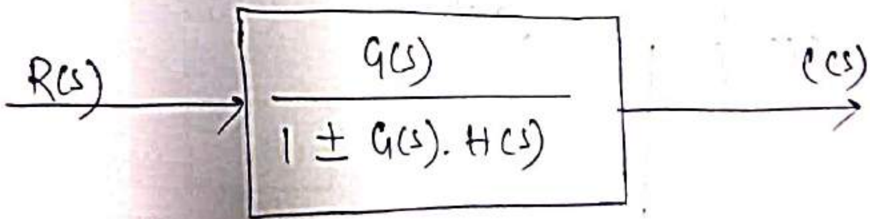
equivalent is

Rule - 3 :

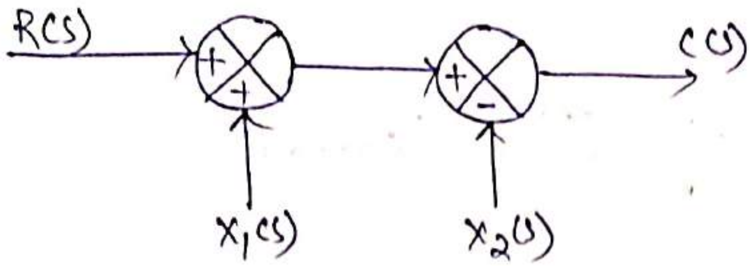


(Loop)

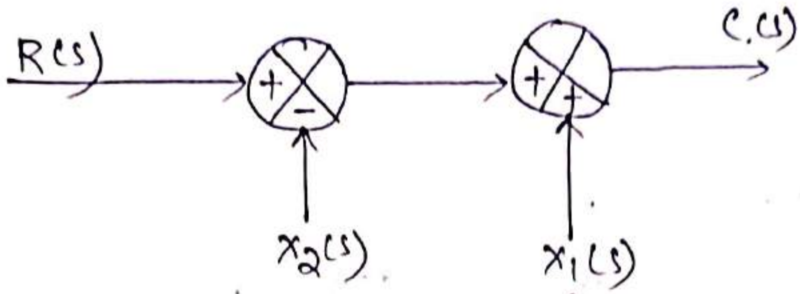
equivalent is



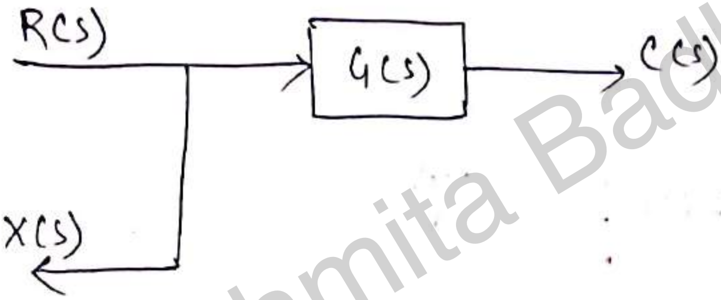
Rule - 4 (Associative law)



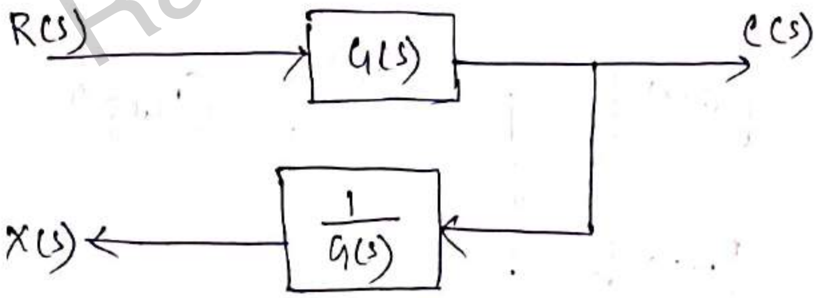
Equivalent is.



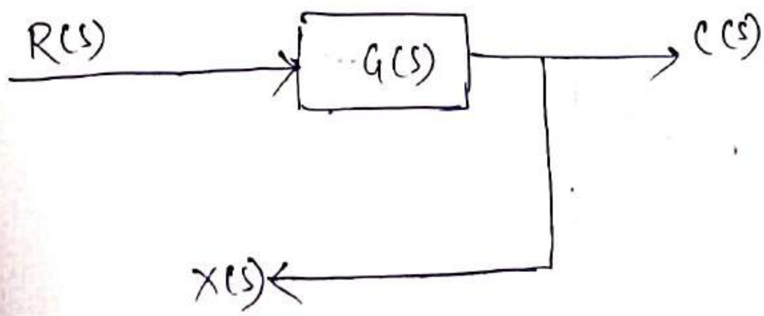
Rule - 5 (Move take off point after a block)



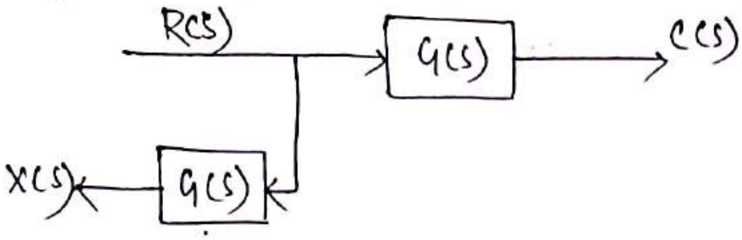
Equivalent is



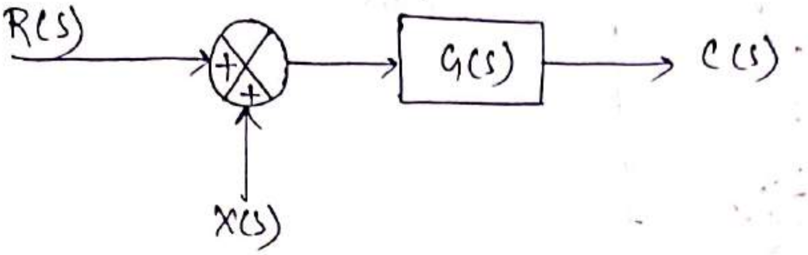
Rule - 6 (Move take off point before a block)



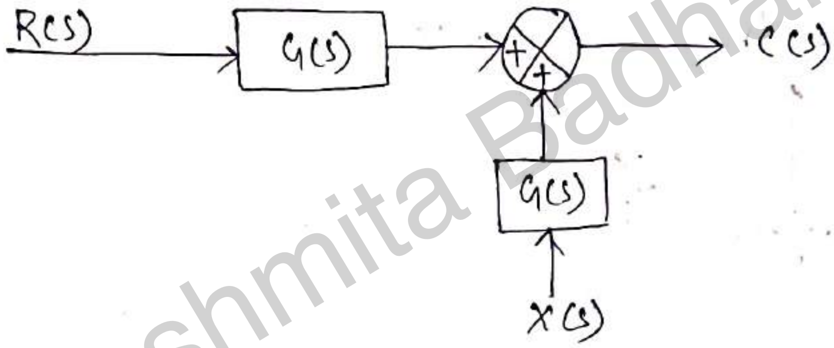
Equivalent is



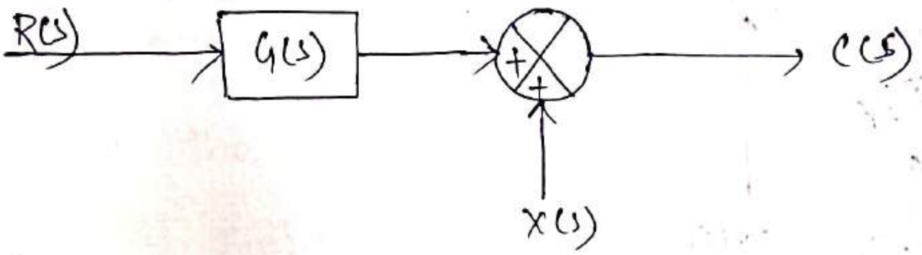
Rule-7: Moving summing point after a block.



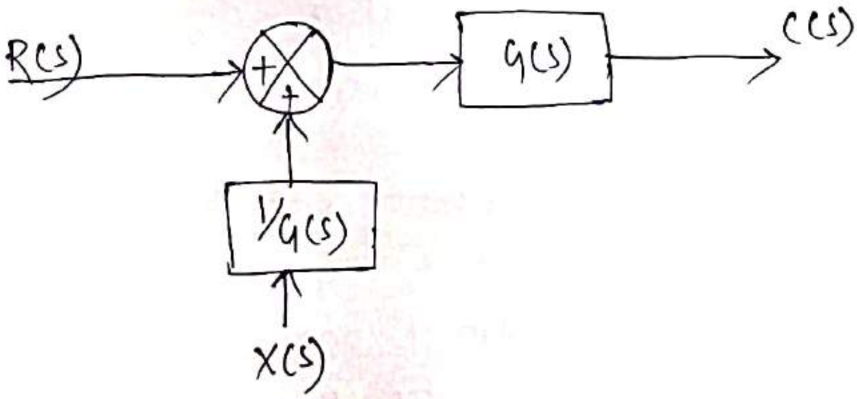
equivalent is



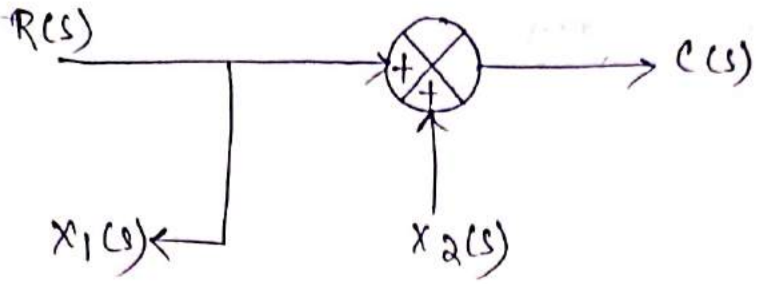
Rule-8: Move summing point before a block.



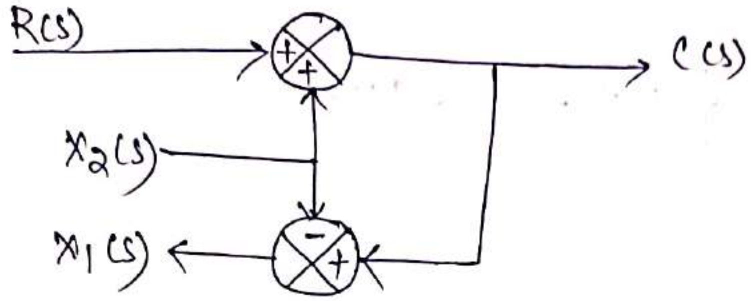
equivalent is



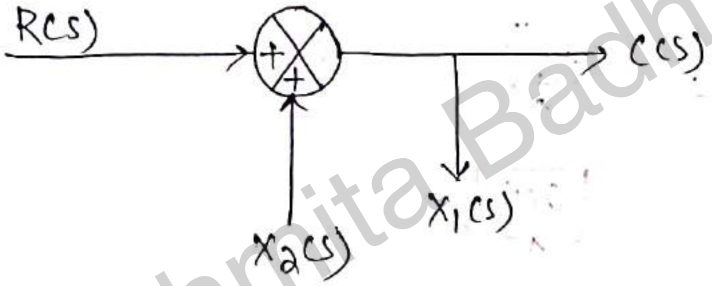
Rule-9: Move take off point after a summing point



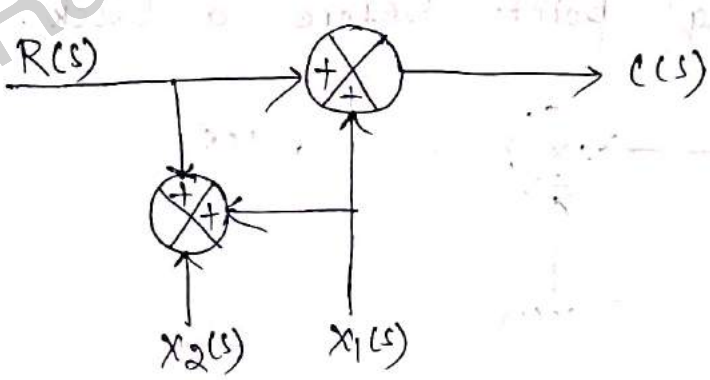
Equivalent is



Rule-10: Move take off point before a summing point.



Equivalent is



Procedure for reduction of Block Diagram :

The following steps are used in block diagram reduction :

1. Combine the cascaded blocks in a path.
2. Combine the parallel blocks.
3. If a block diagram consists of many feedback loops, then find the innermost feedback loop.
4. If the innermost feedback loop doesnot contain any tak-off and/or summing points inside the loop, then reduce it to a single block.
5. If the innermost feedback loop contains any takeoff and/or summing points inside it, then shift these points. After shifting these points, simplify the innermost loop to a single block.
6. It is better to shift the takeoff points towards right and summing points towards left.
7. Reduce all other feedback loop.
8. Repeat above steps according to the requirement until one block with input R(s) and output C(s) is obtained.
9. Find the transfer function $\frac{C(s)}{R(s)}$ for the overall system.

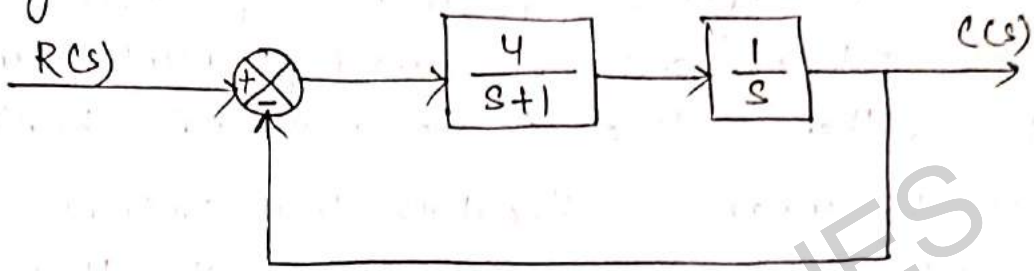
Procedure for reduction of multiple inputs.

1. Consider only one input and set all other inputs to zero.
2. Calculate the output due to the chosen input acting alone.
3. Repeat steps 1 and 2 for each of the remaining inputs one at a time.

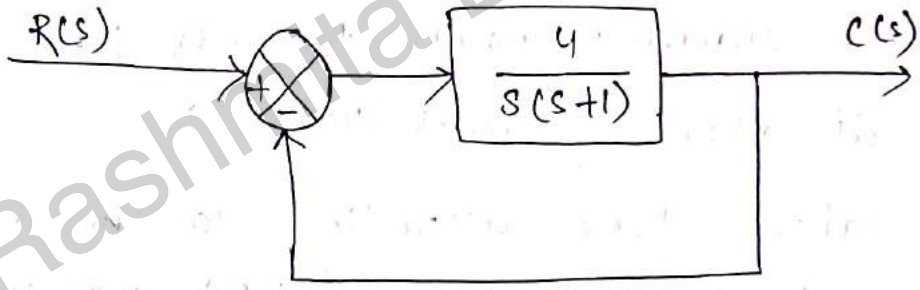
4. Apply the principle of superposition. That is, algebraically add all of the outputs determined in steps 1 to 3.

This sum is the total output of the system with all inputs acting simultaneously.

Problem-1: Find the transfer function of the closed loop system.



Ans: <u>Step-1</u> The two series blocks are combined by the cascading rule.



<u>Step-2</u> Here, $H(s) = 1$

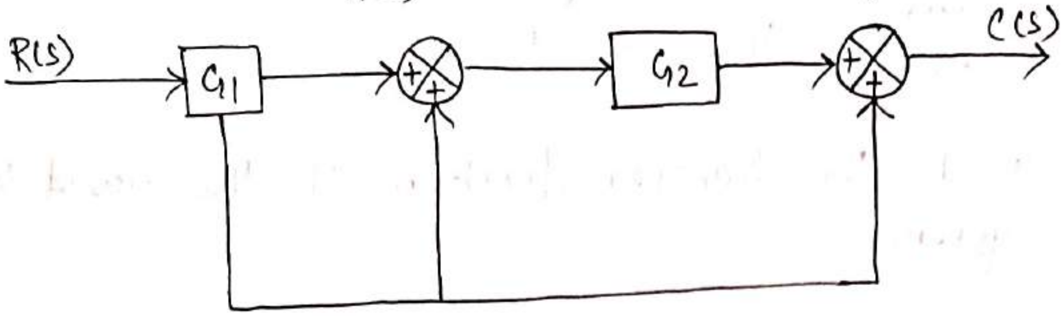
$$G(s) = \frac{4}{s(s+1)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

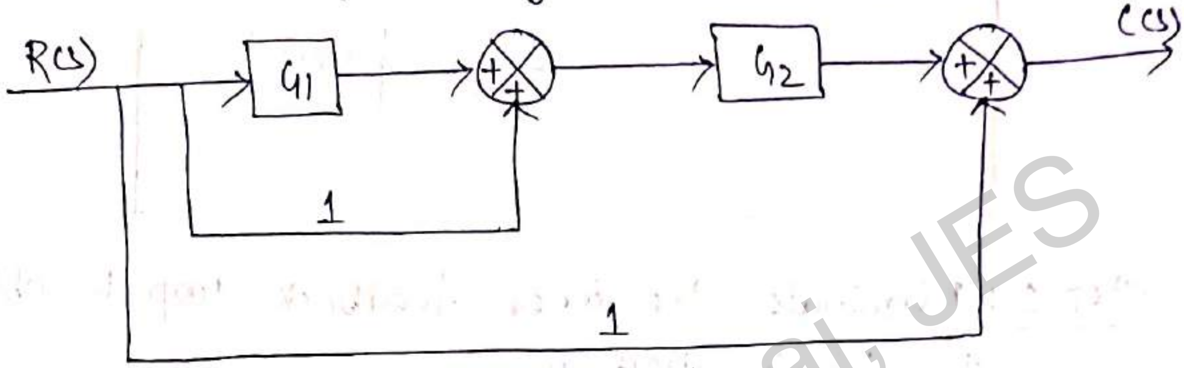
$$= \frac{\frac{4}{s(s+1)}}{1 + \frac{4}{s(s+1)} \times 1} = \frac{\frac{4}{\cancel{s(s+1)}}}{\frac{s(s+1)+4}{\cancel{s(s+1)}}}$$

$$= \frac{4}{s^2+s+4}$$

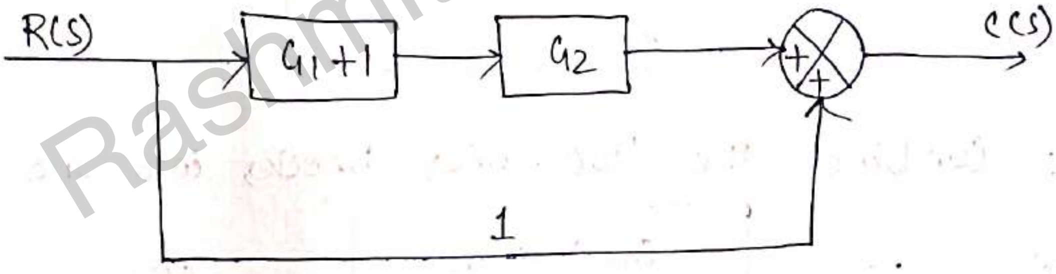
Problem-2: Find $\frac{C(s)}{R(s)}$ of the block diagram.



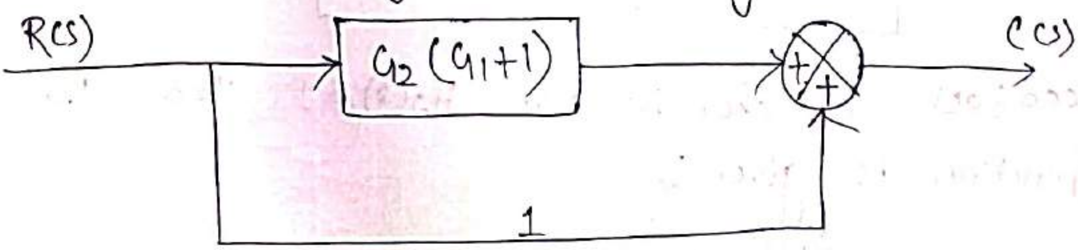
Ans: <u>Step-1</u> Modify the given block diagram



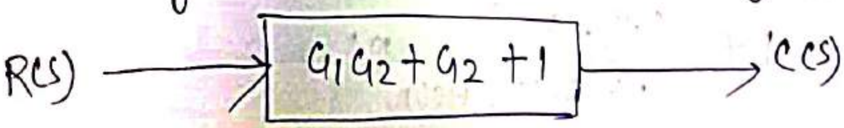
<u>Step-2</u> Eliminate the unity feedback loop containing block G_1



<u>Step-3</u> The two cascaded blocks (G_1+1) and G_2 are combined by the cascading rule.



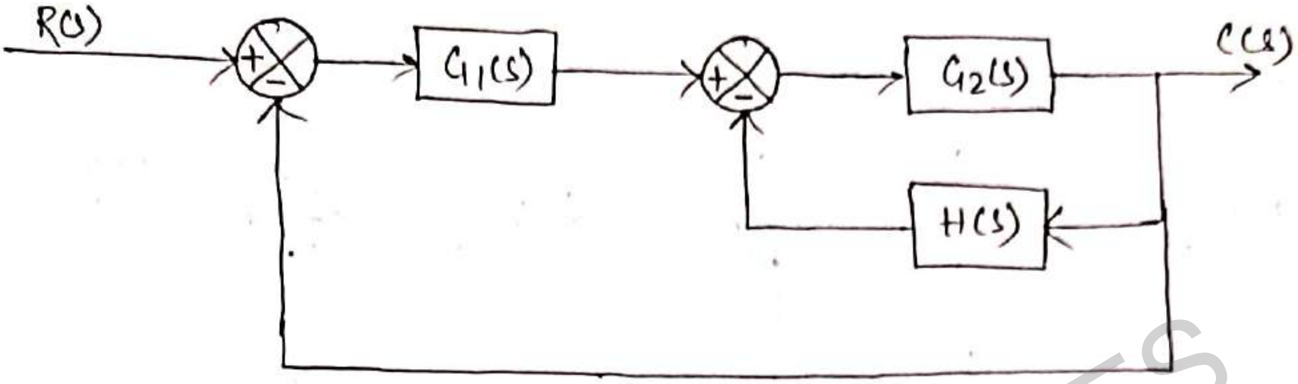
<u>Step-4</u> Adding parallel pairs containing $(G_1G_2+G_2)$ and unity.



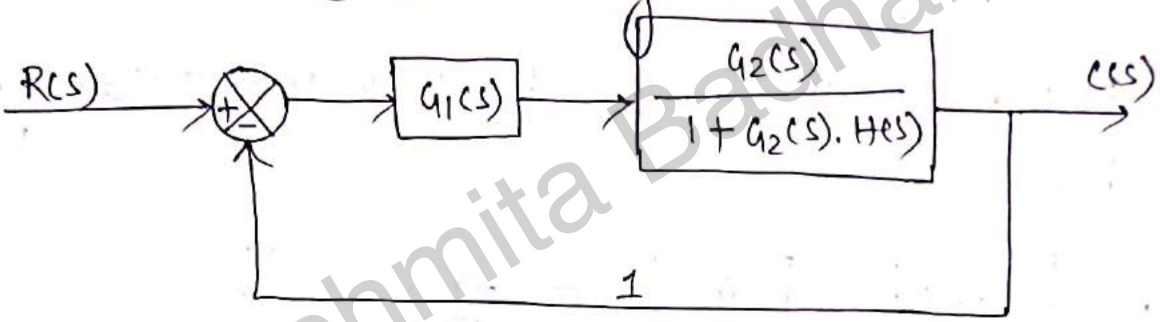
The transfer function is given by

$$\frac{C(s)}{R(s)} = G_1G_2 + G_2 + 1$$

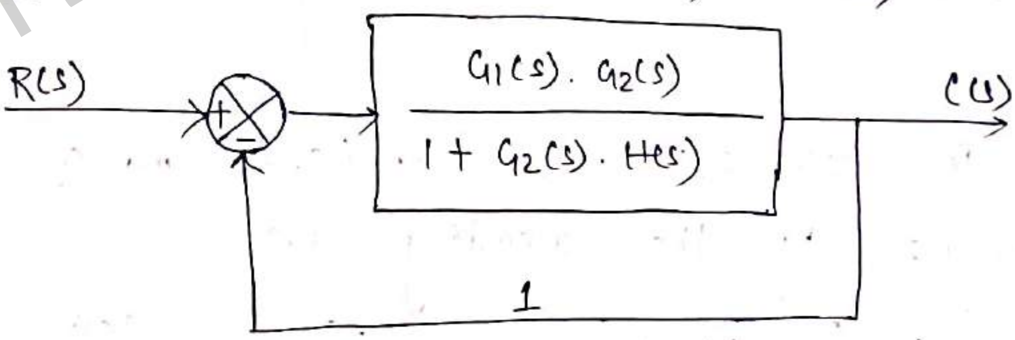
Problem-3 : Find the transfer function of the closed loop system.



Ans: Step-1 : Eliminate the inner feedback loop to obtain the block diagram



Step-2 : Combine the two series blocks into one



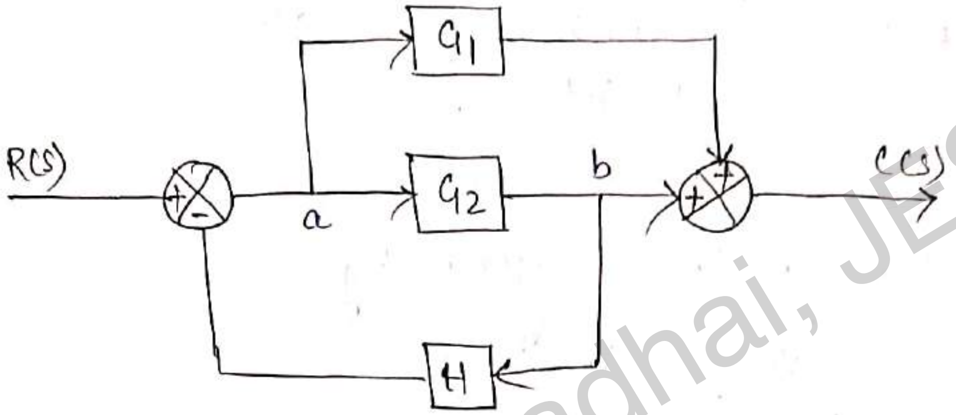
The feedback transfer function $H_2(s) = 1$. Then the transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(s).G_2(s)}{1+G_2(s).H(s)}}{1+\dfrac{G_1(s).G_2(s)}{1+G_2(s).H(s)} \times 1}$$

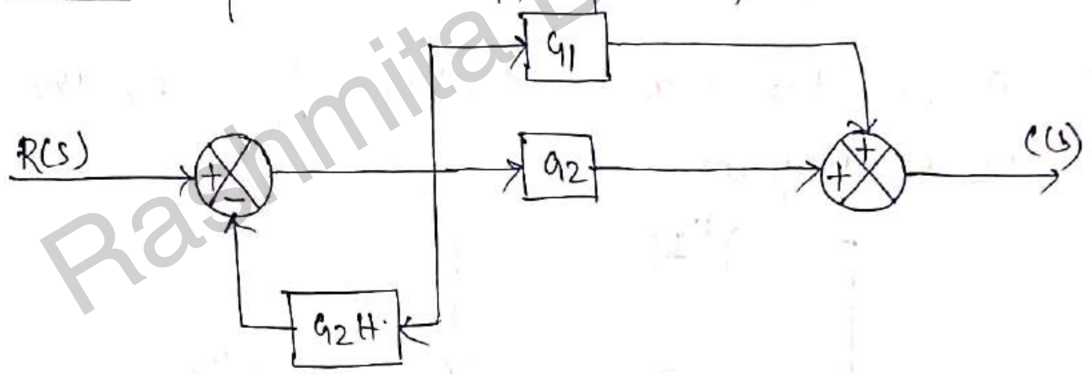
$$= \frac{\dfrac{G_1(S).\ G_2(S)}{1+ \cancel{G_2(S).\ H(S)}}}{\dfrac{1+G_2(S).H(S) + G_1(S).G_2(S)}{1+\cancel{G_2(S).H(S)}}}$$

$$= \frac{G_1(S).\ G_2(S)}{1 + G_1(S).\ G_2(S) + G_2(S).\ H(S)}$$

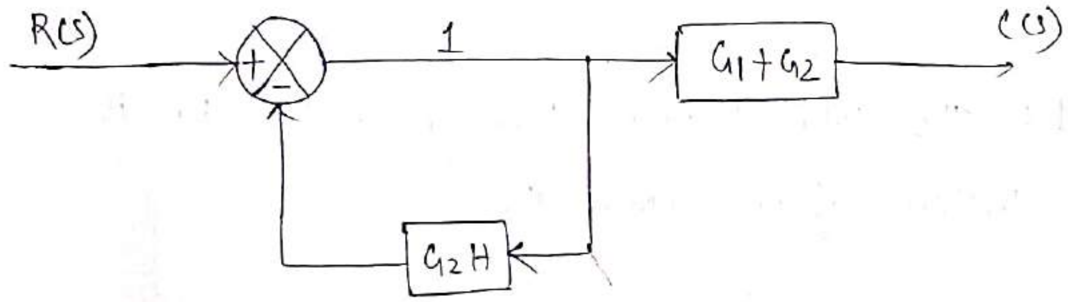
Problem-4: Find the transfer function for the block diagram.



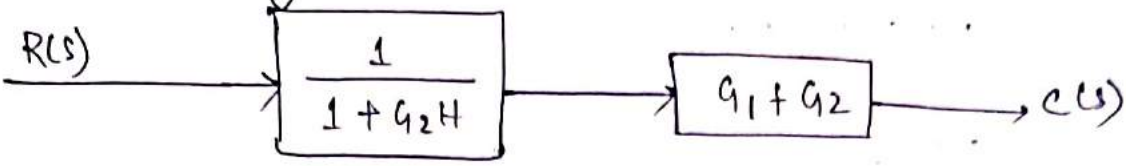
Ans: Step-1: Move take off point from b to a



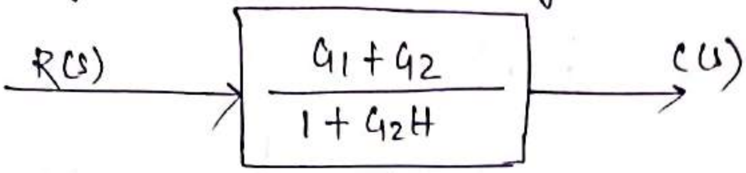
Step-2: Combine two blocks G_1 and G_2 in parallel to give a single block $\boxed{G_1+G_2}$



Step-3: Eliminate the feedback loop with a forward gain of 1 and feedback element H to give a single block.



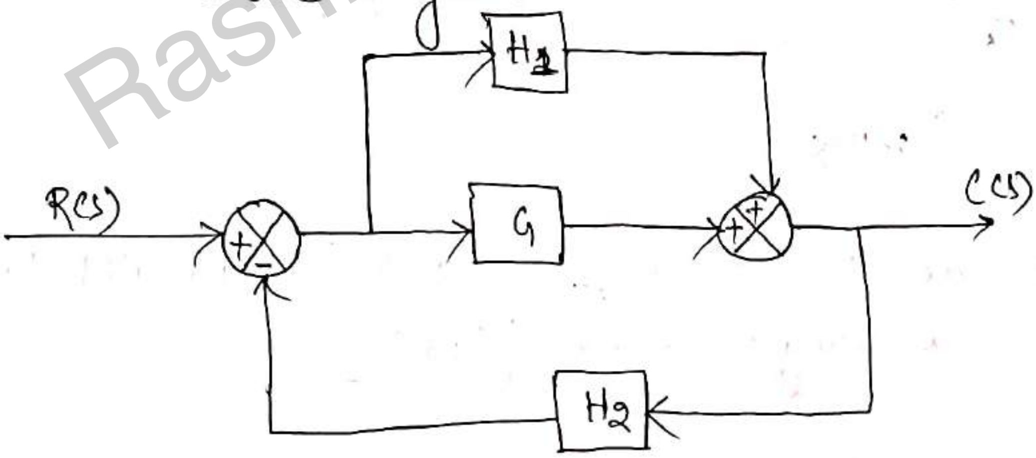
Step-4: The two blocks in cascade are combined to get the block diagram.



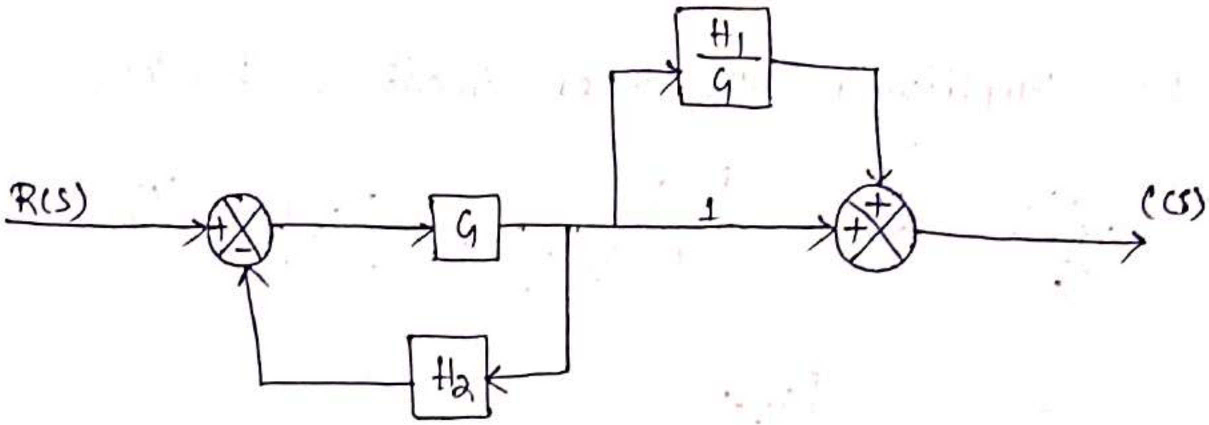
The transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G_1 + G_2}{1 + G_2 H}$$

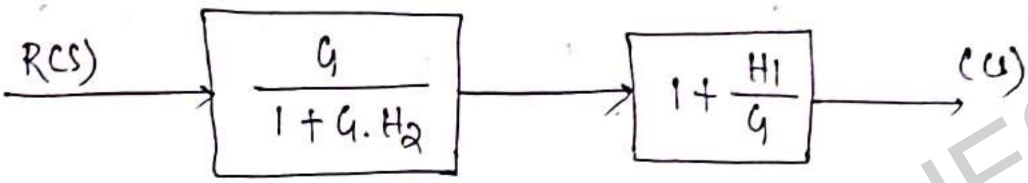
Problem-5: Obtain the transfer function $\frac{C(s)}{R(s)}$ for the block diagram.



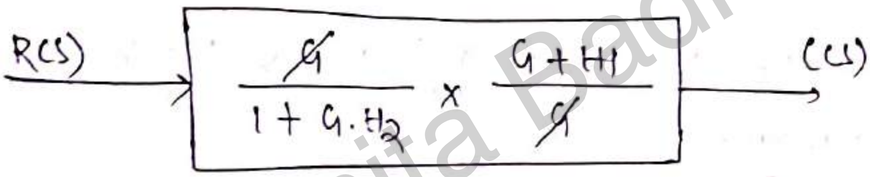
Ans: <u>Step-1</u>: The takeoff point having block H_1 is shifted after block G.



Step-2: Replace the two feedback loops by their equivalent blocks.



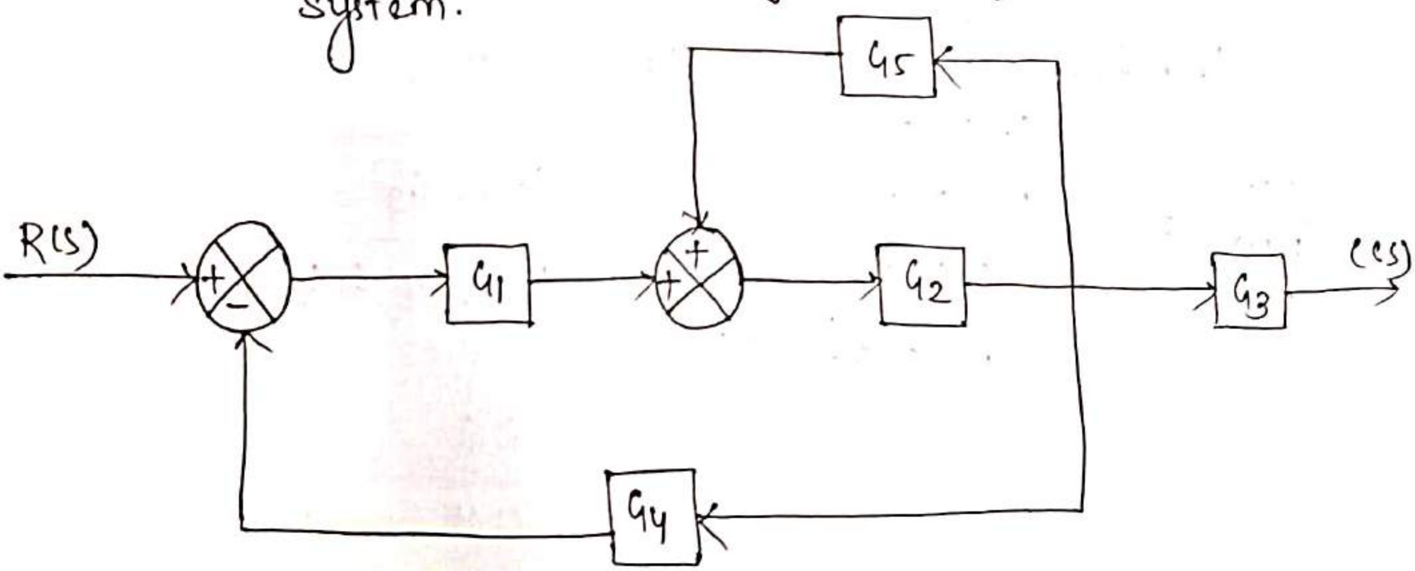
Step-3: Combine the cascaded blocks



Then the transfer function is

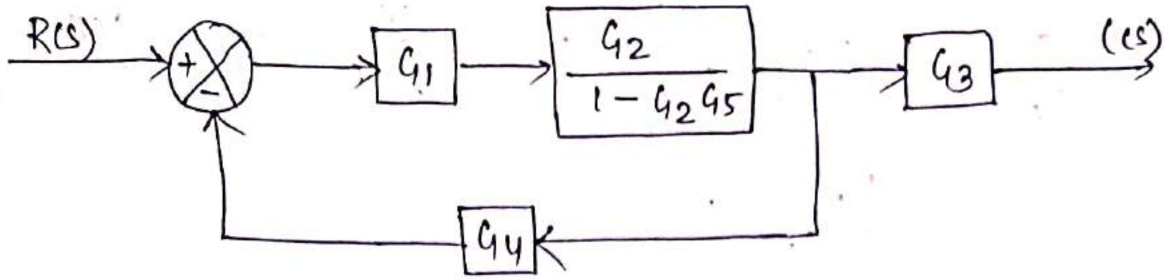
$$\frac{C(s)}{R(s)} = \frac{G + H_1}{1 + GH_2}$$

Problem - 6: Find the transfer function of the closed loop system.

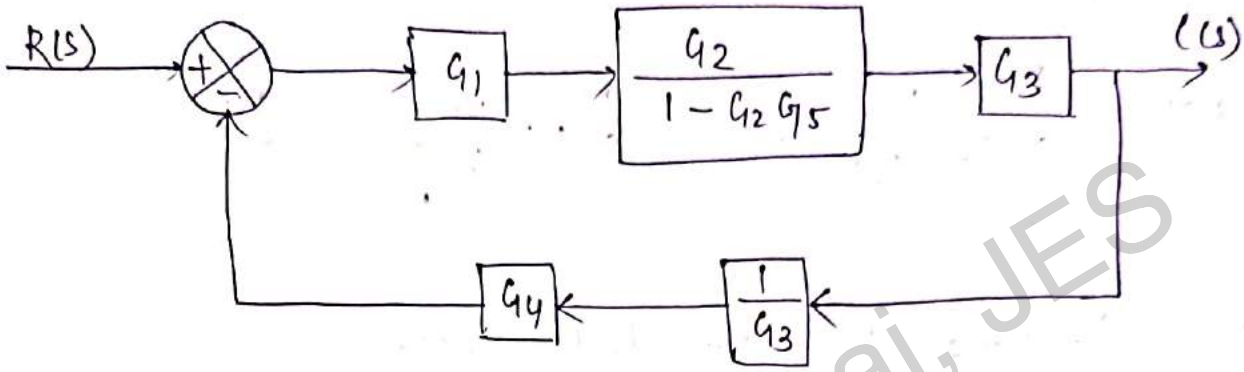


Ans:

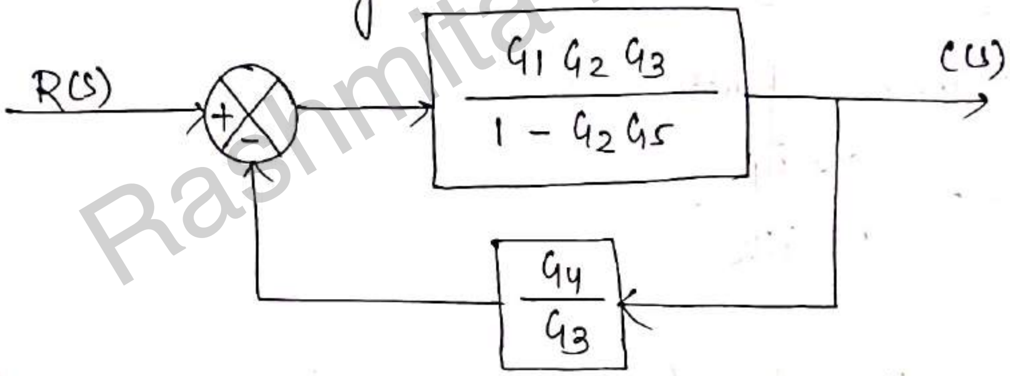
Step-1 : Simplifying the inner feedback blocks.



Step-2 : Shift the takeoff point after the block G_3.



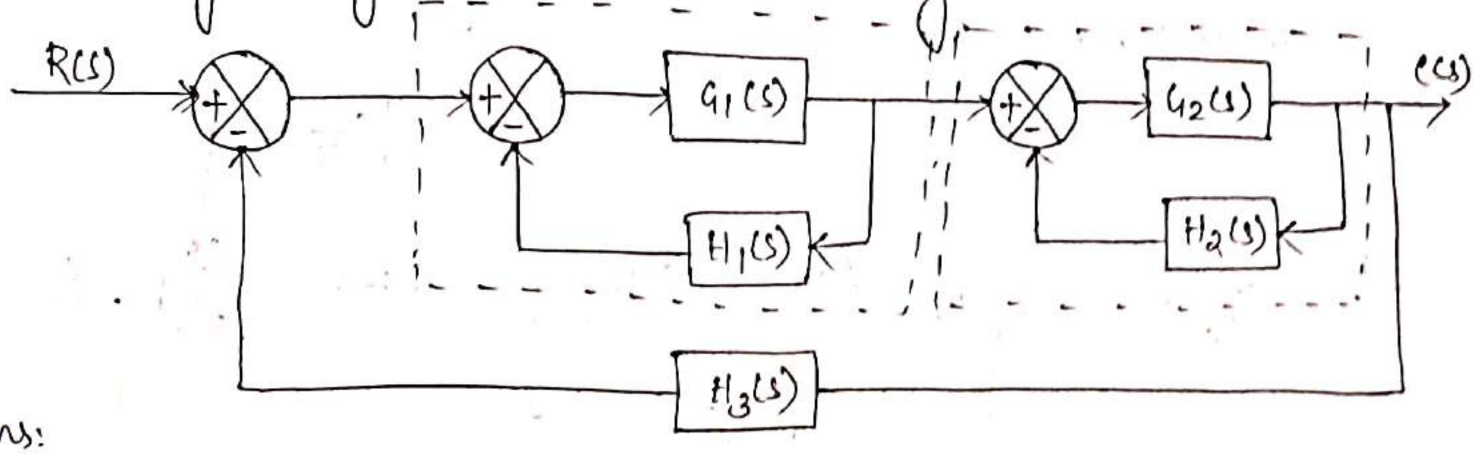
Step-3 : Combine the two sets of series block to get the diagram.



Step-4 : Eliminating the feedback loop, the overall transfer function of the system.

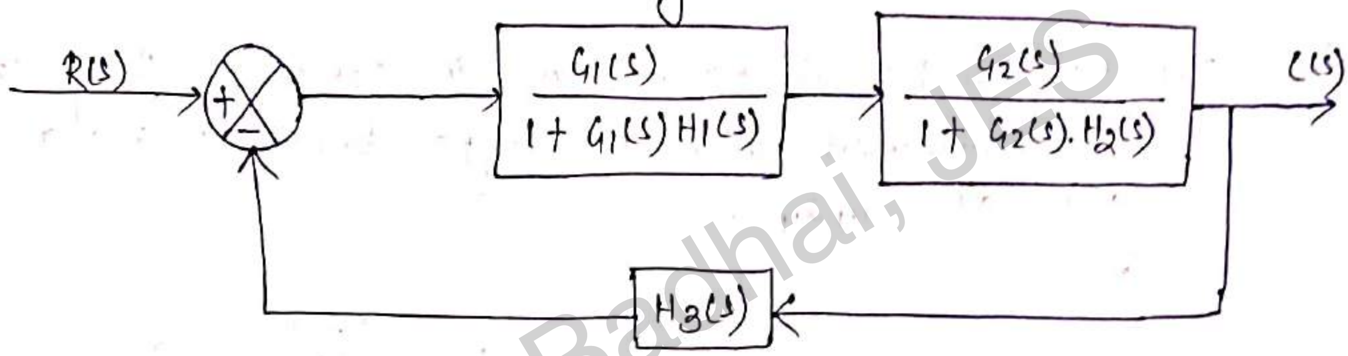
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3}{1-G_2G_5}}{1+\dfrac{G_1G_2\cancel{G_3}}{1-G_2G_5}\times\dfrac{G_4}{\cancel{G_3}}} = \frac{G_1G_2G_3}{1-G_2G_5+G_1G_2G_4}$$

Problem-7: From the block diagram, determine the overall gain by successive block diagram reduction.

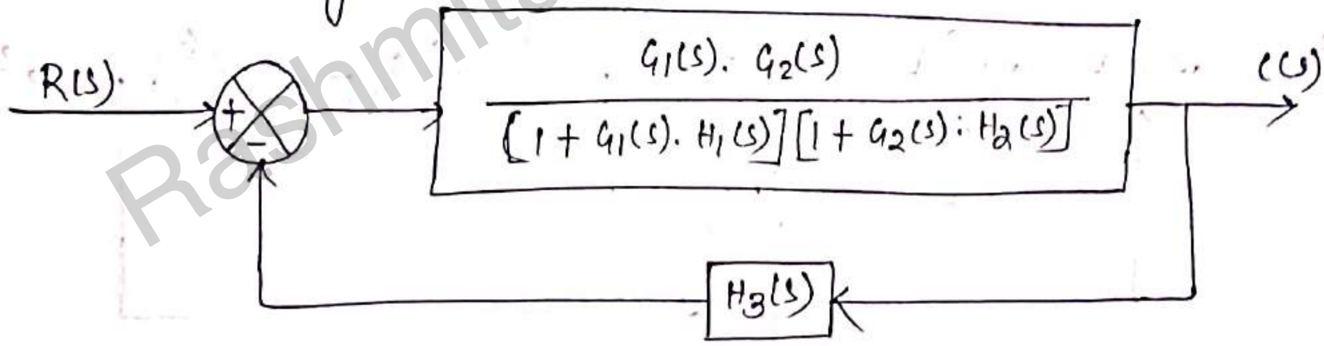


Ans:

Step-1: Eliminating the internal feedback loops to obtain the block diagram.



Step-2: Cascading the two forward blocks into one

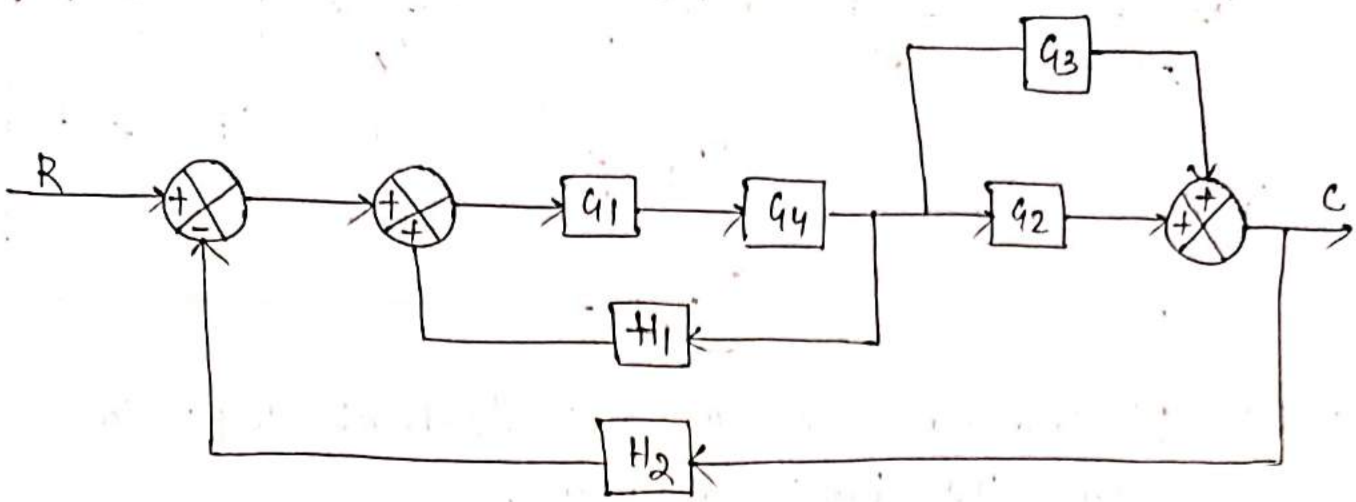


Step-3: Removing the feedback path the overall gain of the system is

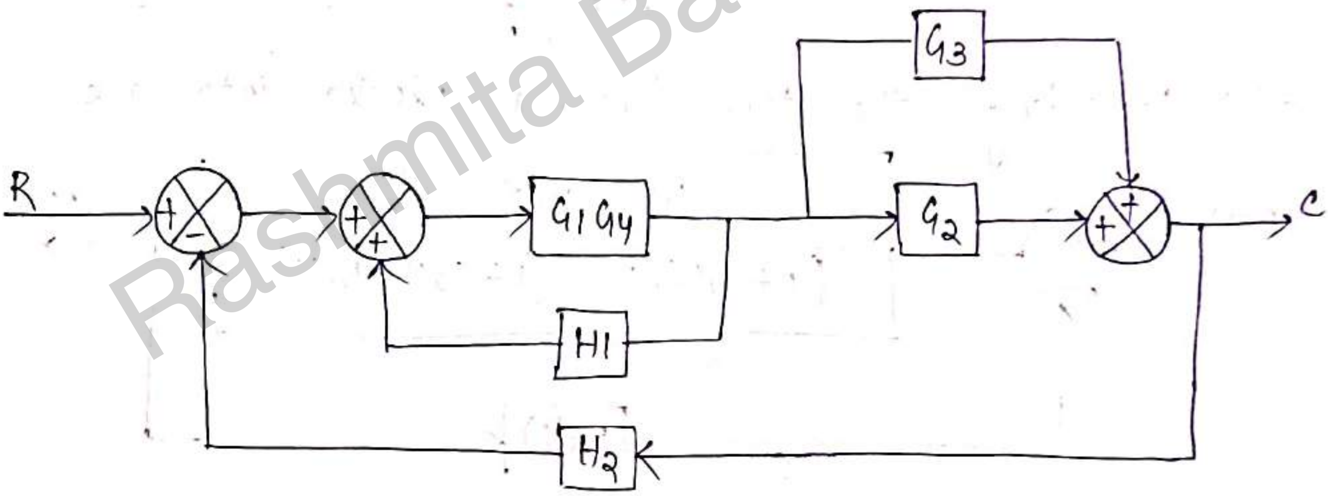
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(s).\ G_2(s)}{[1+G_1(s).\ H_1(s)][1+G_2(s).\ H_2(s)]}}{1+\dfrac{G_1(s).\ G_2(s)}{[1+G_1(s).\ H_1(s)][1+G_2(s).\ H_2(s)]} \times H_3(s)}$$

$$= \frac{G_1(s).\ G_2(s)}{[1+G_1(s).\ H_1(s)][1+G_2(s).\ H_2(s)] + G_1(s).\ G_2(s).\ H_3(s)}$$

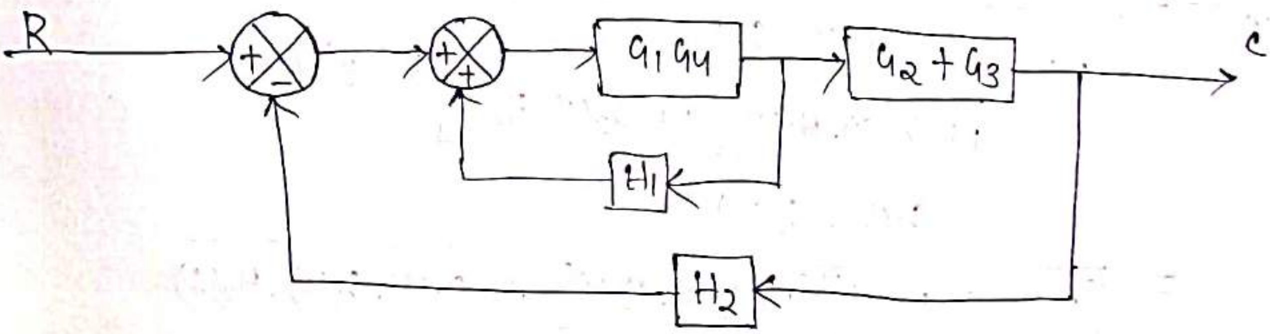
Problem - 8 : From the block diagram, determine the relationship between R and C by successive block reduction.



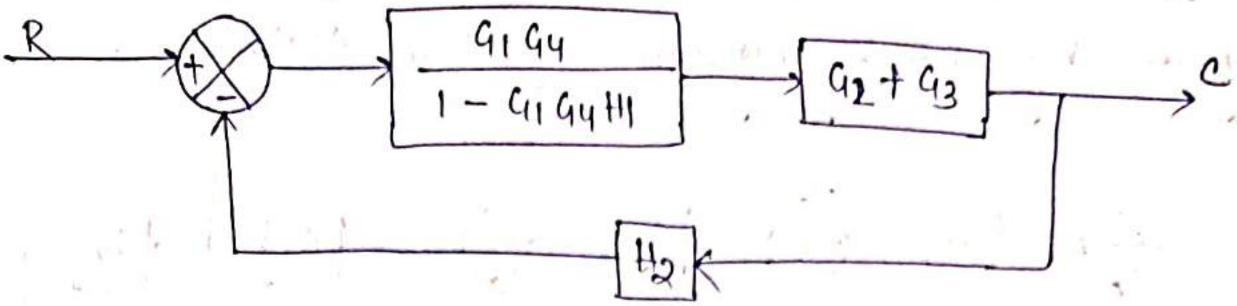
Ans: Step-1: blocks G_1 and G_4 are in cascade. They can be combined into a single block with a gain of $G_1 G_4$.



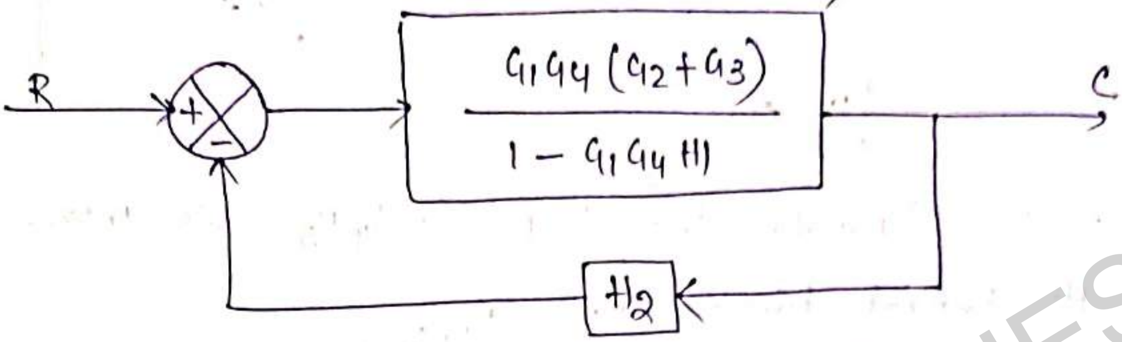
Step-2: Block G_2 and G_3 are in parallel. They can be combined into a single block with a gain of $(G_2 + G_3)$



Step-3: Eliminating the inner feedback loop containing H_1



Step-4: ~~Cascade~~ Combine the blocks in cascade



Step-5: Eliminate the only loop and the transfor function will be.

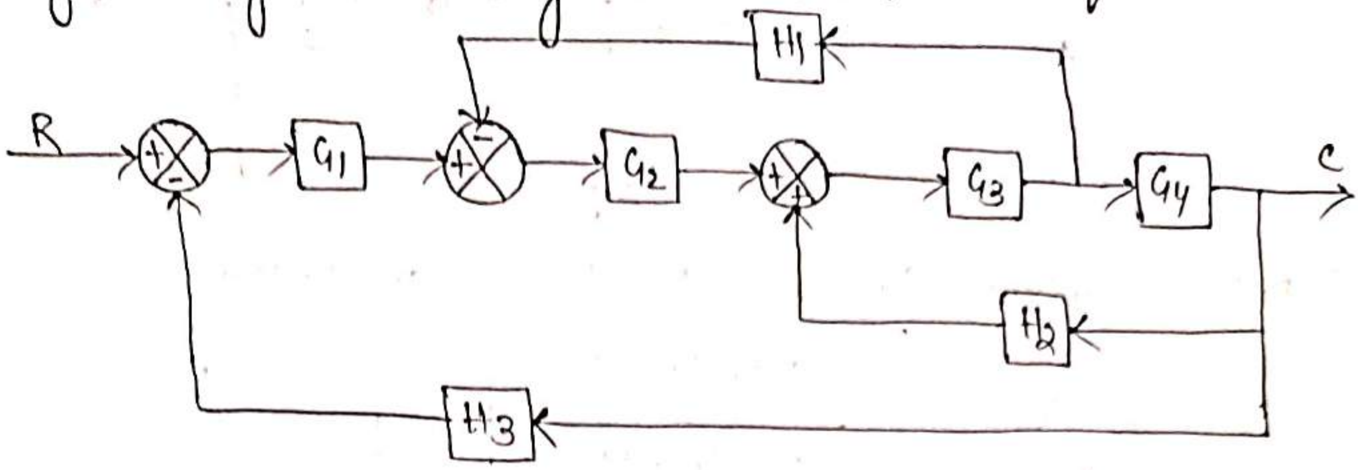
$$\frac{C}{R} = \frac{\dfrac{G_1 G_4 (G_2 + G_3)}{1 - G_1 G_4 H_1}}{1 + \dfrac{G_1 G_4 (G_2 + G_3)}{1 - G_1 G_4 H_1} \cdot H_2}$$

$$= \frac{\dfrac{G_1 G_4 (G_2 + G_3)}{\cancel{1 - G_1 G_4 H_1}}}{\dfrac{1 - G_1 G_4 H_1 + G_1 G_4 H_2 (G_2 + G_3)}{\cancel{1 - G_1 G_4 H_1}}}$$

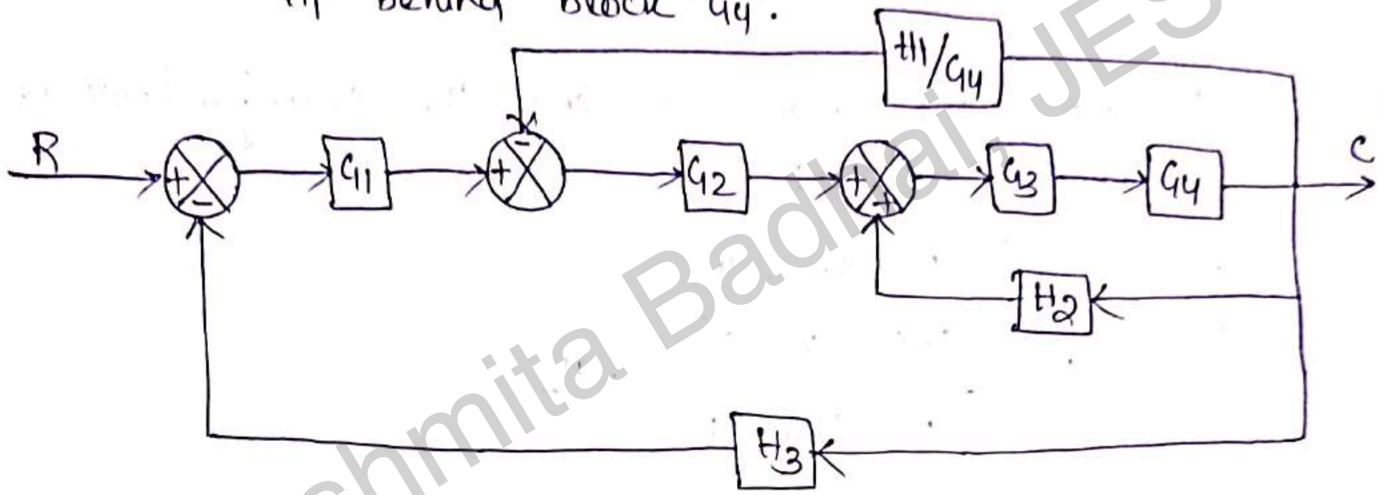
$$\frac{C}{R} = \frac{G_1 G_4 (G_2 + G_3)}{1 - G_1 G_4 H_1 + G_1 G_4 H_2 (G_2 + G_3)}$$

Problem - 9 :

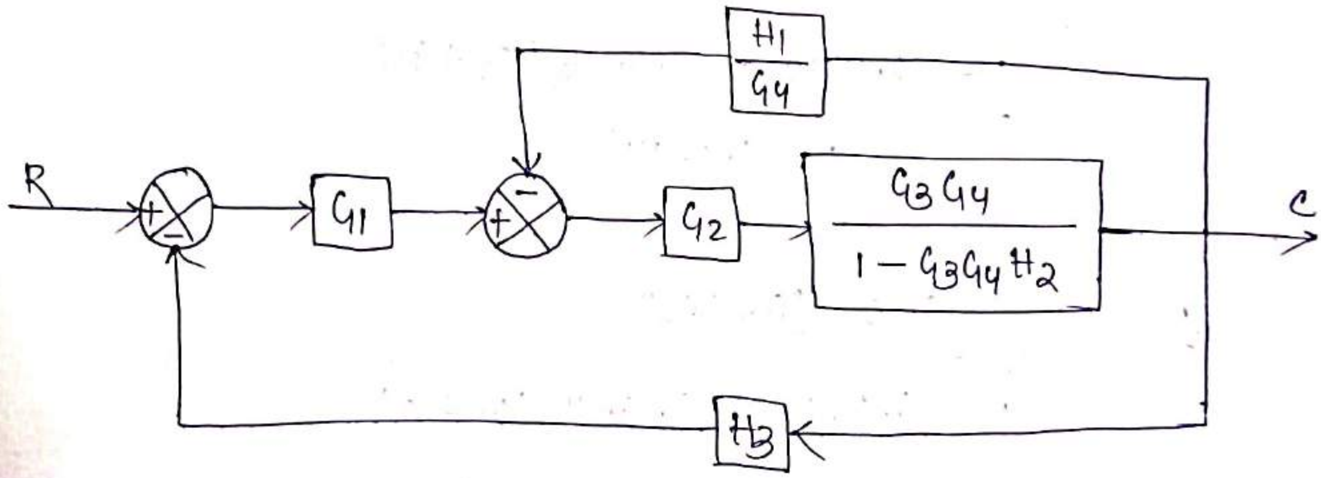
Obtain the overall transfer function of the block diagram by block diagram reduction technique.



Ans: Step-1 : To eliminate the loop $G_3 G_4 H_2$, we move H_1 behind block G_4.



Step - 2 : Combine G_3 and G_4 and eliminate $G_3, G_4 H_2$.



Step - 3 : Blocks G_2 and $\dfrac{G_3 G_4}{1 - G_3 G_4 H_2}$ are in cascade.

Their combination gives a block $\dfrac{G_2 G_3 G_4}{1 - G_3 G_4 H_2}$.

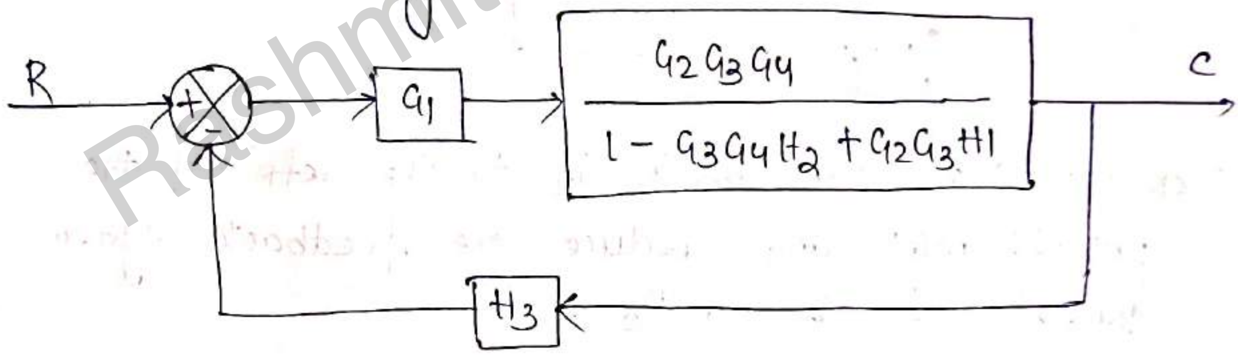
Step-4: The inner loop containing $\frac{H_1}{G_4}$ and $\frac{G_2G_3G_4}{1-G_3G_4H_2}$ is eliminated to give

$$\frac{\dfrac{G_2G_3G_4}{1-G_3G_4H_2}}{1+\dfrac{G_2G_3\cancel{G_4}}{1-G_3G_4H_2}\times\dfrac{H_1}{\cancel{G_4}}}$$

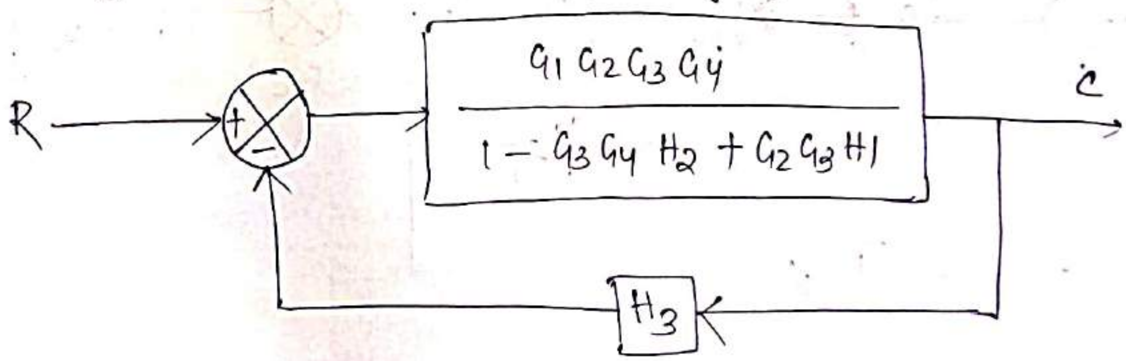
$$=\frac{\dfrac{G_2G_3G_4}{\cancel{1-G_3G_4H_2}}}{\dfrac{1-G_3G_4H_2+G_2G_3H_1}{\cancel{1-G_3G_4H_2}}}$$

$$=\frac{G_2G_3G_4}{1-G_3G_4H_2+G_2G_3H_1}$$

Then the block diagram be



Step-5: Combine the cascaded block and eliminate the feedback loop formed by H_3.

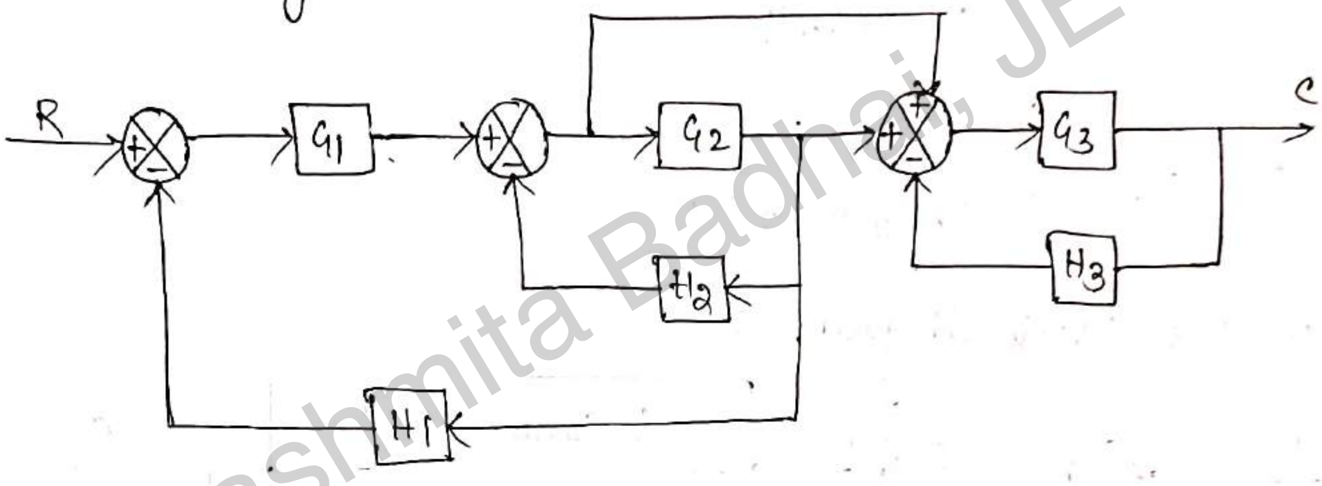


Then the closed loop transfer function is given by

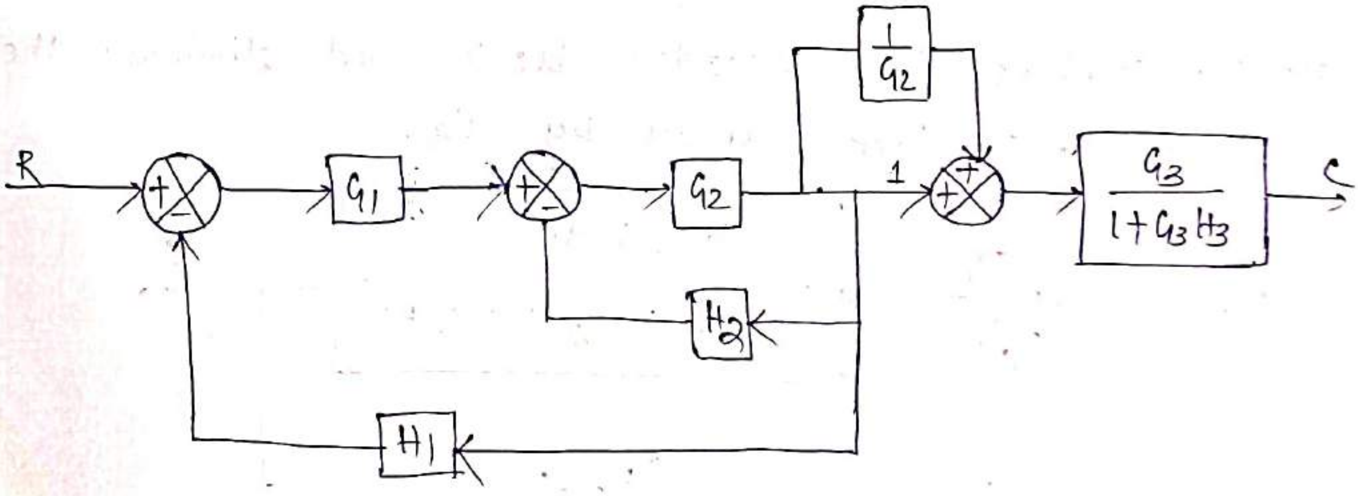
$$\frac{C}{R} = \frac{\dfrac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_2 + G_2 G_3 H_1}}{1 + \dfrac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_2 + G_2 G_3 H_1} \cdot H_3}$$

$$= \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_2 + G_2 G_3 H_1 + G_1 G_2 G_3 G_4 H_3}$$

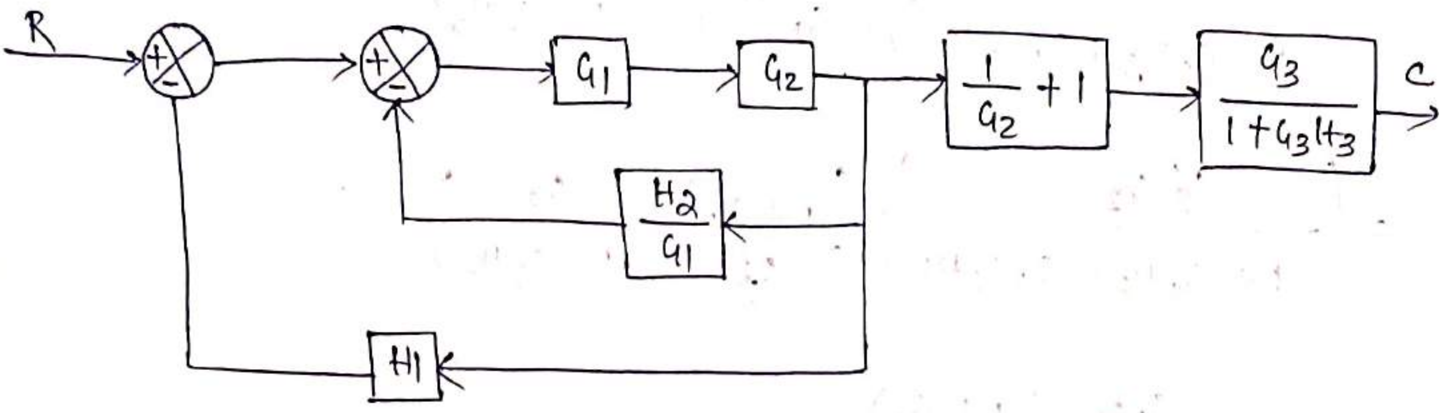
Problem : 10 Using Block diagram reduction technique for the system, find the input-output relationship.



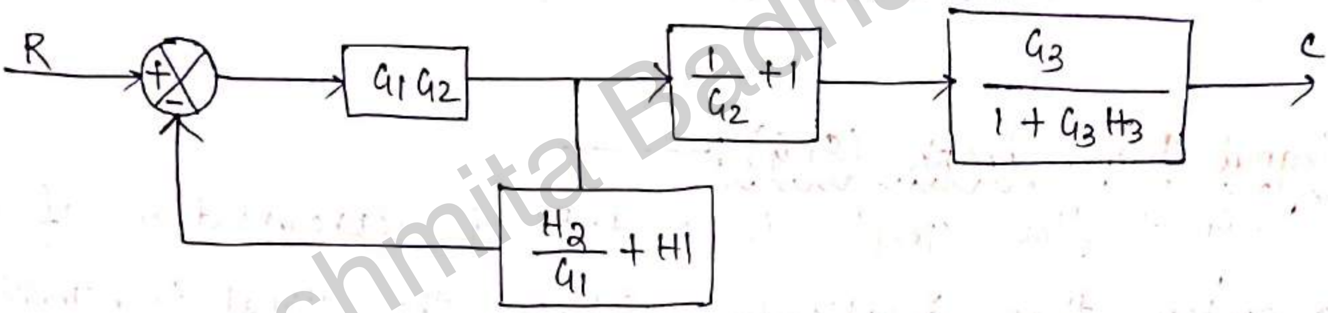
Ans: Step-1: Move the block G_2 to the left of the pickoff point and reduce the feedback system consists of G_3 and H_3 :-



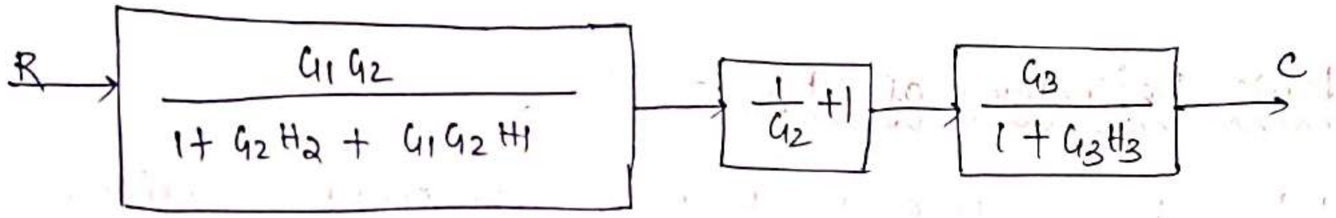
Step-2: Add the parallel pair consisting of $\frac{1}{G_2}$ and unity. Move G_1 to the right of the summing point.



Step-3: Blocks G_1 and G_2 in cascade are combined into single block G_1G_2 and summing points are combined.



Step-4: Eliminate the feedback loop



$$\frac{G_1G_2}{1+G_1G_2\left(\frac{H_2}{G_1}+H_1\right)}$$

$$= \frac{G_1G_2}{1+\cancel{G_1}G_2\left(\frac{H_2+G_1H_1}{\cancel{G_1}}\right)} = \frac{G_1G_2}{1+G_2H_2+G_1G_2H_1}$$

Step-5: The three cascaded blocks are combined into a single block.

$$\left(\frac{G_1 G_2}{1+G_2H_2+G_1G_2H_1}\right)\left(\frac{1}{G_2}+1\right)\left(\frac{G_3}{1+G_3H_3}\right)$$

$$=\left(\frac{\cancel{G_2}\, G_1}{1+G_2H_2+G_1G_2H_1}\right)\left(\frac{1+G_2}{\cancel{G_2}}\right)\left(\frac{G_3}{1+G_3H_3}\right)$$

$$=\frac{G_1G_3(1+G_2)}{(1+G_2H_2+G_1G_2H_1)(1+G_3H_3)}$$

$$\frac{C}{R}=\frac{G_1G_3(1+G_2)}{(1+G_2H_2+G_1G_2H_1)(1+G_3H_3)}$$

Signal Flow Graph (SFG) :—

Signal flow graph is a pictorial representation of a system that graphically displays the signal transmission in it.

Basic Definations of SFG :—

(1) Node :— A node is a point representing a variable or signal.

(2) Branch :— A branch is a directed line segment joining two nodes. The arrow on the branch indicates the direction of signal flow.

(3) Transmittance :— The gain aquired by the signal when it travels from one node to another. Transmittance can be real or complex.

4. Input node (or) source :— It is a node that contains only outgoing branches.

5. Output node (or) sink :— It is a node that contains only incoming branches.

6. Mixed node :— It is a node that has both incoming and outgoing branches.

7. Path :— A path is a traversal of connected branches in the direction of arrows. The path should not cross a node more than once.

Path is of two types.

(1) open path

(2) closed path.

Open path means it starts at one node and ends at other node. If a path started at one node and ended at the same node is called closed path.

8. Forward path :— If path starts at input and ended at output is called forward path.

9. Forward path gain :— In the forward path, the combination of gains is called forward path gain.

10. Individual loop :— If any loop started at a node and ending at the same node without crossing a single node twice that is called as individual loop.

11. Loop gain :— Product of gains of a loop is called loop gain.

12. Non-touching loop :— If the loops does not have common node then those are said to be non-touching loops.

Properties of signal flow graph :—

(1) It is applicable to linear time-invariant systems.

(2) The signal flow is only along the direction of arrows.

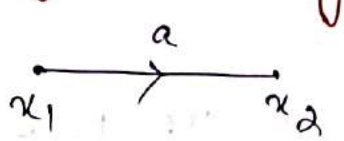
(3) The value of variable at each node is equal to the algebraic sum of all signals entering at that node.

(4) It is given by Mason's gain formula.

(5) It is not be the unique property of the system.

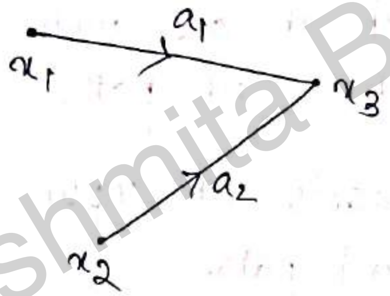
Rules of signal flow graph :—

Rule-1 : (i)



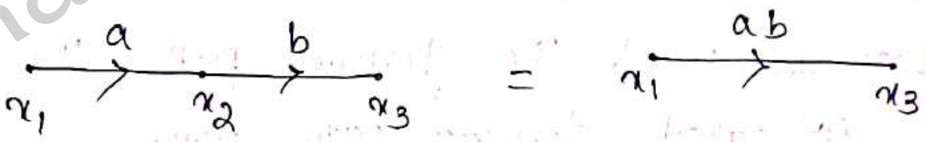
$x_2 = a x_1$

(ii)



$x_3 = a_1 x_1 + a_2 x_2$

Rule-2 :



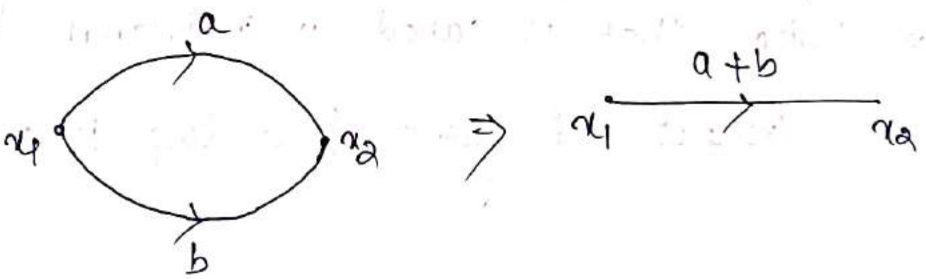
$x_2 = a x_1$

$x_3 = b x_2$

$\Rightarrow x_3 = a x_1 \cdot b$

$\Rightarrow x_3 = a b x_1$

Rule-3 :



$x_2 = a x_1$

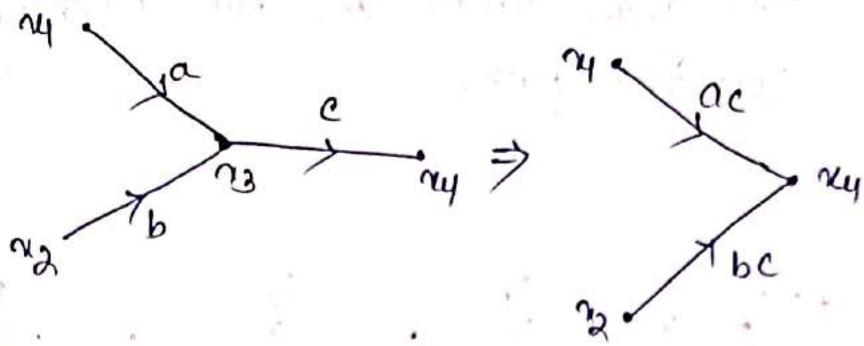
$x_2 = b x_1$

$x_2 = a x_1 + b x_1$

$= (a+b) x_1$

So, $x_2 = (a+b) x_1$

Rule-4:

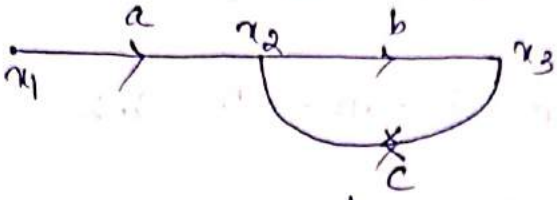


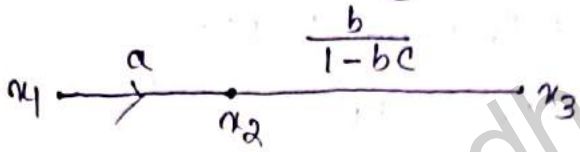
$$x_3 = a x_1 + b x_2$$

$$x_4 = a c x_1 + b c x_2$$

$$\begin{aligned} x_4 &= c x_3 \\ &= c(a x_1 + b x_2) \\ &= a c x_1 + b c x_2 \end{aligned}$$

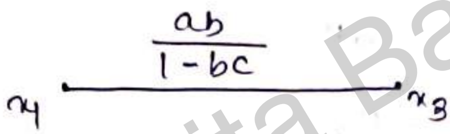
Rule-5:





$$x_2 = a x_1$$

$$x_3 = \frac{b}{1-bc} x_2$$



$$\Rightarrow x_3 = \frac{b}{1-bc} \times a x_1$$

$$= \frac{ab}{1-bc} x_1$$

Mason's Gain Formula :—

let $R(s) \rightarrow$ input of the system

$C(s) \rightarrow$ output of the system

Transfer function of the system $T(s) = \frac{C(s)}{R(s)}$

overall gain

$$\boxed{T = \frac{1}{\Delta} \sum_{K=1}^{N} P_K \Delta_K}$$

where,

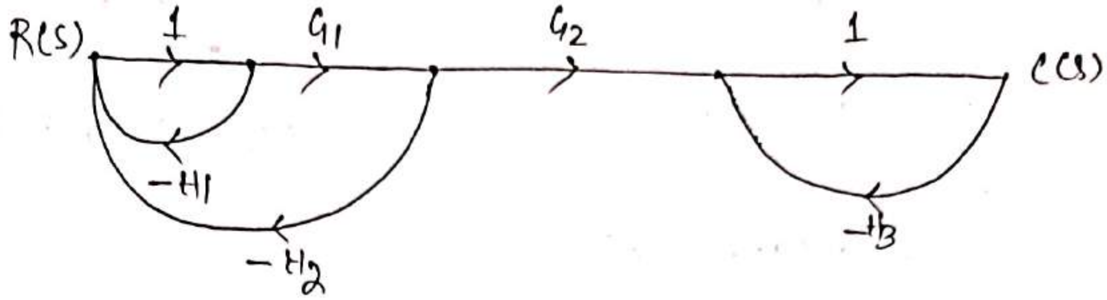
$T = T(s)$ = Transfer function of the system.

P_K = Forward path gain of k^{th} forward path

$\Delta = 1 -$ (sum of individual loop gains) + (sum of (two non touching loops) + (sum of 3 non-touching loops)..

$\Delta_k = 1 -$ loop gains which are not touching to k^{th} forward path.

Eg:



No. of forward paths : N = 1

Path gain of forward paths : $P_1 = G_1 G_2$

Loop gain of individual loops :

$$L_1 = -H_1$$

$$L_2 = -G_1 H_2$$

$$L_3 = -H_3$$

No. of two non-touching loops : L_1 and L_3
L_2 and L_3

$$P_{21} = H_1 H_3$$

$$P_{22} = G_1 H_2 H_3$$

$$\Delta = 1 - (-H_1 - G_1 H_2 - H_3) + (H_1 H_3 + G_1 H_2 H_3)$$

$$\Delta_k = \Delta_1 = 1 - (0) = 1$$

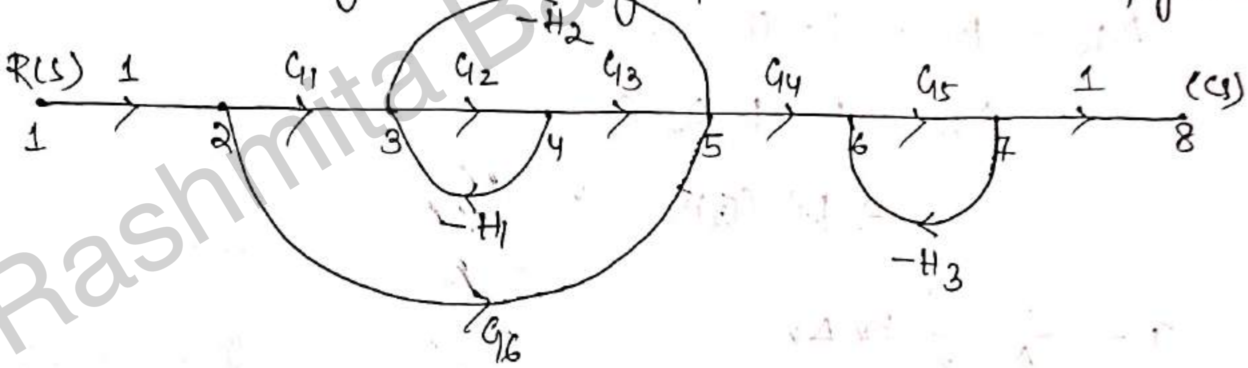
$$T(s) = \frac{1}{1 + H_1 + G_1 H_2 + H_3 + H_1 H_3 + G_1 H_2 H_3} \times G_1 G_2 \times 1$$

$$= \frac{G_1 G_2}{1 + H_1 + G_1 H_2 + H_3 + H_1 H_3 + G_1 H_2 H_3}$$

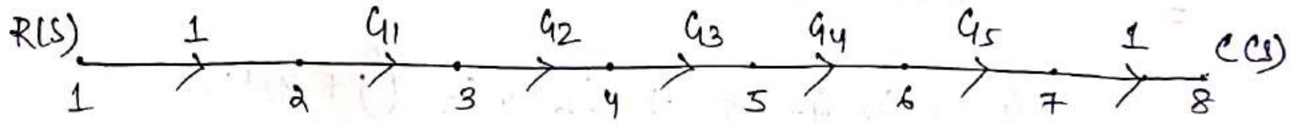
Steps for solving signal flow graph :—

1. Identify all forward paths.
2. Calculate all forward path gain.
3. Identify all feedback loops.
4. Calculate all the loop gain.
5. Identify all feedback loop that don't touch eachother.
6. Calculate the gain of two non-touching loop.
7. Calculate the value of Δ and Δ_k.
8. Put all the values in the Mason's gain formula.
9. Calculate the overall gain.

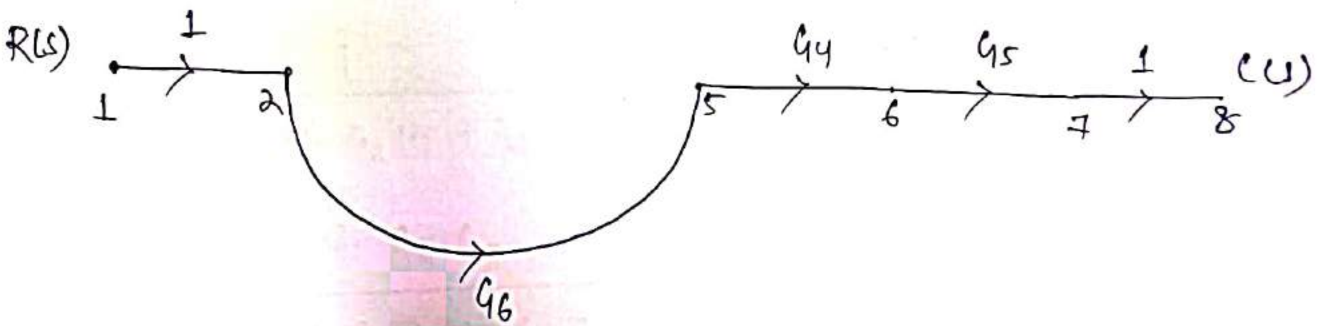
Problem-1 : Find the overall transfer function of the system whose signal flow graph is shown in figure.



Ans : No. of forward paths = 2



$$P_1 = G_1 G_2 G_3 G_4 G_5$$



$$P_2 = G_6 G_4 G_5$$

$\Delta = 1 -$ (sum of individual loops) + (sum of two non-touching loops) + ...

Loops: $L_1 = -G_2H_1$

$L_2 = -G_5H_3$

$L_3 = -G_2G_3H_2$

two non-touching loops : L_1 and L_2

L_3 and L_2

$P_{21} = G_2G_5H_1H_3$

$P_{22} = G_2G_3G_5H_2H_3$

$$\Delta = 1 - (-G_2H_1 - G_5H_3 - G_2G_3H_2) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$

$$= 1 + G_2H_1 + G_5H_3 + G_2G_3H_2 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

$$\Delta_K = \Delta_1 = 1 - 0 = 1$$

$$\Delta_2 = 1 - (-G_2H_1)$$

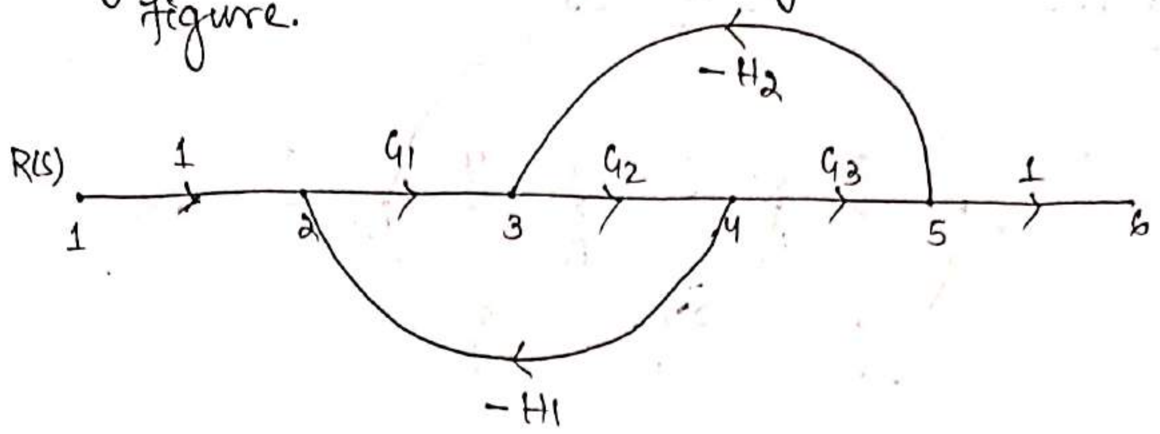
$$= 1 + G_2H_1$$

$$T = \frac{1}{\Delta}\sum_{K=1}^{N} P_K \Delta_K$$

$$= \cancel{G_2G_5H_1H_3} +$$

$$= \frac{G_1G_2G_3G_4G_5 + G_6G_4G_5(1 + G_2H_1)}{1 + G_2H_1 + G_5H_3 + G_2G_3H_2 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3}$$

Problem-2: Find the overall transfer function of the system, whose signal flow graph is shown in the figure.



Ans: No. of forward path = 1

Forward path gain = $G_1 G_2 G_3$

Individual loop gain = $L_1 = -G_1 G_2 H_1$

$L_2 = -G_2 G_3 H_2$

No. of non-touching loop = 0

$\Delta = 1 - (L_1 + L_2) - 0$

$= 1 - (-G_1 G_2 H_1 - G_2 G_3 H_2) - 0$

$\Delta = 1 + G_1 G_2 H_1 + G_2 G_3 H_2$

$\Delta_K = \Delta_1 = 1 - 0$

$= 1$

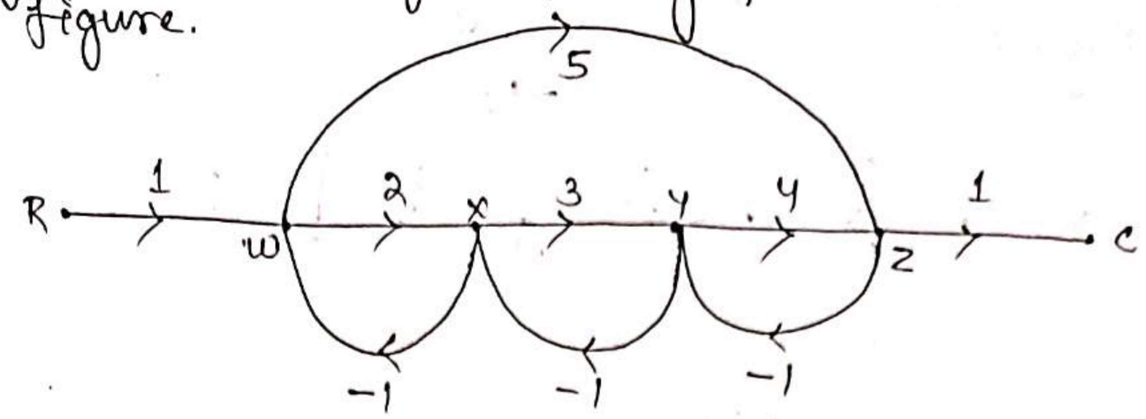
Applying Mason's gain formula:

$$T = \frac{1}{\Delta} \sum_{K=1}^{n} P_K \cdot \Delta_K$$

$$= \frac{1}{1 + G_1 G_2 H_1 + G_2 G_3 H_2} \times G_1 G_2 G_3 \times 1$$

$$= \frac{G_1 G_2 G_3}{1 + G_1 G_2 H_1 + G_2 G_3 H_2}$$

Problem - 3: Find the overall transfer function of the system, whose signal flow graph is shown in the figure.



Ans: No. of forward path = 2

$$P_1 = 1 \times 2 \times 3 \times 4 \times 1 = 24$$

$$P_2 = 1 \times 5 \times 1 = 5$$

Individual loops : $L_1 = -2$ $(w - x - w)$

$L_2 = -3$ $(x - y - x)$

$L_3 = -4$ $(y - z - y)$

$L_4 = -5$ $(w - z - y - x - w)$

L_1 and L_3 are non-touching loop.

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_3) - 0$$

$$= 1 - (-2 - 3 - 4 - 5) + (-2 \times -4)$$

$$= 1 - (-14) + 8$$

$$= 1 + 14 + 8$$

$$= 23$$

Since there are two forward paths, there will be 2 Δ_K.

$$\Delta_1 = 1 - 0$$
$$= 1$$

$$\Delta_2 = 1 - (-3)$$
$$= 1 + 3$$
$$= 4$$

$P_1 = 24 \qquad \Delta_1 = 1 \qquad \Delta = 23$

$P_2 = 5 \qquad \Delta_2 = 4$

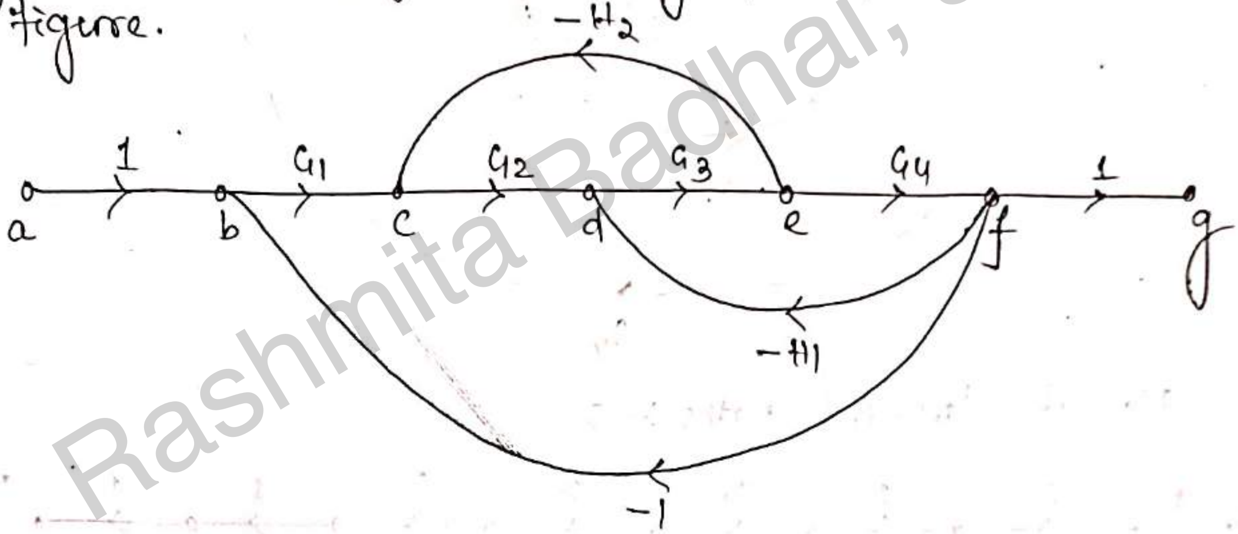
Mason's gain formula $= \frac{C}{R} = \frac{1}{\Delta}\sum_{K=1}^{N} P_K \Delta_K$

$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2.\Delta_2)$$

$$= \frac{1}{23} \times (24 + 20)$$

$$= \frac{44}{23}$$

Problem-4: Find the overall transfer function of the system, whose signal flow graph is shown in the figure.



Ans: No. of forward path = 1

Forward path gain = $G_1 G_2 G_3 G_4$

Individual loop gain ; $L_1 = -G_2G_3H_2$

$L_2 = -G_3G_4H_1$

$L_3 = -G_1G_2G_3G_4$

There are no non-touching loops.

$$\Delta = 1 - (-G_2G_3H_2 - G_3G_4H1 - G_1G_2G_3G_4) + 0$$

$$= 1 + G_2G_3H_2 + G_3G_4H1 + G_1G_2G_3G_4$$

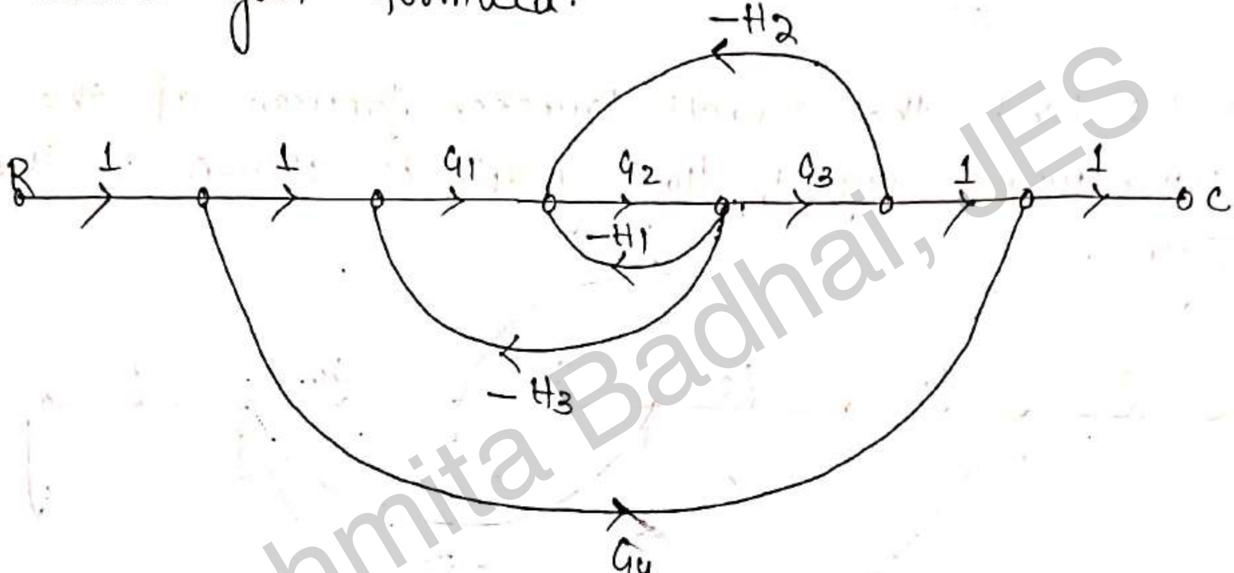
$\Delta_K = 1 - 0$

$\Delta_1 = 1$

Mason's gain formula $= \dfrac{1}{\Delta} \sum_{K=1}^{n} P_K \Delta_K$

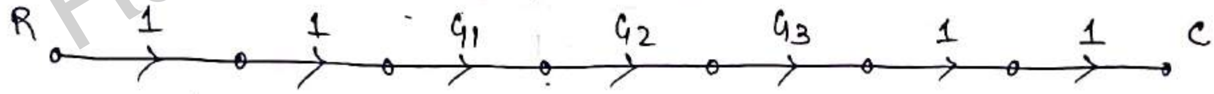
$$= \frac{G_1 G_2 G_3 G_4}{1 + G_2 G_3 H_2 + G_3 G_4 H_1 + G_1 G_2 G_3 G_4}$$

Problem - 5: Find the gain of the given system $\left(\frac{C}{R}\right)$ using Mason's gain formula.

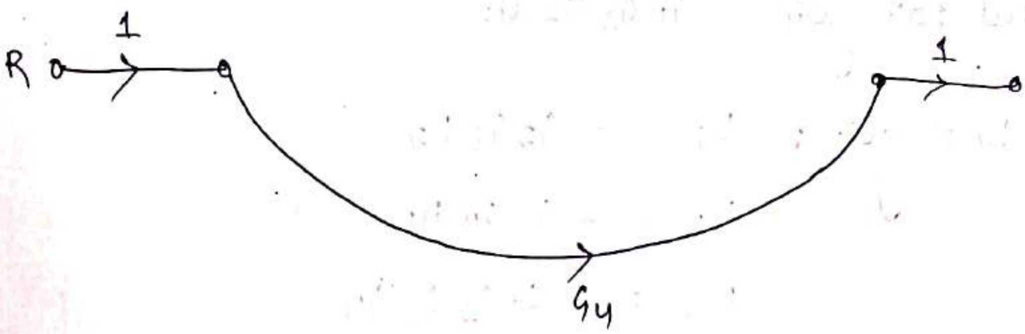


Ans: No. of forward paths :- 2

P_1



$P_1 = G_1 G_2 G_3$



$P_2 = G_4$

Individual loop gain : $L_1 = -G_2H_1$

$L_2 = -G_1G_2H_3$

$L_3 = -G_2G_3H_2$

There is no non-touching loops.

So, $\Delta = 1 - [-G_2H_1 - G_1G_2H_3 - G_2G_3H_2] + 0$

$= 1 + G_2H_1 + G_1G_2H_3 + G_2G_3H_2$

$\Delta_K = \Delta_1 = 1$

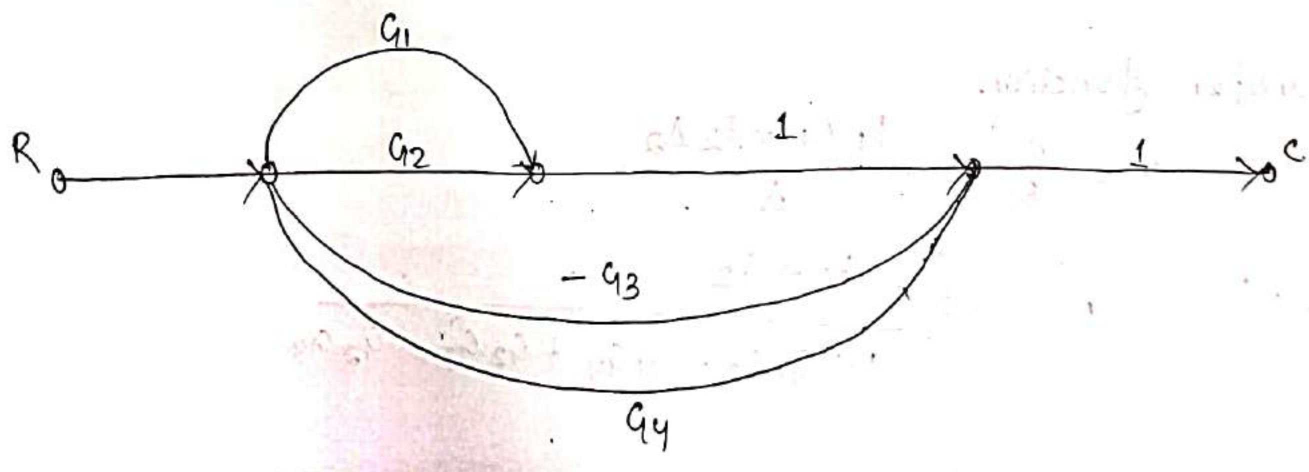
$\Delta_2 = 1 - [-G_2H_1 - G_2G_3H_2 - G_1G_2H_3]$

$= 1 + G_2H_1 + G_2G_3H_2 + G_1G_2H_3$

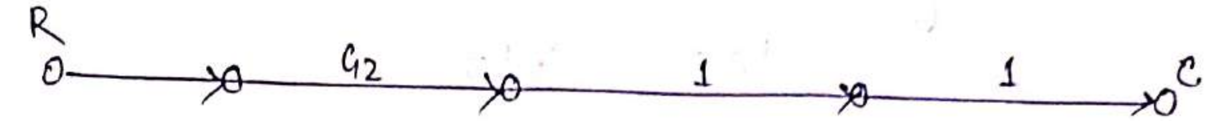
$$T = \frac{1}{\Delta}\sum_{K=1}^{N} P_K \Delta_K$$

$$= \frac{G_1G_2G_3 + G_4(1 + G_2H_1 + G_2G_3H_2 + G_1G_2H_3)}{1 + G_2H_1 + G_2G_3H_2 + G_1G_2H_3}$$

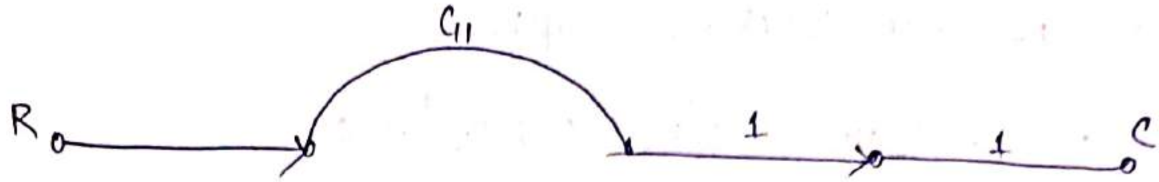
Problem-6 : Find the gain of the given system $\left(\frac{C}{R}\right)$ using Mason's gain formula.



Ans: Number of forward path $= 2$



$P_1 = G_2$



$P_2 = G_1$

There are four loops.

Loop gains are $L_1 = -G_1 G_3$

$L_2 = G_1 G_4$

$L_3 = -G_2 G_3$

$L_4 = G_2 G_4$

There are no non-touching loops

$$\Delta = 1 - [-G_1 G_3 + G_1 G_4 - G_2 G_3 + G_2 G_4]$$

$$= 1 + G_1 G_3 - G_1 G_4 + G_2 G_3 - G_2 G_4$$

Forward paths 1 and 2 touch all the loops.

Therefore, $\Delta_1 = 1 - 0 = 1$

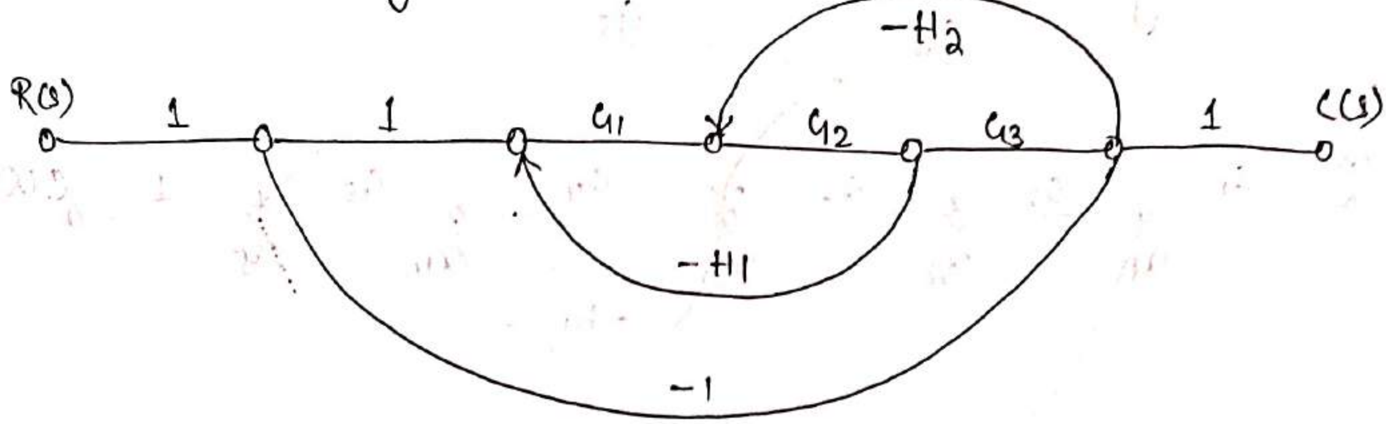
$\Delta_2 = 1 - 0 = 1$

Transfer function

$$T = \frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 + G_2}{1 + G_1 G_3 - G_1 G_4 + G_2 G_3 - G_2 G_4}$$

Problem-7: Find the gain of the given system $\left(\frac{C}{R}\right)$ using Mason's gain formula.



Ans: Number of forward path = 1

$$P_1 = G_1 G_2 G_3$$

There are three loops. Loop gains $L_1 = -G_1 G_2 H_1$

$$L_2 = -G_2 G_3 H_2$$

$$L_3 = -G_1 G_2 G_3$$

There are ~~three~~ no non touching loops

$$\Delta = 1 - [-G_1G_2H_1 - G_2G_3H_2 - G_1G_2G_3]$$

$$= 1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$$

Forward path touches all the loops. Therefore

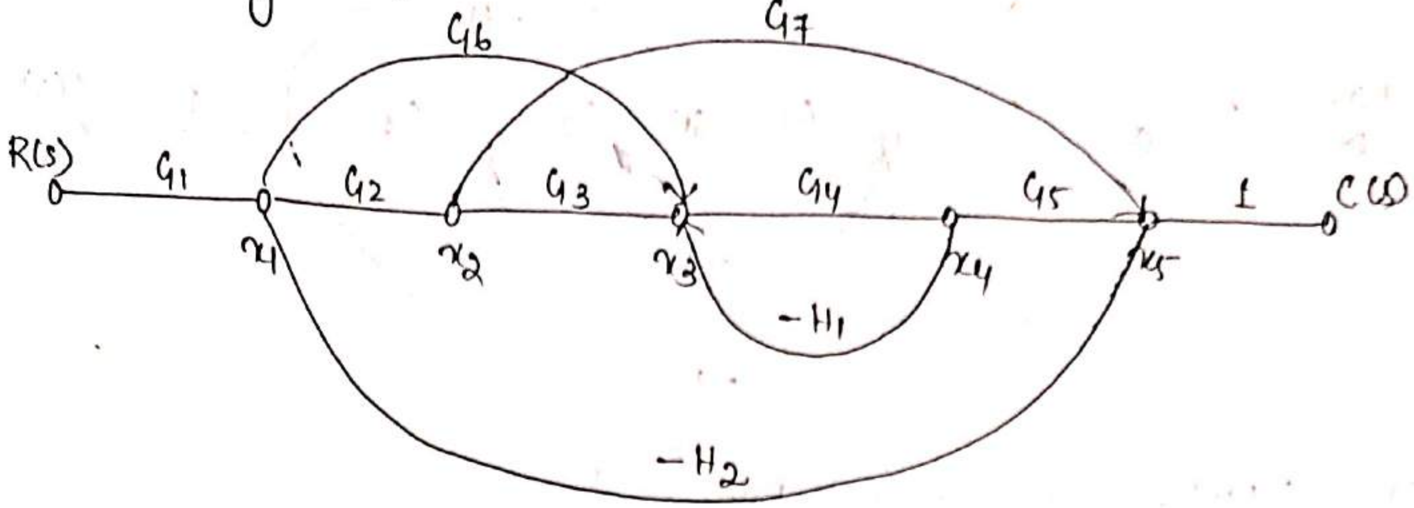
$$\Delta_K = 1$$

Transfer function,

$$T = \frac{C}{R} = \frac{P_1 \Delta_K}{\Delta}$$

$$= \frac{G_1 G_2 G_3}{1 + G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

Problem-8: Find the gain of the given system $\left(\frac{C}{R}\right)$ using Mason's gain formula.



Ans: There are three forward paths.

The gain of the forward path are:

$$P_1 = G_1 G_2 G_3 G_4 G_5$$

$$P_2 = G_1 G_6 G_4 G_5$$

$$P_3 = G_1 G_2 G_7$$

There are four loops with loop gains:

$$L_1 = -G_4 H_1$$

$$L_2 = -G_2 G_7 H_2$$

$$L_3 = -G_6 G_4 G_5 H_2$$

$$L_4 = -G_2 G_3 G_4 G_5 H_2$$

There is one combination of loops L_1 and L_2 which are non touching with loop gain product

$$L_1 L_2 = G_2 G_7 H_2 G_4 H_1$$

$$\Delta = 1 - [-G_4 H_1 - G_2 G_7 H_2 - G_6 G_4 G_5 H_2 - G_2 G_3 G_4 G_5 H_2] + G_2 G_4 G_7 H_1 H_2$$

$$= 1 + G_4 H_1 + G_2 H_2 G_7 + G_6 G_4 G_5 H_2 + G_2 G_3 G_4 G_5 H_2 + G_2 G_4 G_7 H_1 H_2$$

Forward path 1 and 2 touch all the four loops.

Therefore, $\Delta_1 = 1$, $\Delta_2 = 1$

Forward path 3 is not in touch with loop 1 .

Hence, $\Delta_3 = 1 + G_4 H_1$

So, the transfer function,

$$T = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_4G_5G_6 + G_1G_2G_3(1+G_4H_1)}{1 + G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_2G_4G_7H_1H_2}.$$

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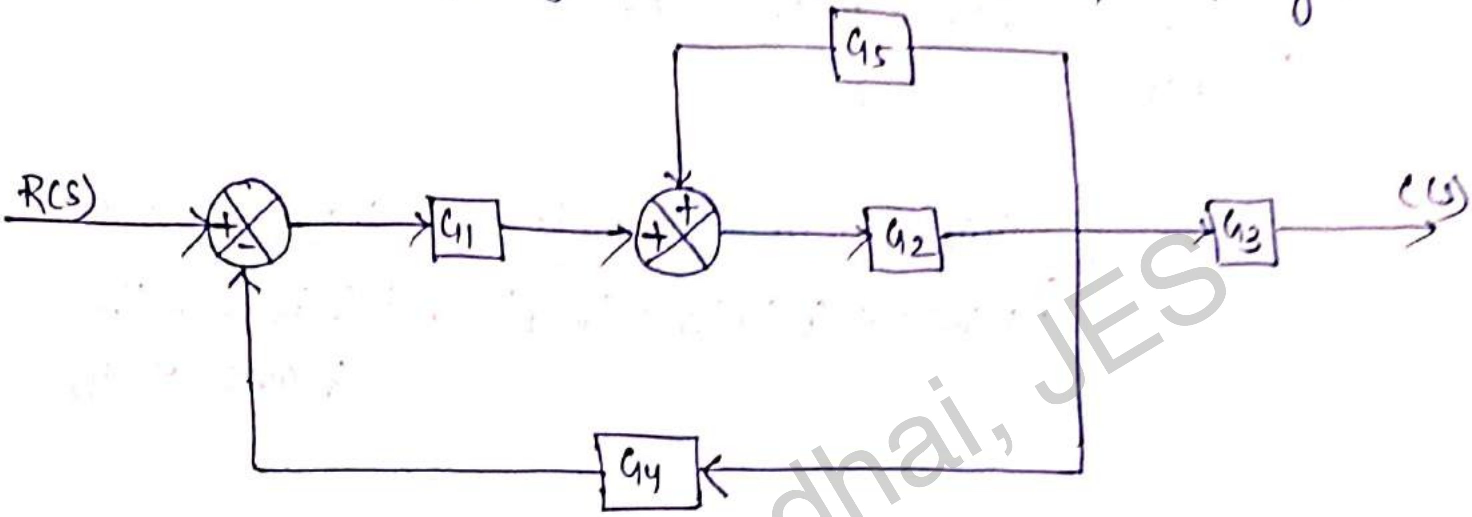
Assignment - 4

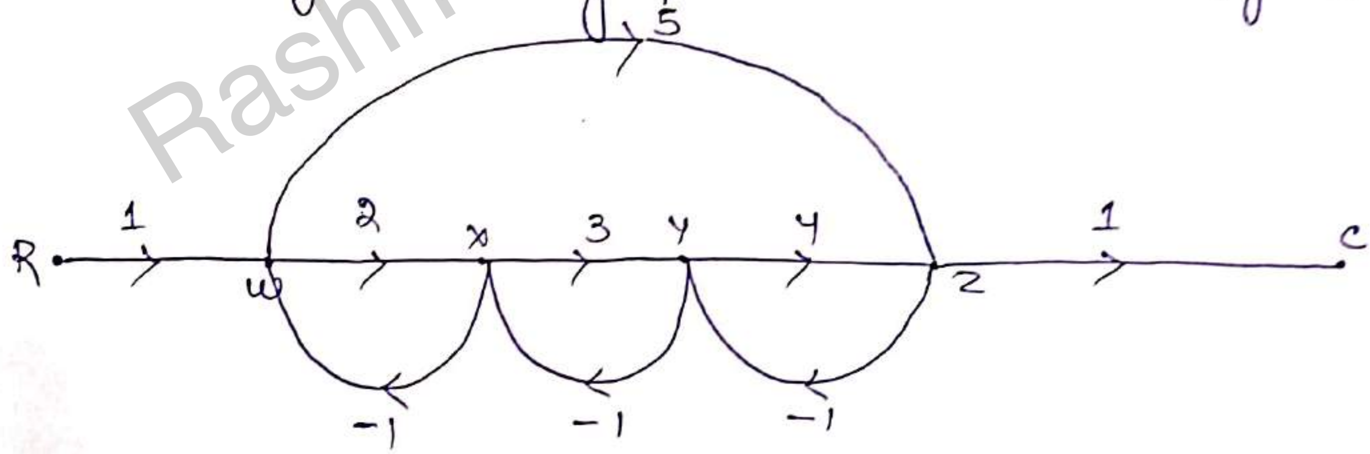
1. What do you mean by block diagram in control systems?

2. Explain the canonical form of closed loop system?

3. Find the transfer function of the closed loop system



4. Find the overall transfer function of the system, whose signal flow graph is shown in the figure.



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Chapter - 5

Time Domain Analysis of Control systems

Syllabus: Defination of Time, Stability, Steady-state response, accuracy, In-sencitivity and robustness. System Time response, Analysis of steady state error, Types of input and steady state error (step, ramp, parabolic), Parameters of first order system and second order systems, Derivation of time response specification (Delay time, Rise time, peak time, setting time, peak overshoot).

Definations :—

Time domain is the analysis of mathematical functions, physical signals with respect to time.
In the time domain, the signal or function's value is known for all real numbers, for the case of continuous time or at various separate instants in the case of discrete time.

Defination of time :— Time is the indefinite continued progress of existence and events that occur in an apparently irreversible succession from the past, through the present, into the future.

Defination of stability :— A system is said to be stable, if its output is under control. Otherwise, it is said to be unstable. A stable system produces a bounded output for a given bounded input.
we can classify the systems based on stability as

(1) Absolutely stable system
(2) Conditionally stable system
(3) Marginally stable system.

If a system is stable for all the range of system component values, then it is known as the absolutely stable system.

If the system is stable for a certain range of system component values, then it is known as conditionally stable system.

If the system is stable by producing an output signal with constant amplitude and constant frequency of oscillations for bounded input, then it is known as marginally stable system.

Defination of steady state response:—

The steady state response of the system is the response of the system for a given input that remains after the transient has died out.

Defination of accuracy:—

In a set of measurements, accuracy is closeness of the measerements to a specific value, while precision is the closeness of the measurements to each other.

Defination of Transient Accuracy:—

Transient analysis calculates a circuit's response over a period of time defined by the user. The accuracy of the transient analysis is dependent on the size of internal time steps, which together make up the complete simulation time known as the Run time or stop time.

Robustness :—

Robustness is the property of being strong and healthy in constitution. When it is transposed into a system, it refers to the ability of tolerating perturbation that might affect the system's functional body.

System Time Response :—

Time response c(t) is the variation of output with respect to time. The part of time response that goes to zero after large interval of time is called transient response $C_{tr}(t)$. The part of time response that remains after transient response is called steady state response $C_{ss}(t)$.

If the output of control system for an input varies with respect to time is called the time response of the control system.

The time response consists of two parts.

(1) Transient response

(2) Steady state response.

Mathematically

$$C(t) = C_{tr}(t) + C_{ss}(t)$$

Transient Response :—

After applying input to the control system, output takes certain time to reach steady state. So the output will be in transient state till it goes to a steady state. Therefore, the response of the control system during the transient state is known as Transient response.

The transient response will be zero for large values of 't'. Ideally, this value of 't' is infinity and practically, it is five times constant.

Steady state response :—

The part of the time response that remains even after the transient response has zero value for large values of 't' is known as steady state response. This means, the transient response will be zero even during the steady state.

Example :—

let us find the transient and steady state terms of the time response of the control system.

$$c(t) = 10 + 5e^{-t}$$

Here the second term $5e^{-t}$ will be zero as t denotes infinity. So, this is the transient term. And the first term 10 remains even t approaches infinity. So, this is the steady state term.

Analysis of steady state error :—

For a step excitation the difference between the desired output and the final value of the output of a system is term as steady state error of the system.

The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as e_{ss}.

We can find steady state error using the final value theorem.

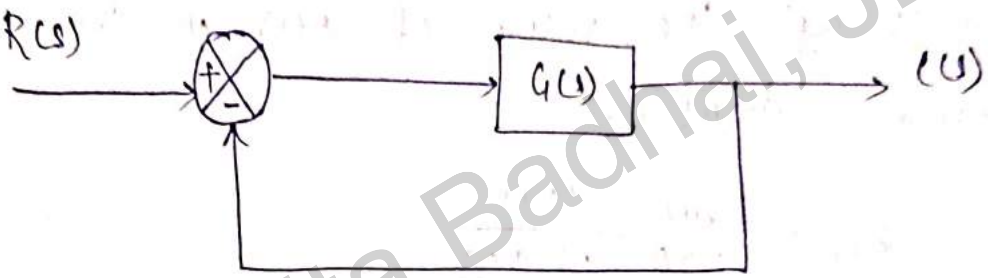
$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s\,E(s)$$

where,

$E(s)$ is the laplace transform of the error signal $e(t)$.

Steady state errors for unity feedback systems:—

consider the following block diagram of closed loop control system, which is having unity negative feedback.



Where,

$R(s)$ is the laplace transform of the reference input signal $r(t)$

$C(s)$ is the laplace transform of the output signal $c(t)$.

We know, The transfer function of the unity negative feedback closed loop control system as

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$\Rightarrow C(s) = \frac{R(s) \cdot G(s)}{1 + G(s)}$$

The output of the summing point is.

$$E(s) = R(s) - C(s)$$

By substituting the value of $C(s)$

$$E(s) = R(s) - \frac{R(s) \cdot G(s)}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s) + \cancel{R(s) \cdot G(s)} - \cancel{R(s) \cdot G(s)}}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1+G(s)}$$

By substituting the value of $E(s)$ in the steady state error formula.

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}$$

The following table shows the steady state error and the error constants for standard input signals like unit step, unit ramp & unit parabolic signals.

Unit step signal :—

Steady state error : $\frac{1}{1+K_p}$

Error constant, $K_p = \lim_{s \to 0} G(s)$

K_p = Position error constant.

Unit Ramp signal :

Steady state error $e_{ss} = \frac{1}{K_v}$

$$K_v = \lim_{s \to 0} s\,G(s)$$

K_v = velocity error constant

Unit parabolic signal :

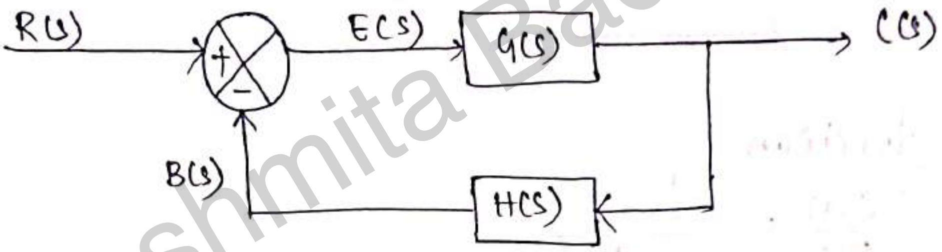
Steady state error $e_{ss} = \frac{1}{K_a}$

$$K_a = \lim_{s \to 0} s^2\,G(s)$$

K_a = acceleration error constant

Steady state error for a negative feedback system :—

A simple closed loop control system with negative feedback is



$$E(s) = R(s) - B(s)$$

$$B(s) = C(s) \cdot H(s)$$

$$C(s) = E(s) \cdot G(s)$$

$$E(s) = R(s) - C(s) \cdot H(s)$$

$$\Rightarrow E(s) = R(s) - E(s) \cdot G(s) \cdot H(s)$$

$$\Rightarrow E(s) + E(s) \cdot G(s) \cdot H(s) = R(s)$$

$$\Rightarrow E(s)\left[1 + G(s) \cdot H(s)\right] = R(s)$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s) \cdot H(s)}$$

Therefore, steady state error depends on two factors.

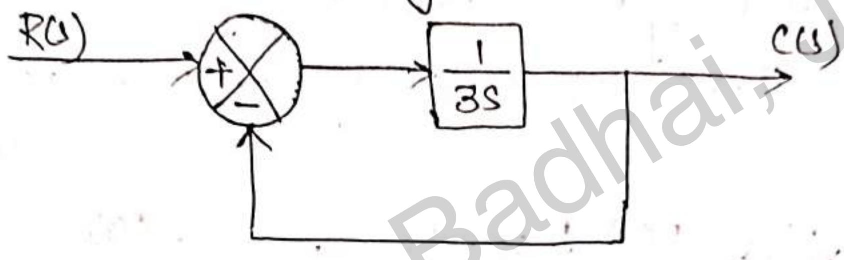
(i) Types and magnitude of $R(s)$

(ii) Open loop transfer function $G(s).H(s)$

Order of system :—

The order of a system is the highest power of 's' in the denominator of system transfer function. Accordingly a system is categorised as first order system, second order system etc.

Example : Find the order of the system represented by following block diagram.



Ans: Transfer function

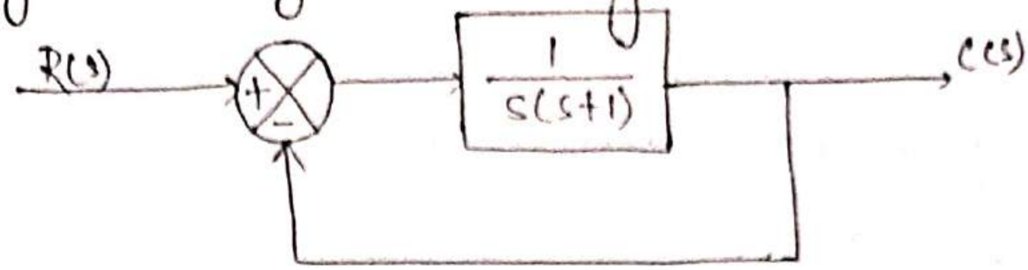
$$\frac{C(s)}{R(s)} = \frac{\frac{1}{3s}}{1+\frac{1}{3s}}$$

$$= \frac{\cancel{1/3s}}{\frac{3s+1}{\cancel{3s}}}$$

$$= \frac{1}{3s+1}$$

The highest power of 's' in denominator is one. So it is a first order system.

Example : Determine the order of the system represented by following block diagram.



Ans:

$$G(s) = \frac{1}{s(s+1)}$$

$$H(s) = 1$$

$$\text{Transfer function} = \frac{G(s)}{1 + G(s).H(s)}$$

$$= \frac{\frac{1}{s(s+1)}}{1 + \frac{1}{s(s+1)}.1}$$

$$= \frac{\frac{1}{\cancel{s(s+1)}}}{\frac{s(s+1)+1}{\cancel{s(s+1)}}}$$

$$= \frac{1}{s^2 + s + 1}$$

The highest power of 's' in denominator of transfer function is two, so it is a second order system.

Example :- The differential equation governing a control system is given by

$$\frac{d^2c}{dt^2} + 5\frac{dc}{dt} + 8c = 3r$$

Find the transfer function and order of the system.

Ans: Taking the laplace transfer of the differential equation

$$s^2 C(s) + 5s\, C(s) + 8\, C(s) = 3R(s)$$

$$\Rightarrow (s^2 + 5s + 8)\cdot C(s) = 3R(s)$$

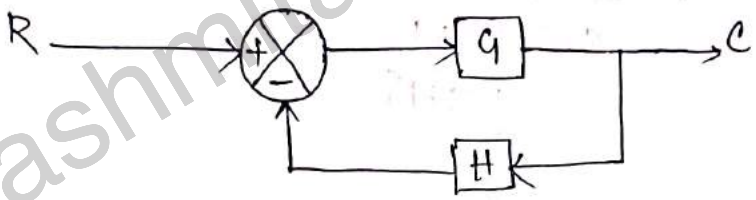
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{3}{s^2 + 5s + 8}$$

The highest power of s in denominator is 2; Therefore it is a second order system.

Types of input and steady state error (Step, ramp, parabolic):—

Types of control system :—

Consider a general feedback control system as represented by block diagram.



The open loop transfer function is GH. The type of control system depend on power of term 's' in the denominator of GH.

GH can be generally expressed as

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots}{s^m (1+T_a s)(1+T_b s)\ldots}$$

If $m=0$, then the system is type '0'.

For $m = 1, 2, \ldots$ categorised system as type 1, type 2 ... system respectively.

m	Type of system
0	Type '0' system
1	Type '1' system
2	Type '2' system
3	Type '3' system.

Example: The open loop transfer function of some system are given below. Name the type of the system.

(a) $\frac{10}{(s+1)(s^2+3s+4)}$ (b) $\frac{9}{s(s+1)(s+2)}$ (c) $\frac{3(s+5)}{s^2(s+3)(s+8)(s+9)(s+10)}$

Ans: (a) $G(s).H(s) = \frac{10}{(s+1)(s^2+3s+4)}$

s^m in denominator is $1 = s^0$

So, $m=0$, it is a type '0' system.

(b) $G(s).H(s) = \frac{9}{s(s+1)(s+2)}$

s^m in denominator is s^1

So, $m=1$, it is a type '1' system.

(c) $G(s).H(s) = \frac{3(s+5)}{s^2(s+3)(s+8)(s+9)(s+10)}$

s^m in denominator is s^2

So, $m=2$, it is a type '2' system.

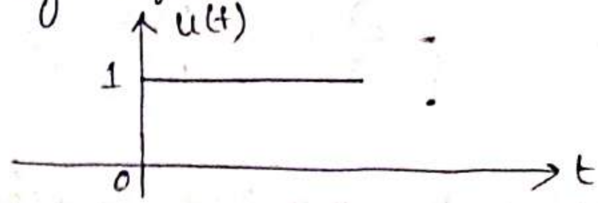
→ Types of inputs :—

The type of inputs are step, ramp and parabolic signals. These signals are used to know the performance of the control systems using time response of the output.

(1) Unit step signal :—

A unit step function is represented by u(t) and is expressed mathematically by equation

$$u(t) = 1 \;;\; t \geq 0$$
$$= 0 \;;\; t < 0$$



Laplace transform of a unit step function is

$$\mathcal{L}\{u(t)\} = \int_0^\infty u(t) \cdot e^{-st} \, dt$$

$$= \int_0^\infty e^{-st} \, dt$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^\infty$$

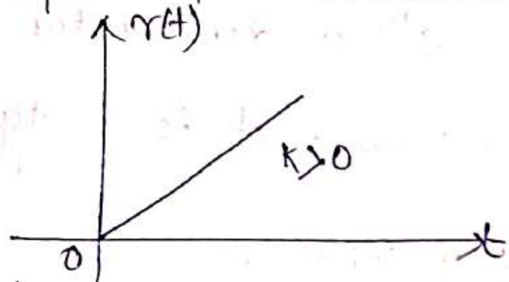
$$= \frac{1}{s}$$

Laplace transform of a step function of magnitude K is equal to $\frac{K}{s}$.

(2) Unit Ramp signal :—

A ramp function varies linearly with time. It is represented as r(t) and is expressed mathematically as

$$r(t) = K(t) \;;\; t \geq 0$$
$$= 0 \;;\; t < 0$$



Laplace transform of ramp function is given by

$$\mathcal{L}\{r(t)\} = \int_0^\infty r(t) \cdot e^{-st} \, dt$$

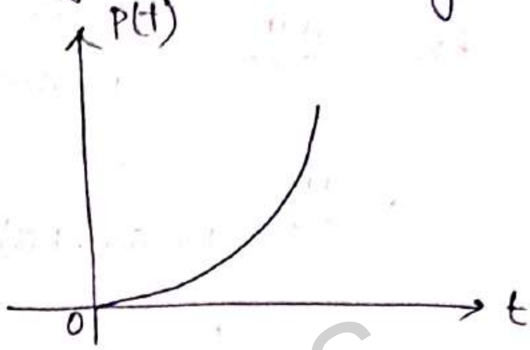
$$= \int_0^\infty Kt \, e^{-st} \, dt \quad = \frac{K}{s^2}$$

(3) Unit parabolic function :—

Parabolic function used as a test input, the ~~input~~ unit parabolic signal exists for all the positive values of 't' including zero. And its value increases non-linearly with respect to 't' during this interval. The value of the unit parabolic signal is zero for all the negative values of 't'.

$$P(t) = \frac{t^2}{2} \; ; \; t \geq 0$$

$$= 0 \; ; \; t < 0$$



Laplace transform of parabolic signal is given by

$$\mathcal{L}\left\{\frac{t^2}{2}\right\} = \frac{1}{s^3}$$

Steady state error :—

The steady state error can be expressed by two types of constants.

(i) Static error coefficients

(ii) Dynamic error coefficients.

Static error coefficient :—

Static error coefficient depends on open loop transfer function $G(s).H(s)$. There are three types of static error coefficients.

(i) Static position error constant (K_p) : It is defined as

$$K_p = \lim_{s \to 0} G(s).H(s)$$

(ii) Static velocity error constant (K_v) : It is defined as

$$K_v = \lim_{s \to 0} s.G(s).H(s)$$

(iii) Static Acceleration constant (K_a) : It is defined as

$$K_a = \lim_{s \to 0} s^2.G(s).H(s)$$

Steady state error for different types of system :—

(a) Steady state error with unit step input.

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{R(s)}{1+G(s)\cdot H(s)}$$

For unit step input, $R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1/s}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{1}{1+G(s)\cdot H(s)}$$

$$= \frac{1}{1+\lim_{s\to 0} G(s)\cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{1+K_p}}$$

where, $K_p = \lim_{s\to 0} G(s)\cdot H(s)$.

(b) Steady state error with unit ramp input.

For unit ramp input $R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s\to 0} \frac{s\left(\frac{1}{s^2}\right)}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{1}{s[1+G(s)\cdot H(s)]}$$

$$= \lim_{s\to 0} \frac{1}{s+s\cdot G(s)\cdot H(s)}$$

$$= \frac{1}{\lim_{s\to 0} s\, G(s)\cdot H(s)}$$

where, $K_v = \lim_{s\to 0} s\cdot G(s)H(s)$

$$= \frac{1}{K_v}$$

(c) Steady state error with unit parabolic signal:

For unit parabolic input $R(s) = \frac{1}{s^3}$

$$e_{ss} = \lim_{s \to 0} \frac{s\left(\frac{1}{s^3}\right)}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^2(1 + G(s) \cdot H(s))}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s) \cdot H(s)}$$

$$= \frac{1}{\lim_{s \to 0} s^2 G(s) \cdot H(s)}$$

$$= \frac{1}{K_a}$$

where, $K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s)$.

For unit step input:

let $G(s) \cdot H(s) = \frac{K(1+sT_1)(1+sT_2) \dots}{s^m (1+sT_a)(1+sT_b) \dots}$

For type zero system, $m = 0$

$$K_p = \lim_{s \to 0} G(s) \cdot H(s) = K$$

$$e_{ss} = \frac{1}{1+K}$$

For type one system, $m = 1$

$$K_p = \lim_{s \to 0} G(s) \cdot H(s) = \infty$$

$$e_{ss} = \frac{1}{1+\infty} = 0$$

For type two system, $m = 2$

$$k_p = \lim_{s \to 0} G(s) \cdot H(s) = \infty$$

$$e_{ss} = \frac{1}{1 + k_p}$$

$$= \frac{1}{1 + \infty} = 0$$

So, steady state error due to unit step input is $\frac{1}{1+K}$ for '0' type system and zero for higher type systems.

For unit ramp input :—

For type '0' system, $m = 0$

$$K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s) = 0$$

$$e_{ss} = \frac{1}{0} = \infty$$

For type '1' system, $m = 1$

$$K_v = \lim_{s \to 0} s \cdot G(s) \cdot H(s).$$

$$= K$$

$$e_{ss} = \frac{1}{K}$$

For type '2' system, $m = 2$

$$K_v = \lim_{s \to 0} s\, G(s) \cdot H(s) = \infty$$

$$e_{ss} = \frac{1}{\infty} = 0$$

So, steady state error due to unit ramp input is infinite for type '0' system, $\frac{1}{K}$ for type 1 system and zero for second and higher order system.

For unit parabolic input :-

For type 0 system, $m=0$

$$K_a = \lim_{s \to 0} s^2 G(s).H(s) = 0$$

$$e_{ss} = \frac{1}{K_a} = \infty$$

For type 1 system, $m=1$

$$K_a = \lim_{s \to 0} s^2 G(s).H(s) = 0$$

$$e_{ss} = \frac{1}{0} = \infty$$

For type-2 system, $m=2$

$$K_a = \lim_{s \to 0} s^2 G(s).H(s) = K$$

$$e_{ss} = \frac{1}{K}$$

Types of System	Error Constant			Steady state error (e_{ss})		
				$\frac{1}{1+K_p}$	$\frac{1}{K_v}$	$\frac{1}{K_a}$
	$K_p = \lim_{s \to 0} G(s).H(s)$	$K_v = \lim_{s \to 0} s\, G(s).H(s)$	$K_a = \lim_{s \to 0} s^2 G(s).H(s)$	Unit Step input	Unit Ramp input	Unit parabolic input
0	K	0	0	$\frac{1}{1+K}$	∞	∞
1	∞	K	0	0	$\frac{1}{K}$	∞
2	∞	∞	K	0	0	$\frac{1}{K}$
3	∞	∞	∞	0	0	0

Example : The forward path transfer function G(s) and feedback transfer function H(s) of a control system are

$$G(s) = \frac{10}{(s+2)(s^2+10s+10)}$$

$$H(s) = \frac{5}{s+4}$$

Find the steady state error for following inputs.

(i) $r(t) = 4$

(ii) $r(t) = 4t$

(iii) $r(t) = 4t^2$

Ans: Open loop transfer function $= G(s) \cdot H(s)$

$$= \frac{10}{(s+2)(s^2+10s+10)} \cdot \frac{5}{s+4}$$

$$= \frac{50}{(s+2)(s+4)(s^2+10s+10)}$$

(i) $r(t) = 4$

$$R(s) = \frac{4}{s}$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{s \cdot \frac{4}{s}}{1 + G(s) \cdot H(s)}$$

$$= \frac{4}{1 + \lim_{s \to 0} G(s) \cdot H(s)}$$

$$\lim_{s\to 0} G(s).H(s) = \frac{50}{\lim_{s\to 0} (s+2)(s+4)(s^2+10s+10)}$$

$$= \frac{50}{80} = \frac{5}{8}$$

$$\therefore e_{ss} = \frac{4}{1+\frac{5}{8}} = \frac{4}{\frac{8+5}{8}} = \frac{4\times 8}{13}$$

$$= \frac{32}{13}$$

(ii) $r(t) = 4t$

$R(s) = \frac{4}{s^2}$

$$e_{ss} = \lim_{s\to 0} s.\frac{R(s)}{1+G(s).H(s)}$$

$$= \lim_{s\to 0} \frac{s.\frac{4}{s^2}}{1+G(s).H(s)}$$

$$= \lim_{s\to 0} \frac{4}{s+s.G(s).H(s)}$$

$$= \frac{4}{\lim_{s\to 0} s.G(s).H(s)}$$

$$= \frac{4}{K_v}$$

It is a type '0' system

$$K_v = \lim_{s\to 0} s.G(s).H(s)$$

$$= 0$$

$$\therefore e_{ss} = \frac{4}{0}$$

$$= \infty$$

(iii) $r(t) = 4t^2$

$$R(s) = 2 \cdot \frac{4}{s^3} = \frac{8}{s^3}$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{R(s)}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{s \frac{8}{s^3}}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{8}{s^2 + s^2 G(s) \cdot H(s)}$$

$$= \frac{8}{\lim_{s \to 0} s^2 G(s) \cdot H(s)}$$

$$= \frac{8}{K_a}$$

It is a type '0' system.

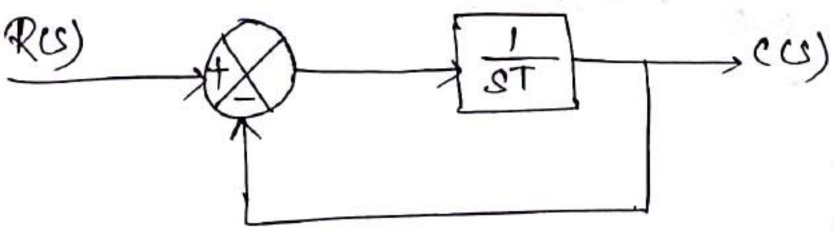
$K_a = 0$

So, $e_{ss} = \frac{8}{0} = \infty$

Parameters of first order system and second order system:-

First order system :—

The block diagram of closed loop unity feedback system is



(Block diagram of first order system)

Here, $G(s) = \frac{1}{sT}$

$H(s) = 1$

$$\text{Transfer function} = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}$$

$$= \frac{\frac{1}{sT}}{1+\frac{1}{sT}\cdot 1}$$

$$= \frac{\frac{1}{\cancel{sT}}}{\frac{sT+1}{\cancel{sT}}}$$

$$= \frac{1}{sT+1}$$

The highest power of s in denominator is one. So, this represents a first order system.

Responses of first order system :—

1. Unit step response of first order system :—

This response is obtained by applying unit step input to a first order system.

$$\text{Transfer function } \frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

For unit step input $r(t) = 1$

$R(s) = \frac{1}{s}$

$$\text{output } C(s) = \frac{1}{sT+1} \cdot R(s)$$

$$= \frac{1}{s(sT+1)} = \frac{1}{s} - \frac{1}{sT+1}$$

Time response of the system is obtained by taking inverse laplace transform.

$$\mathcal{L}^{-1} C(s) = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{T}{sT+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{T}{sT+1}\right\}$$

$$= 1 - \mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{T}}\right\}$$

$$= 1 - e^{-\frac{t}{T}}$$

Case-1: Time $t = T$ (time constant)

The value of output $c(t)$ when $t = T$ is given by

$$c(t) = 1 - e^{-\frac{T}{T}}$$

$$= 1 - e^{-1}$$

$$= 0.632$$

$$= 63.2\%$$

T is known as the time constant of the system. Time constant is defined as the time required by the signal to attain 63.2% of its final or steady state value.

→ A system is faster when time constant is smaller.

Case-2: $t = 2T$

Then $c(t) = 1 - e^{-\frac{2T}{T}}$

$$= 1 - e^{-2}$$

$$= 0.864 \text{ or } 86.4\%$$

Case-3:— $t = 4T$

Then $c(t) = 1 - e^{-\frac{4T}{T}}$

$= 1 - e^{-4}$

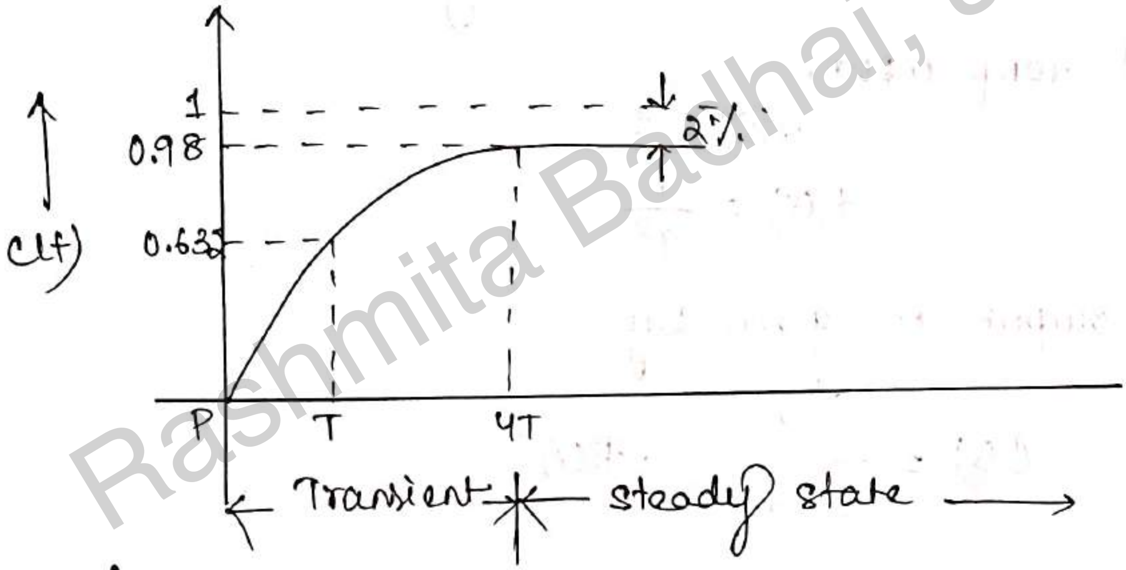
$= 0.98$

$= 98\%$

For a time of 4T, the output is 98%. This time is known as settling time (t_s).

$$t_s = 4T$$

Graphical representation :—



(unit step response of first order system)

Steady state error :—

The error of a control system is given by

$e(t) = r(t) - c(t)$

$= 1 - \left(1 - e^{-\frac{t}{T}}\right)$

$= e^{-\frac{t}{T}}$

The steady state error (e_{ss}) is the value of error of the system when time approaches infinity.

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$= \lim_{t \to \infty} e^{-\frac{t}{T}}$$

$$= 0$$

2. Unit Ramp Response of first order system :—

The response is obtained by applying a unit ramp input to a first order system.

For unit ramp input,

$$r(t) = t$$

$$R(s) = \frac{1}{s^2}$$

The output is given by

$$C(s) = \frac{1}{sT+1} \cdot R(s)$$

$$= \frac{1}{sT+1} \times \frac{1}{s^2}$$

$$= \frac{1-sT}{s^2} + \frac{T^2}{sT+1}$$

$$= \frac{1}{s^2} - \frac{T}{s} + T \cdot \frac{1}{s+\frac{1}{T}}$$

The time response of the system is obtained by taking laplace inverse

$$\mathcal{L}^{-1}[C(s)] = \mathcal{L}^{-1}\left[\frac{1}{s^2} - T\frac{1}{s} + T\cdot\frac{1}{s+\frac{1}{T}}\right]$$

$$\Rightarrow c(t) = t - T + Te^{-\frac{t}{T}}$$

Error $e(t) = r(t) - c(t)$

$$= t - \left(t - T + Te^{-\frac{t}{T}}\right)$$

$$= T - Te^{-\frac{t}{T}}$$

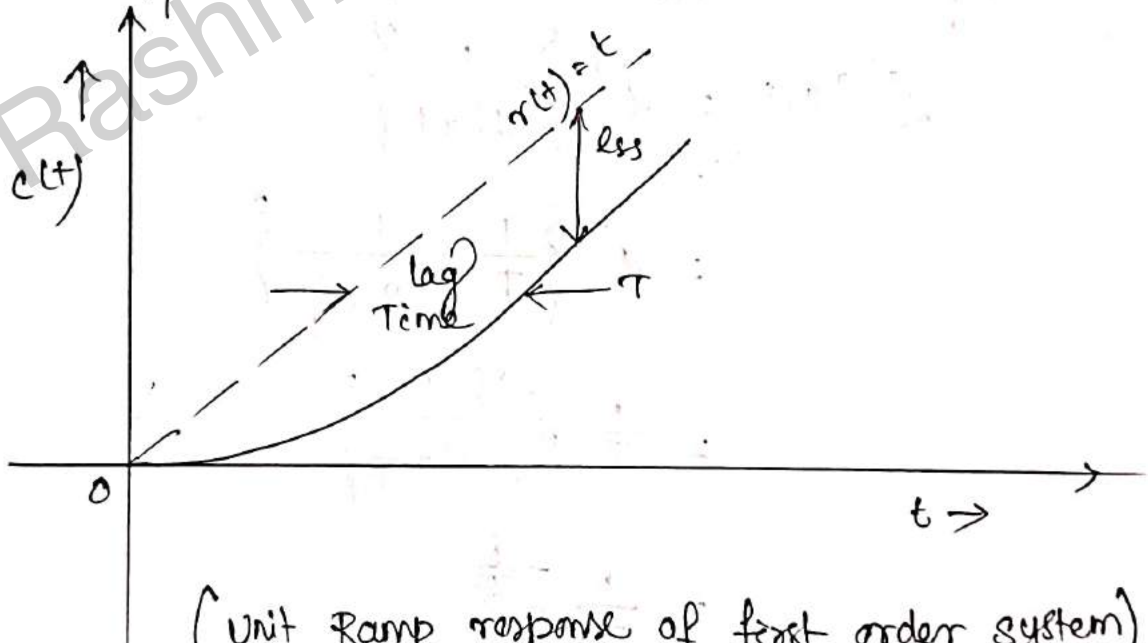
steady state error is given by

$$e_{ss} = \lim_{t\to\infty} T - Te^{-\frac{t}{T}}$$

$$= T - 0$$

$$= T$$

Graphical Representation :—



(Unit Ramp response of first order system)

The response indicates that the output matches the input but lag behind the input by time T. Time T is called lag time.

The time lag and steady state error will be less for a smaller time constant of the system.

3. Unit Impulse response of a first order system :—

The response is obtained when a unit impulse input is applied to a first order control system.

Here, input $r(t) = \delta(t)$

$$R(s) = 1$$

The output of the system is given by

$$C(s) = \frac{1}{sT+1} \cdot R(s)$$

$$= \frac{1}{sT+1}$$

The time response of the system is obtained by taking laplace inverse

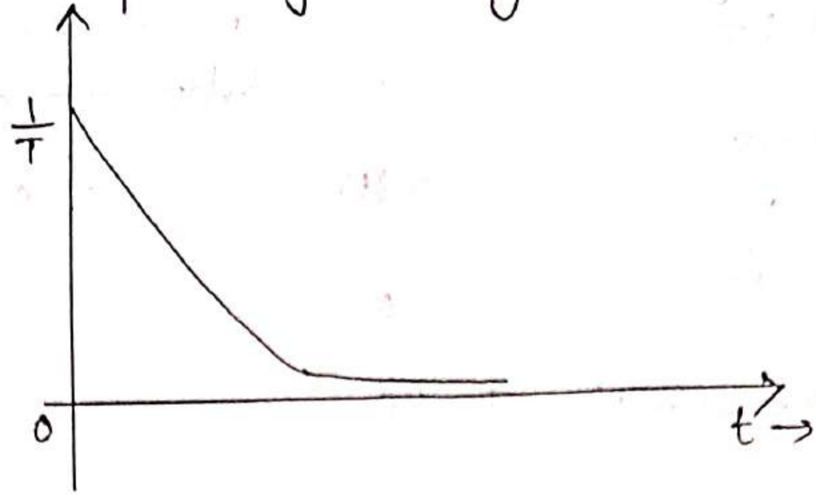
$$\mathcal{L}^{-1}\{C(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{sT+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{T} \cdot \frac{1}{s+\frac{1}{T}}\right\}$$

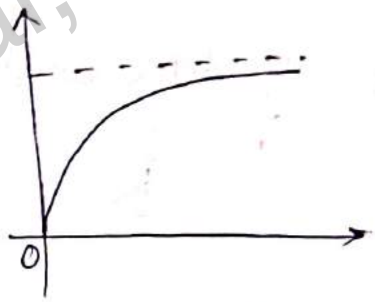
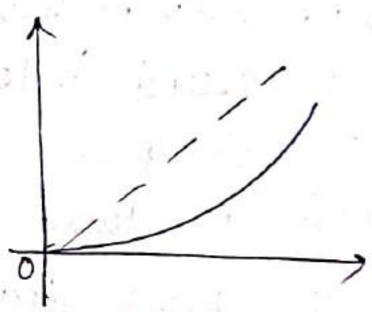
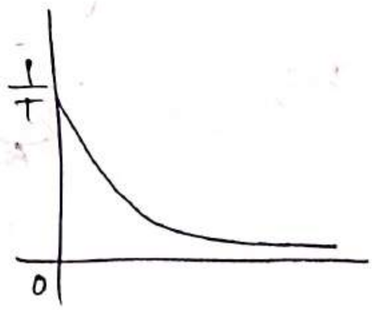
$$= \frac{1}{T}\mathcal{L}^{-1}\left\{\frac{1}{s+\frac{1}{T}}\right\}$$

$$= \frac{1}{T}e^{-\frac{t}{T}}$$

The time response of the system is

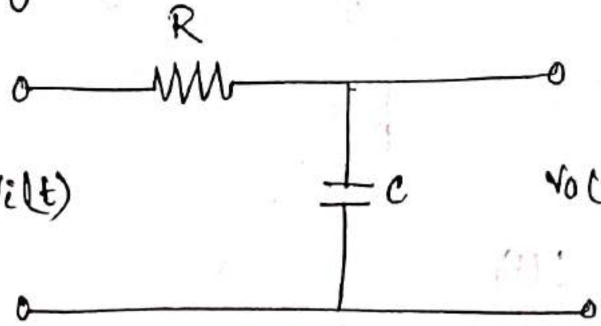


Time Responses of first order system :—

Input		Output	
r(t)	R(s)	c(t)	Graphical Representation
Unit Step $r(t) = 1$	$\frac{1}{s}$	$1 - e^{-\frac{t}{T}}$	
Unit Ramp $r(t) = t$	$\frac{1}{s^2}$	$1 - T - Te^{-\frac{t}{T}}$	
Unit Impulse	1	$\frac{1}{T} e^{-\frac{t}{T}}$	

Examples of 1st order system :—

① RC circuit :—

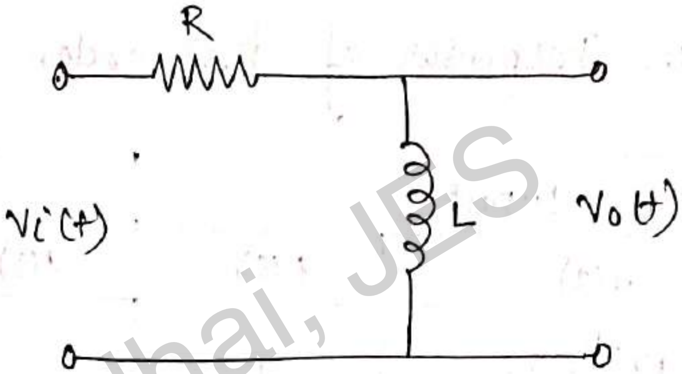


Transfer function

$$= \frac{V_o(s)}{V_i(s)} = \frac{1}{1+RCs}$$

Value of time constant $T = RC$.

② RL circuit :—



Transfer function

$$= \frac{V_o(s)}{V_i(s)} = \frac{Ls}{R+Ls}$$

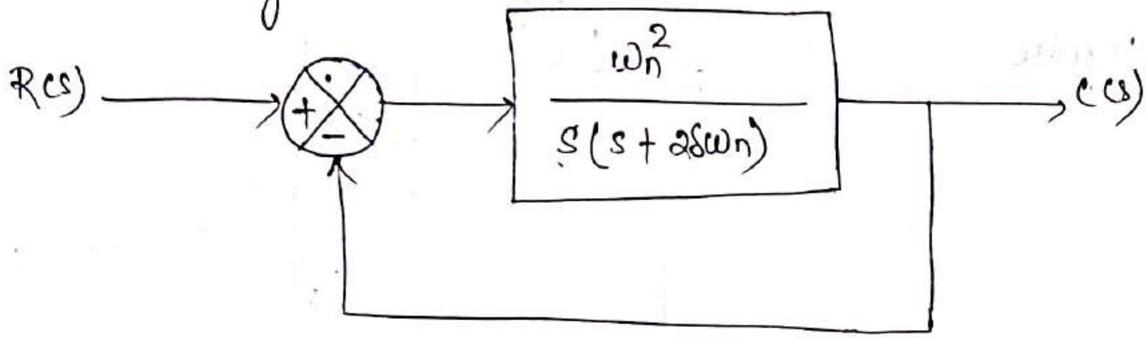
Value of time constant

$$T = \frac{L}{R}$$

Second Order System :—

A second order control system is one in which the highest power of s in the denominator of transfer function is two.

The block diagram representation of a general second order system is



This block diagram is unity feedback closed loop control system with forward path transfer function $G(s)$ and feedback transfer function $H(s)$ as given

$$G(s) = \frac{\omega_n^2}{s(s + 2\delta\omega_n)}$$

$$H(s) = 1$$

Where, ω_n = undamped natural frequency

δ = damping factor.

The closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

$$= \frac{\frac{\omega_n^2}{s(s+2\delta\omega_n)}}{1 + \frac{\omega_n^2}{s(s+2\delta\omega_n)} \cdot 1}$$

$$= \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Here,

Characteristics equation is

$$s^2 + 2\delta\omega_n s + \omega_n^2 = 0$$

Derivation of time Response Specification :—

(Delay time, Rise time, peak time, settling time, peak overshoot)

The time response exhibits damped oscillations with overshoot and undershoot before reaching steady state. These are the quantities which are used to express transient response of the system.

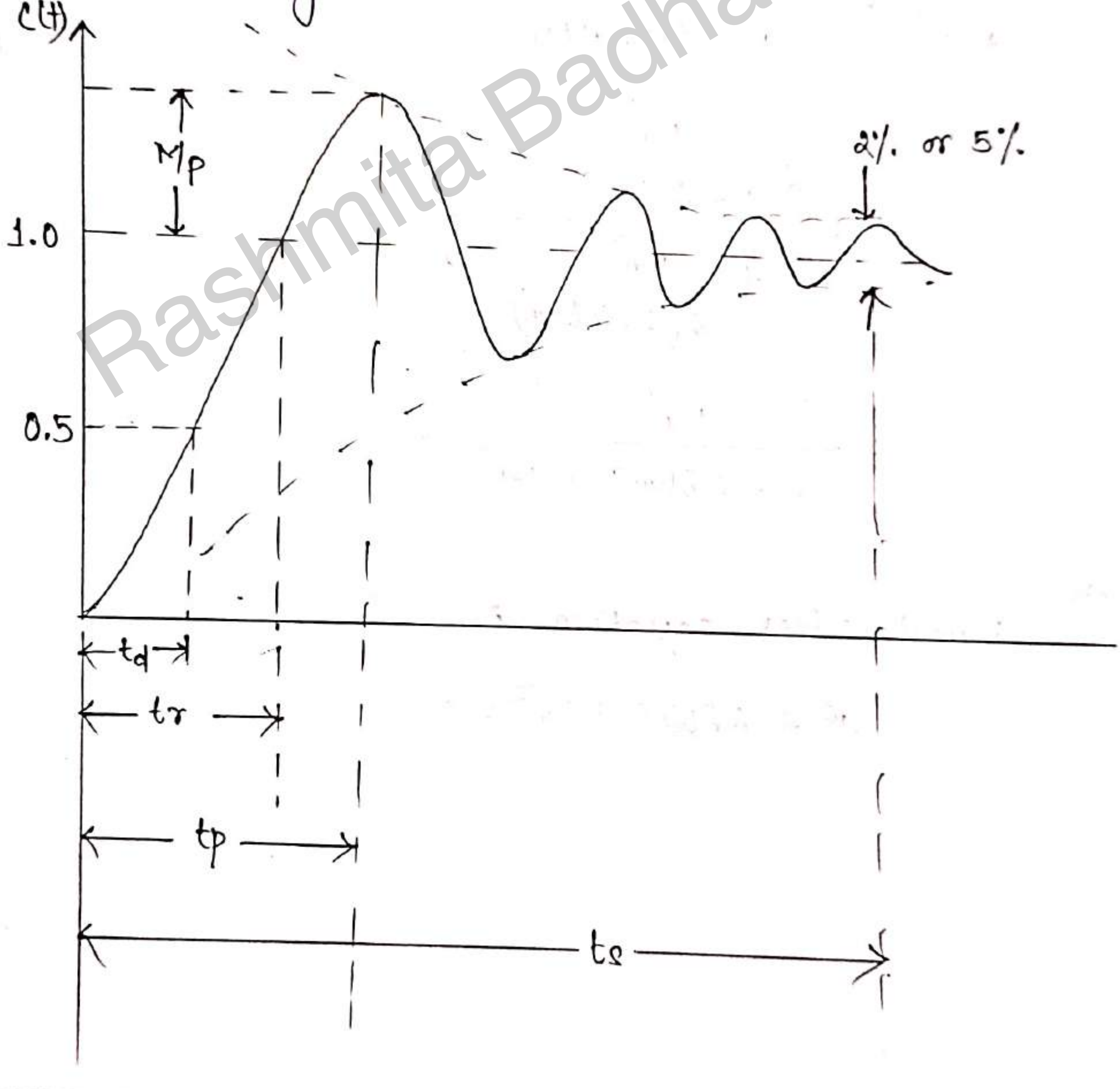
(i) Delay time (t_d)

(ii) Rise time (t_r)

(iii) Peak time (t_p)

(iv) Peak overshoot (M_p)

(v) Settling time (t_s)



1. Delay time :— (t_d)

It is defined as the time required for the response to reach half of the final value for the first time.

2. Rise time :— (t_r)

It is defined as the time required for the response to reach final value for the first time.

Expression :—

The time response equation is

$$c(t) = 1 - \frac{e^{-\delta \omega_n t_r}}{\sqrt{1-\delta^2}} \sin\left[\omega_d t_r + \phi\right]$$

To obtain t_r, $c(t) = 1$.

Then, $$1 = 1 - \frac{e^{-\delta \omega_n t_r}}{\sqrt{1-\delta^2}} \sin\left[\omega_d t_r + \phi\right]$$

$$\Rightarrow \frac{e^{-\delta \omega_n t_r}}{\sqrt{1-\delta^2}} \sin(\omega_d t_r + \phi) = 0$$

So, $\frac{e^{-\delta \omega_n t_r}}{\sqrt{1-\delta^2}} \neq 0$, therefore

$$\sin(\omega_d t_r + \phi) = 0$$

$$\Rightarrow \sin(\omega_d t_r + \phi) = \sin n\pi$$

Puting $n = 1$; $$\sin(\omega_d t_r + \phi) = \sin \pi$$

$$\Rightarrow \omega_d t_r + \phi = \pi$$

$$\Rightarrow t_r = \frac{\pi - \phi}{\omega_d}$$

$$\Rightarrow t_r = \frac{\pi - \phi}{\omega_n \sqrt{1-\delta^2}}$$ where, $\phi = \tan^{-1} \frac{\sqrt{1-\delta^2}}{\delta}$

3. Peak Time :— (t_p)

It is defined as the time required to reach to first peak of the time response.

Expression for t_p :—

$$c(t) = 1 - \frac{e^{-\delta \omega_n t}}{\sqrt{1-\delta^2}} \sin(\omega_d t + \phi)$$

For maximum value, the first differential of the function is equated to zero.

$$\therefore \frac{d\,c(t)}{dt} = 0$$

$$\Rightarrow -\frac{1}{\sqrt{1-\delta^2}} \left[e^{-\delta \omega_n t} \cos(\omega_d t + \phi) \cdot \omega_d + \sin(\omega_d t + \phi) \cdot e^{-\delta \omega_n t} (-\delta \omega_n) \right] = 0$$

$$\Rightarrow -\frac{e^{-\delta \omega_n t}}{\sqrt{1-\delta^2}} \left[\cos(\omega_d t + \phi) \cdot \omega_d - \sin(\omega_d t + \phi)\, \delta \omega_n \right] = 0$$

Since, $e^{-\delta \omega_n t} \neq 0$

$$\therefore \cos(\omega_d t + \phi) \cdot \omega_d - \sin(\omega_d t + \phi)\, \delta \omega_n = 0$$

$$\Rightarrow \cos(\omega_d t + \phi) \cdot \omega_n \sqrt{1-\delta^2} - \sin(\omega_d t + \phi)\, \delta \omega_n = 0$$

$$\Rightarrow \cos(\omega_d t + \phi) \sqrt{1-\delta^2} - \sin(\omega_d t + \phi)\, \delta = 0$$

$$\Rightarrow \cos(\omega_d t + \phi) \sin\phi - \sin(\omega_d t + \phi) \cdot \cos\phi = 0$$

$$\Rightarrow \sin(\omega_d t + \phi - \phi) = 0$$

$$\Rightarrow \sin \omega_d t = 0$$

Time for various peak of overshoot are

$$\sin \omega_d t = \sin n\pi$$

$t = t_p$ for first peak and $n = 1$

$$\sin \omega_d t_p = \sin n\pi = \sin \pi$$

$$\Rightarrow \omega_d t_p = \pi$$

$$\Rightarrow t_p = \frac{\pi}{\omega_d}$$

$$\Rightarrow \boxed{t_p = \frac{\pi}{\omega_n \sqrt{1-\delta^2}}}$$

4. Peak overshoot :— (M_p)

The maximum positive deviation of output from the steady state value is called peak overshoot.

$$M_p = c(t)_{max} - c(\infty)$$

$$= c(t)_{max} - 1 \qquad (\because \text{steady state value} = c(\infty) = 1)$$

Generally maximum overshoot or peak overshoot is expressed in percentage of steady state value and is given by

$$\%\, M_p = \frac{c(t)_{max} - c(\infty)}{c(\infty)} \times 100$$

$$= \frac{c(t)_{max} - 1}{1} \times 100$$

$$= M_p \times 100$$

Expression for M_p :—

The maximum value of $c(t)$ is obtained at the peak time t_p.

$$c(t_p) = 1 - \frac{e^{-\delta \omega_n t_p}}{\sqrt{1-\delta^2}} \sin(\omega_d t_p + \phi)$$

$$= 1 - \frac{e^{-\delta\omega_n \frac{\pi}{\omega_n\sqrt{1-\delta^2}}}}{\sqrt{1-\delta^2}} \sin\left(\omega_d \cdot \frac{\pi}{\omega_n\sqrt{1-\delta^2}} + \phi\right)$$

$$= 1 - \frac{e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}}{\sqrt{1-\delta^2}} \sin(\pi + \phi)$$

$$= 1 + \frac{e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}}{\sqrt{1-\delta^2}} \sin\phi$$

$$= 1 + \frac{e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}}{\cancel{\sqrt{1-\delta^2}}} \cdot \cancel{\sqrt{1-\delta^2}} \qquad \left(\because \sin\phi = \sqrt{1-\delta^2}\right)$$

$$= 1 + e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}$$

$$M_p = \frac{c(t_p) - 1}{1}$$

$$= 1 + e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} - 1$$

$$= e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}$$

$$\% M_p = M_p \times 100$$

$$= e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}} \times 100$$

5. Settling time (t_s) :—

It is the time required for the response to reach within the specified range of final value (2% or 5%).

Time constant of the system $= T = \frac{1}{\delta\omega_n}$

If the range is 2% then

$$\text{Settling time} = 4T = \frac{4}{\delta \omega_n}$$

If the range is 5% then

$$\text{Settling time} = 3T = \frac{3}{\delta \omega_n}$$

Transient response specifications	Magnitude
1. Delay Time (t_d)	—
2. Rise Time (t_r)	$\frac{\pi - \phi}{\omega_n\sqrt{1-\delta^2}}$ where $\tan\phi = \frac{\sqrt{1-\delta^2}}{\delta}$
3. Peak Time (t_p)	$\frac{\pi}{\omega_n\sqrt{1-\delta^2}}$
4. Peak overshoot (M_p)	$M_p = e^{-\frac{\pi\delta}{\sqrt{1-\delta^2}}}$ $\% M_p = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \times 100$
5. Settling time (t_s)	$t_s = \frac{4}{\delta\omega_n}$ for 2% settling range $t_s = \frac{3}{\delta\omega_n}$ for 5% settling range.

Characteristics of system for different damping ratio (δ) :—

For $\delta > 1$

→ Over damped.

→ Roots are negative, real and distinct.

→ Not oscillating

→ No over shoot

→ largest t_r (rise time)

For $\delta = 1$

→ Critically damped.

→ Roots are negative, real and repeatitive

→ Sustation oscillation

→ No overshoot

→ less t_r (rise time)

For $\delta < 1$

→ under damped

→ Roots are complex with negative real part.

→ Oscillating

→ Overshoot

→ Less t_r (rise time).

Problem:- The open loop transfer function of a system with unity feedback is given by $G(s) = \frac{10}{(s+2)(s+5)}$

Determine the damping ratio, undamped natural frequency of oscillation. What is the percentage overshoot of the response to a unit step input.

Ans:
$$G(s) = \frac{10}{(s+2)(s+5)} \quad ; \quad H(s) = 1$$

The closed loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

$$= \frac{\frac{10}{(s+2)(s+5)}}{1 + \frac{10}{(s+2)(s+5)} \times 1} = \frac{\frac{10}{\cancel{(s+2)(s+5)}}}{\frac{(s+2)(s+5)+10}{\cancel{(s+2)(s+5)}}}$$

$$= \frac{10}{s^2 + 5s + 2s + 10 + 10}$$

$$= \frac{10}{s^2 + 7s + 20}$$

So, the characteristic equation is $s^2 + 7s + 20 = 0$

Standard characteristic equation is $s^2 + 2\delta\omega_n s + \omega_n^2 = 0$

By comparing these two equations we get,

(1) Natural frequency : (ω_n)

$$\omega_n^2 = 20 \Rightarrow \omega_n = \sqrt{20} = 4.472 \text{ rad/sec.}$$

(2) Damping ratio (δ)

$$2\delta\omega_n = 7$$

$$\Rightarrow 2 \times \delta \times 4.472 = 7$$

$$\Rightarrow \delta = \frac{7}{2 \times 4.472}$$

$$\Rightarrow \delta = 0.7826$$

(3) Percentage overshoot :—

$$M_p = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} = e^{-\frac{\pi \times 0.7826}{\sqrt{1-(0.7826)^2}}} \times 100$$

$$= 1.92\ \%$$

Assignment - 5

1. What is peak time ?

Ans: If the signal is overdamped, then rise time is counted as the time required by the response to rise from 10% to 90% of its final value. Peak time (tp) is simply the time required by response to reach its first peak.

2. What is settling time ?

3. What is time response ?

4. What is rise time in control system ?

5. Derivation of time response specification for delay time, rise time, peak time, settling time, peak overshoot of a second order system.

6. What are the different type of input and derive steady state error for step response, ramp and parabolic input ..

— o —

Chapter-6

Feedback Characteristics of control systems

Syllabus: Effect of parameter variation in open loop system and closed loop system. Introduction to Basic control action and Basic modes of feedback control: proportional, Integral and derivative. Effect of feedback on overall gain, stability, Realisation of controllers (P, PI, PD, PID) with op-amp.

Effect of parameter variation in open loop system and closed loop system :—

Effect of parameter variation :—

Feedback reduces error, reduces the sensitivity of the system to parameter variation.

Parameter may vary due to some change in condition and it's variation effect the performance of the system. So, it is necessary to make the system insensitive to parameter variation.

Effect of parameter variation on overall gain of a degenerative feedback control system :—

The overall gain or transfer function of a degerative feedback control system depends upon these parameters.

(i) variation in parameters of plant

(ii) variation in parameters of feedback system.

(iii) Disturbance signals.

The term sensitivity is a measure of the effectiveness of feedback on reducing the influence of any of the above described parameters.

Eg: It is used to describe the relative variations in the overall transfer function of a system $T(s)$ due to variation in $G(s)$.

$$\text{Sensitivity} = \frac{\%\text{ change in output } (T(s))}{\%\text{ change in input } (G(s))}$$

Effect of variation in $G(s)$ on $T(s)$ of a degenerative feedback control system :—

In an open loop system :—

$$C(s) = G(s).R(s)$$

Let, due to parameter variation in plant $G(s)$ changes to $[G(s)+\Delta G(s)]$ such that, $|G(s)| >> |\Delta G(s)|$.

The output of the openloop system then changes to

$$C(s) + \Delta C(s) = [G(s) + \Delta G(s)]\ R(s)$$

$$\Rightarrow C(s) + \Delta C(s) = G(s).R(s) + \Delta G(s).R(s)$$

$$\Rightarrow \Delta C(s) = \Delta G(s).R(s) \qquad \left(\because C(s) = G(s).R(s)\right)$$

In closed loop system :—

$$C(s) = \frac{G(s)}{1+G(s).H(s)}.R(s)$$

Let, due to parameter variation in plant, $G(s)$ changes to $[G(s)+\Delta G(s)]$ such that $|G(s)| >> |\Delta G(s)|$

The output of the closed loop system then changes to,

$$C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1 + [G(s) + \Delta G(s)]H(s)} \cdot R(s)$$

$$\Rightarrow C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1 + G(s).H(s) + \Delta G(s).H(s)} R(s)$$

Since, $|G(s)| >> |\Delta G(s)|$, then $G(s).H(s) + \Delta G(s).H(s)$,

Therefore, $\Delta G(s).H(s)$ is neglected. now,

$$C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1 + G(s).H(s)} R(s)$$

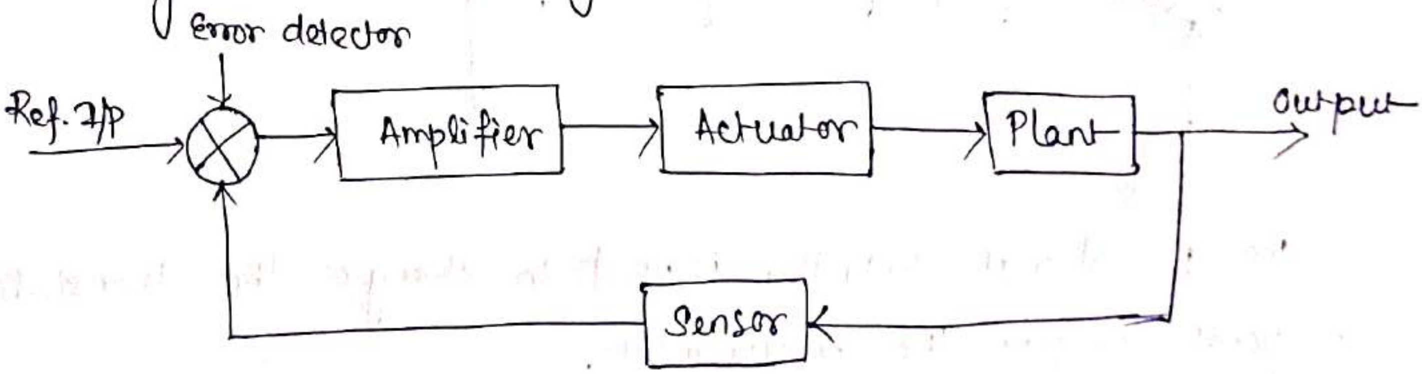
$$\Rightarrow C(s) + \Delta C(s) = \frac{G(s)}{1 + G(s).H(s)} R(s) + \frac{\Delta G(s)}{1 + G(s).H(s)} \cdot R(s)$$

$$\Rightarrow \Delta C(s) = \frac{\Delta G(s)}{1 + G(s).H(s)} \cdot R(s)$$

Introduction to Basic control action :—

It is the value of controlled variable, compare the actual value to the desired value and its deviation and produces control signal that will reduce the deviation to zero or to a smallest possible value and produces the control signal is called mode of control or basic control action.

Eg: Mechanical, Hydraulic, Electromechanical.



Basic Modes of feedback control : (Proportional, Integral and derivative) :—

1. Proportional controller :—

The proportional controller produces an output, which is proportional to error signal.

$$u(t) \propto e(t)$$

$$\Rightarrow u(t) = K_p\, e(t)$$

Applying laplace transform on both the sides.

$$U(s) = K_p\, E(s)$$

$$\Rightarrow \frac{U(s)}{E(s)} = K_p$$

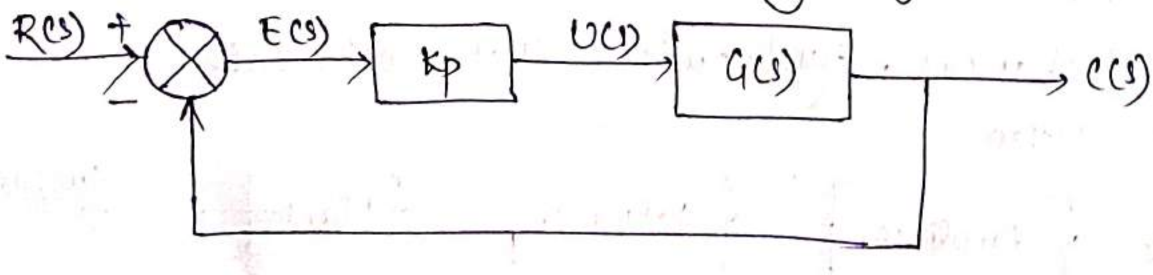
Therefore, the transfer function of the proportional controller is K_p.

where, $U(s)$ is the laplace transform of the actuating signal $u(t)$.

$E(s)$ is the laplace transform of the error signal $e(t)$

K_p is the proportionality constant.

The block diagram of the unity negative feedback closed loop control system along with the proportional controller is shown in the following figure.



The proportional controller is used to change the transient response as per the requirement.

2. Derivative Controller :—

The derivative controller produces an output, which is derivative of the error signal.

$$u(t) = K_D \frac{de(t)}{dt}$$

Applying laplace transform on both sides

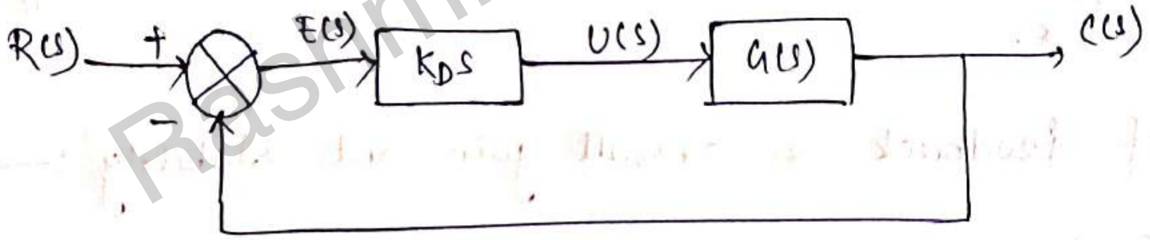
$$U(s) = K_D s E(s)$$

$$\Rightarrow \frac{U(s)}{E(s)} = K_D s$$

Therefore the transform function of the derivative controller is $K_D s$.

Where, K_D is derivative constant.

The block diagram of the unity negative feedback closed loop control system along with the derivative controller is shown in the following figure.



→ The derivative controller is used to make the unstable control system into a stable one.

3. Integral controller :—

The integral controller produces an output, which is integral of the error signal.

$$u(t) = K_I \int e(t)\,dt$$

Apply laplace transform on both the sides :

$$U(s) = \frac{K_I E(s)}{s}$$

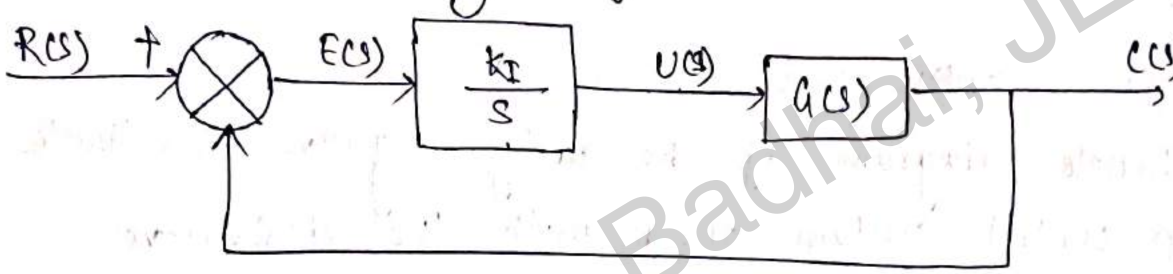
$$\Rightarrow \frac{U(s)}{E(s)} = \frac{K_I}{s}$$

Therefore,

The transfer function of the integral controller is $\frac{K_I}{s}$.

where, K_I is the integral constant.

The block diagram of the unity negative feedback closed loop control system along with the integral controller is shown in the following figure.



→ The integral controller is used to decrease the steady state error.

Effect of feedback on overall gain and stability:—

1. Effect of feedback on overall gain:—

The overall transfer function in an open loop system is

$$G(s) = \frac{C(s)}{R(s)}$$

closed loop $$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}$$

Hence, the gain is reduced by a factor of

$$\frac{1}{1+G(s).H(s)}$$

2. Effect of feedback on stability :—

For open loop system,

$$G(s) = \frac{K}{s+t}$$

So, the pole is located at $s = -t$

For closed loop system,

$$\frac{C(s)}{R(s)} = \frac{K}{s+(t+K)}$$

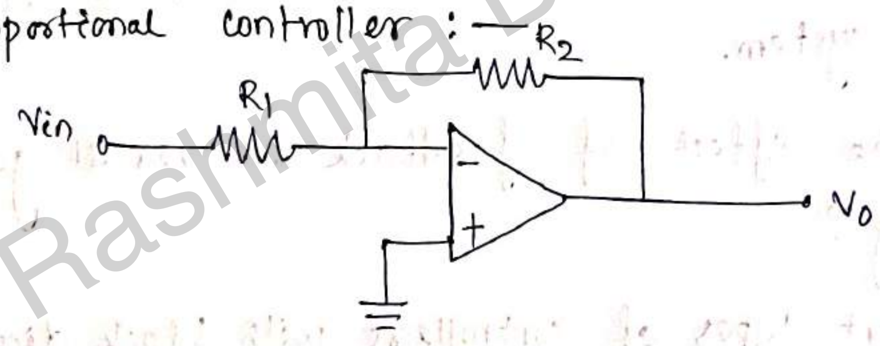
So, the pole is located at $-(t+K)$

Hence feedback controls the time response by adjusting the location of poles.

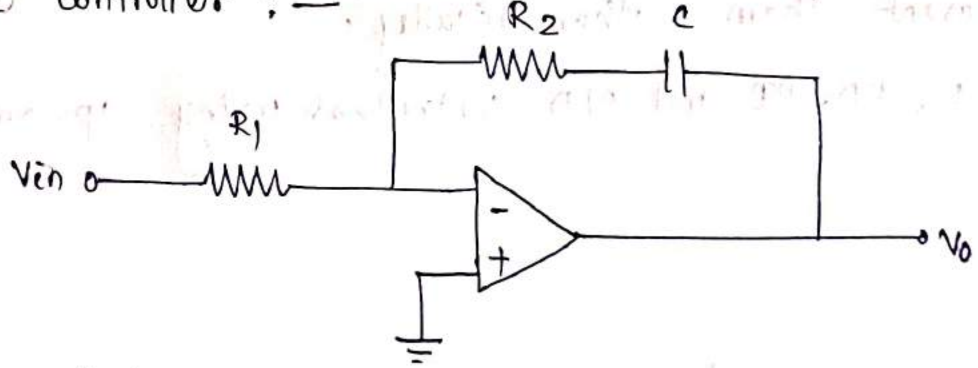
The stability depends on the location of pole. Hence, we can say feedback effects the stability.

Realisation of controllers (P, PI, PD, PID) with op-amp :—

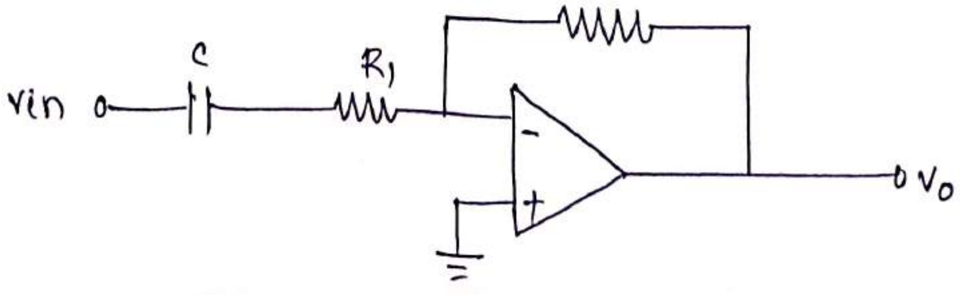
(1) Proportional controller :—



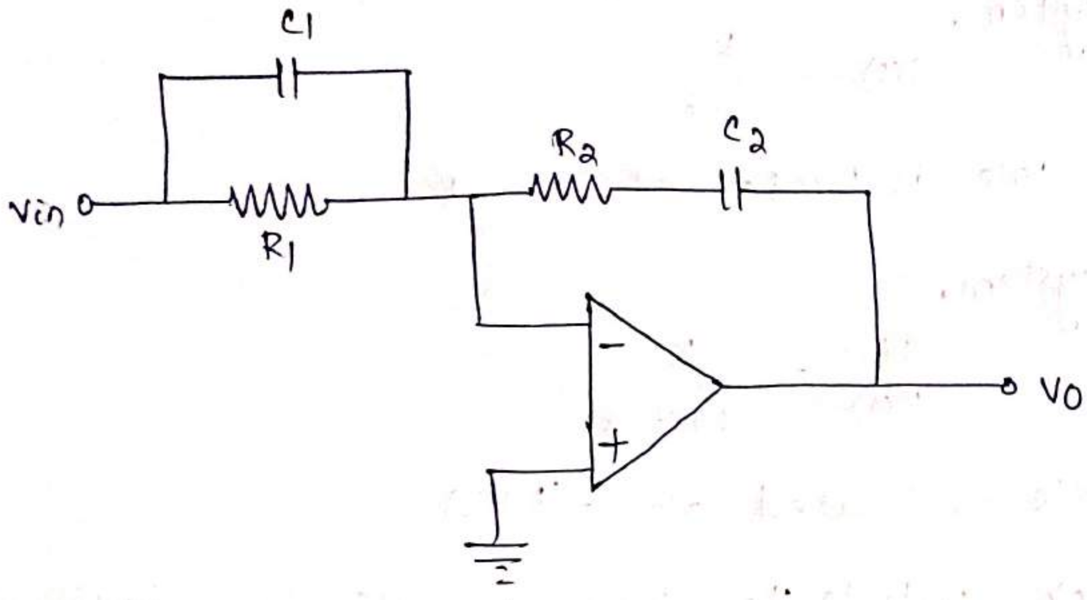
(2) PI controller :—



(3) PD controller :—



4. PID controller :—



Assignment - 6

1. What are the different types of controllers?
2. What is proportional controller?
3. What happens when a derivative controller is applied to a control system.
4. What is the effect of feedback on overall gain and stability?
5. State different types of controllers with block diagram and represent them mathematically.
6. Realise P, PD, PI and PID controllers using op-amp.

Chapter-7

Stability concept and Root locus Method

Syllabus: Effect of location of poles on stability, Routh Hurwitz stability criterion, steps for Root locus method, Root locus method of design (simple problem)

Stability is a very important characteristics of the transient performance of a system. Any working system is designed considering its stability. Therefore all instruments are stable with in a boundary of parameter variations.

A linear time invariant (LTI) system is stable if the following two conditions are satisfied.

(i) When the system is excited by a bounded input, output is also bounded.

(ii) In the absence of the input, the output tends towards zero irrespective of initial conditions. This type of stability is called asymptotic stability.

Effect of location of poles on stability :—

Location of poles has direct effect on stability. The entire S-plane is divided into three categories

(1) left half plane (LHP)

(2) Jw axis

(3) Right half plane (RHP)

(1) Left half plane poles :—

If poles are present (i) on real axis and simple
(ii) on real axis and multiple
(iii) Complex conjugate and simple

For all these conditions the system is stable.

2. $J\omega$ axis :—

If the poles are (i) complex and simple

(ii) complex and multiple

(iii) at origin and simple

(iv) at origin and multiple

In these case the system is always unstable.

3. Right half plane poles :—

If the poles are (i) one real axis and simple

(ii) on real axis and multiple

(iii) complex conjugate and simple

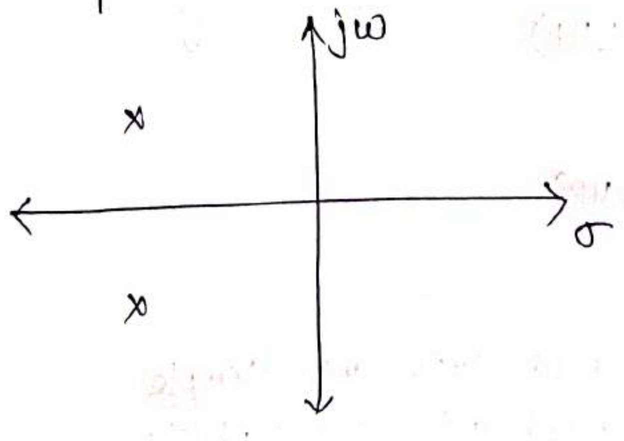
In this case, the system is always unstable.

Stability of control System :—

A linear time invariant (LTI) system is stable, unstable or marginally stable depending upon the situation.

A. Stable System :—

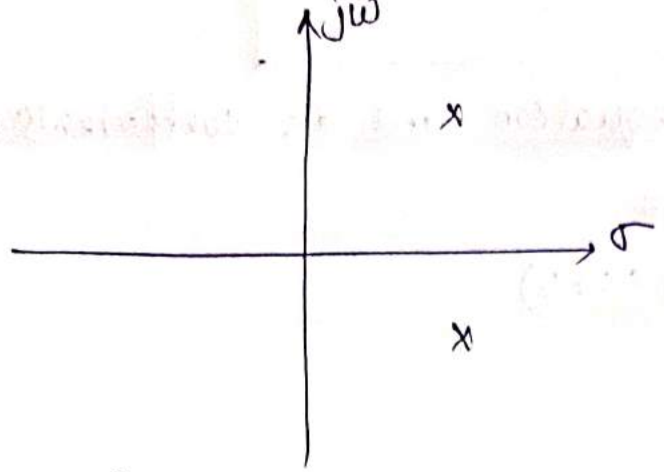
All the roots of characteristics equation lie on the left half of s-plane, then for a bounded input, the response is bounded and the system is stable.



(root lie on left half of s-plane)

B. Unstable System :—

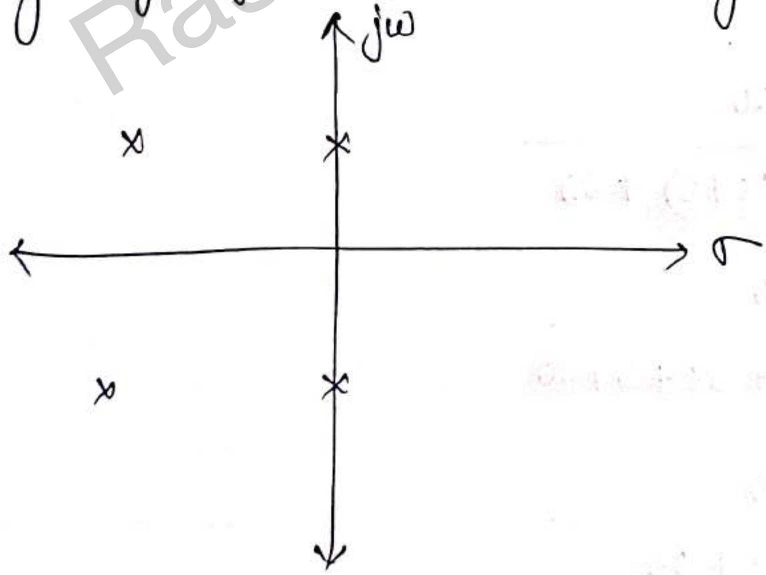
If any root of characteristic equation lies on the right half of s-plane, then the system response is unbounded for a bounded input and the system is unstable.



(Roots on right half of s-plane)

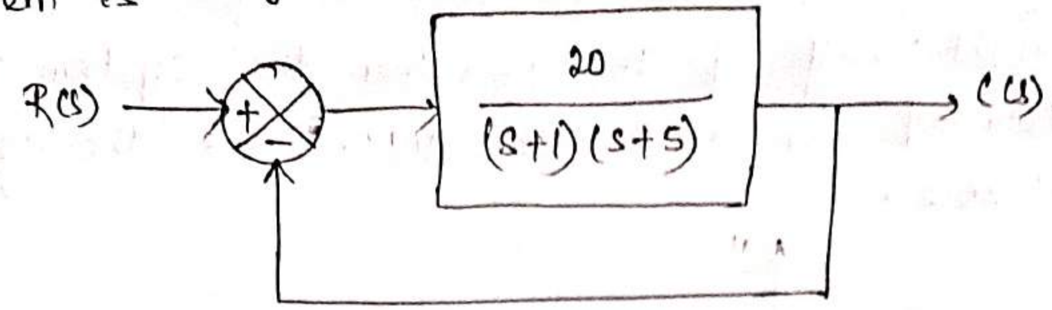
C. Marginally Stable system :—

If roots of characteristics equation lie on left half of s-plane except one or more repeated roots on imaginary (jω axis), then the system is marginally stable.



(roots in imaginary axis and left half of s-plane)

Problem : Block diagram of a unity feedback control system is



Find the characteristics equation and its coefficients.

Ans : Here,

$$G(s) = \frac{20}{(s+1)(s+5)}$$

$$H(s) = 1$$

The overall transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)}$$

$$= \frac{\frac{20}{(s+1)(s+5)}}{1+\frac{20}{(s+1)(s+5)}\cdot 1}$$

$$= \frac{20}{(s+1)(s+5)+20}$$

$$= \frac{20}{s^2+s+5s+5+20}$$

$$= \frac{20}{s^2+6s+25}$$

So, the characteristics equation is $s^2+6s+25=0$

Coefficients of this equation are 1, 6 and 25.

Problem: Characteristic equation of a control system is

$$s^2 + 6s + 8 = 0$$

Find the following:

(i) Order of the system.

(ii) Coefficients of characteristics equation

(iii) Roots of characteristics equation

(iv) Poles of transfer function

(v) Stability of system.

Ans: Characteristic equation is $s^2 + 6s + 8 = 0$

(i) Highest power of 's' is 2. So it is a second order system.

(ii) Coefficients of characteristic equation are 1, 6 and 8.

(iii) $$s^2 + 6s + 8 = 0$$

$$\Rightarrow (s+2)(s+4) = 0$$

$$\Rightarrow s = -2 \text{ and } s = -4$$

Roots of characteristic equation are -2 and -4.

(iv) Poles of transfer function is same as roots of characteristic equation.

Poles are -2 and -4.

(v) Roots of characteristics equation are real and negative, hence, they lie in the left half of s-plane, so, the system is stable.

Problem : State the stability of following system whose roots of characteristics equation are given below.

(i) $-2, 3$ (ii) $-2.5, -8.2$

(iii) $\pm j2$ (iv) $-3 \pm j4, -2, -4$

(v) $\pm j3.5, 5$ (vi) $\pm j6, -2, -3$

Ans: (i) $-2, 3$ → unstable

(ii) $-2.5, -8.2$ → Stable

(iii) $\pm j2$ → marginally stable

(iv) $-3 \pm j4, -2, -4$ → stable

(v) $\pm j\, 3.5, 5$ → unstable

(vi) $\pm j6, -2, -3$ → marginally stable.

Routh - Hurwitz Stability Criterion :—

(I) Hurwitz Stability criterion :—

The characteristics equation of a n^{th} order system is given by $a_0 s^n + a_1 s^{n-1} + \dots + a_n = 0$

Hurwitz stability criteria is based on formulation of Hurwitz determinant.

It is a $n \times n$ determinant.

In Hurwitz determinant, each row is obtained by substituting $n = 1, 2, \dots n$ where indices with negative numbers or larger than n are replaced by zero.

$$\begin{array}{ll} 1^{st}\text{ row} & n=1 \\ 2^{nd}\text{ row} & n=2 \\ 3^{rd}\text{ row} & n=3 \\ & \\ n^{th}\text{ row} & \end{array} \begin{vmatrix} a_1 & a_0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & a_0 & 0 & 0 & \cdots & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 & a_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ a_{2n-1} & a_{2n-2} & \cdots & & & & a_{n-1} & a_n \end{vmatrix}$$

These determinants are.

$$\Delta_1 = a_1$$

$$\Delta_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ a_5 & a_4 & a_3 \end{vmatrix}$$

Mathematically,

The necessary and sufficient condition for stability as per Hurwitz criterion is

$$\Delta_1 = a_1 > 0$$

$$\Delta_2 > 0$$

$$\Delta_3 > 0$$

$$\vdots$$

$$\Delta_n > 0$$

If $\Delta_{n-1} = 0$ then the system is marginally stable.

Problem: Characteristic equation of a fourth order system is

$$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

Form Hurwitz determinant and find the stability of the system applying Hurwitz criteria.

Ans: Characteristic equation is

$$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

The order of the system is 4.

Hurwitz determinant is

$$\begin{vmatrix} a_1 & a_0 & 0 & 0 \\ a_3 & a_2 & a_1 & a_0 \\ a_5 & a_4 & a_3 & a_2 \\ a_7 & a_6 & a_5 & a_4 \end{vmatrix}$$

Here, $a_0 = 1$, $a_1 = 8$, $a_2 = 18$, $a_3 = 16$, $a_4 = 5$.

Coefficient higher than n i.e.

$$a_5 = a_6 = a_7 = 0$$

So, the determinant is formed as

$$\begin{vmatrix} 8 & 1 & 0 & 0 \\ 16 & 18 & 8 & 1 \\ 0 & 5 & 16 & 18 \\ 0 & 0 & 0 & 5 \end{vmatrix}$$

$\Delta_1 = 8 > 0$

$$\Delta_2 = \begin{vmatrix} 8 & 1 \\ 16 & 18 \end{vmatrix} = 8 \times 18 - 16 \times 1 = 128 > 0$$

$$\Delta_3 = \begin{vmatrix} 8 & 1 & 0 \\ 16 & 18 & 8 \\ 0 & 5 & 16 \end{vmatrix}$$

$$= 8(18\times16 - 8\times5) - 16(1\times16 - 0\times5) + 0(1\times8 - 18\times0)$$

$$= 1728 > 0$$

$$\Delta_4 = \begin{vmatrix} 8 & 1 & 0 & 0 \\ 16 & 18 & 8 & 1 \\ 0 & 5 & 16 & 18 \\ 0 & 0 & 0 & 5 \end{vmatrix}$$

$$= 5 \begin{vmatrix} 8 & 1 & 0 \\ 16 & 18 & 8 \\ 0 & 5 & 16 \end{vmatrix}$$

$$= 5 \times 1728 > 0$$

All four determinants are positive. Hence, the system is stable as per Hurwitz stability criteria.

2. Routh Stability Criterion :—

Routh Array :— The routh array is determined by arranging coefficients of characteristics equation in two rows.

The characteristics equation is

$$a_0 s^n + a_1 s^{n-1} + \dots + a_n = 0$$

s^n : a_0 a_2 a_4 . . .

s^{n-1} : a_1 a_3 a_5

$$
\begin{array}{lllll}
s^n : & a_0 & a_2 & a_4 & \cdots \\
s^{n-1} : & a_1 & a_3 & a_5 & \cdots \\
s^{n-2} : & b_1 & b_2 & b_3 & \cdots \\
s^{n-3} : & c_1 & c_2 & c_3 & \cdots \\
s^2 : & & & & \\
s^1 : & & & & \\
s^0 : & a_n & & &
\end{array}
$$

$$b_1 = \frac{\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix}}{a_1} = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{\begin{vmatrix} a_0 & a_4 \\ a_1 & a_5 \end{vmatrix}}{a_1} = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$c_1 = \frac{\begin{vmatrix} a_1 & a_3 \\ b_1 & b_2 \end{vmatrix}}{b_1} = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{\begin{vmatrix} a_1 & a_5 \\ b_1 & b_3 \end{vmatrix}}{b_1} = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

Problem : Determine the stability of a system with characteristic equation

$$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

Ans: All the coefficients of the equation are positive and there is no missing terms. So, it satisfy the necessary condition for stability.

Forming Routh array.

$$\begin{array}{llll}
s^4 & 1 & 18 & 5 \\
s^3 & 8 & 16 & 0 \\
s^2 & \frac{8\times18 - 16\times1}{8} = 16 & \frac{8\times5 - 1\times0}{8} = 5 & \\
s^1 & \frac{16\times16 - 8\times5}{16} = \frac{27}{2} & & \\
s^0 & \frac{\frac{27}{2}\times5 - 0}{\frac{27}{2}} = 5 & &
\end{array}$$

The elements of first column of Routh array are all positive and hence the system is stable.

Problem: Find the stability of a system with characteristics equation

$$3s^4 + 10s^3 + 5s^2 + 5s + 2 = 0$$

Ans: The characteristics equation satisfies the necessary condition for stability given all the coefficient are positive and non-zero. Routh array is formed as.

s^4	3	5	2
s^3	$\cancel{10}\ 2$	$\cancel{5}\ 1$	0 ← Dividing by
s^2	$\frac{2\times5-3\times1}{2}=\frac{7}{2}$	$\frac{2\times2-3\times0}{2}=2$	0
s^1	$\frac{\frac{7}{2}\times1-2\times2}{\frac{7}{2}}=\frac{-1}{7}$	0	
s^0	2		

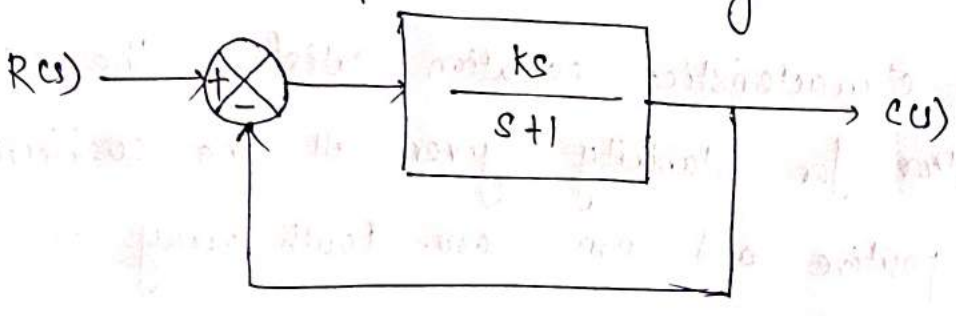
By observing the 1st column of Routh array, it is found that two sign changes (term $\frac{7}{2}$ to $-\frac{1}{7}$ and $-\frac{1}{7}$ to 2). So the system is unstable.

Steps for root locus Method :—

Root locus of a control system is a plot of the roots of the characteristic equation when gain k is varied from zero to infinity.

Root locus for first order system :—

Consider a simple first order system.



(A first order system)

The open loop transfer function of this system is

$$G(s).H(s) = \frac{ks}{s+1}$$

The open loop transfer function has a pole at $s=-1$ and zero at $s=0$.

The characteristic equation of the system is

$$1 + G(s).H(s) = 0$$

$$\Rightarrow 1 + \frac{ks}{s+1} = 0$$

$$\Rightarrow \frac{s+1+ks}{s+1} = 0$$

$$\Rightarrow s(k+1)+1 = 0$$

$$\Rightarrow s = -\frac{1}{k+1}$$

The root of characteristic equation is $s = -\frac{1}{k+1}$

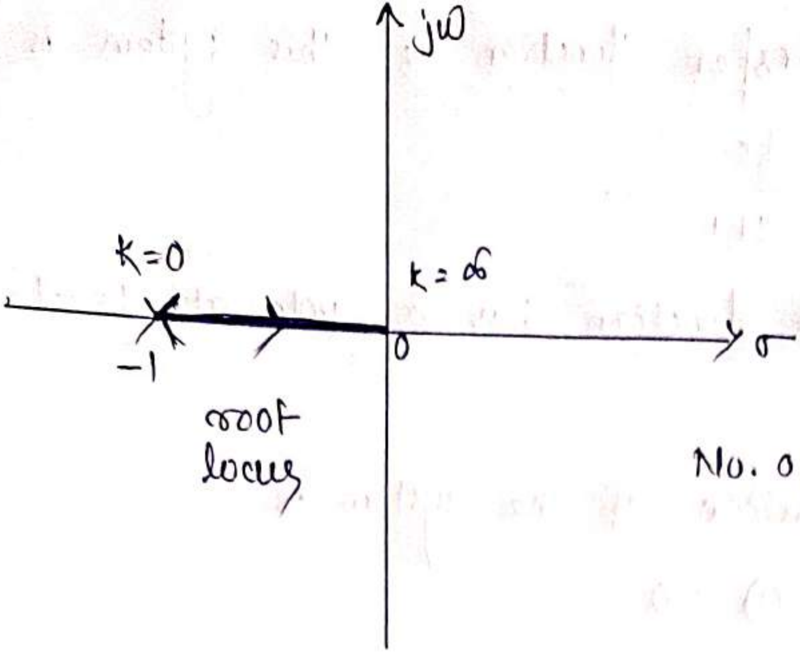
When k varies from 0 to ∞, the root is negative and real and hence lie on σ axis on left half of s-plane.

for $k=0$, $s=-1$; starting point of root locus

for $k=\infty$, $s=0$; end point of root locus.

So, the root locus starts at $s=-1$ for $k=0$ and ends at $s=0$ for $k=\infty$.

We can say that root locus starts from open loop pole and ends at open loop zero.



No. of open loop pole $= p = 1$

No. of open loop zeros $= z = 1$

No. of root locus branches $N = 1$.

Procedure to plot root locus :—

The following steps are to be taken to plot root locus of a system having an open loop transfer function $G(s).H(s)$ following rules.

1. The root locus is symmetrical about the real axis.
2. Originating point : Root locus starts from open loop poles for $k = 0$.
3. Terminating point : Root locus terminates at either on open loop pole or infinity for $k = \infty$.
4. Number of branches of root locus : (N)

$$N = P \quad \text{if } P > Z$$

$$N = Z \quad \text{if } Z > P$$

$$N = P = Z \quad \text{if } P = Z$$

5. Root loci on real axis : A section of real axis is a part of root locus if sum of number of open loop poles and zeros to the right of this section is odd.

6. Centroid angle of asymptotes :

Asymptotes : (p-z) branches of root loci tend to infinity along a set of straight line known as asymptotes.

(a) No. of asymptotes = p-z

(b) Centroid of asymptotes (σ_a) : All asymptotes radiating out from a single point on real axis. This point is called centroid (σ_A)

$$\sigma_A = \frac{\text{Sum of real part of open loop poles} - \text{sum of real part of open loop zeros.}}{p-z}$$

(c) Angle of asymptotes (ϕ) : The asymptotes make an angle with real axis and is given by

$$\phi = \frac{(2n+1)180}{p-z} \quad (\text{where}, n = 0, 1, 2 \ldots)$$

7. Intersection of root locus with imaginary axis :
This is determined using Routh criteria.

8. Break away point : The break away point is determined from the solution of $\frac{dk}{ds} = 0$.

9. Angle of departure from complex pole : It is given by

$$\phi_d = 180° + \theta$$

10. Angle of arrival at complex zeros : It is given by

$$\phi_a = 180° - \theta$$

The steps given above are sufficient to plot root locus for a given open loop transfer function.

Problem: Open loop transfer function of a control system is

$$G(s).H(s) = \frac{K}{s(s+6)}$$

Draw the root locus.

Ans: The transfer function is

$$G(s).H(s) = \frac{K}{s(s+6)}$$

There are no open loop zero. The open loop poles are at $s=0$ and $s=-6$.

1. the root locus starts at $s=0$ and $s=-6$ for $k=0$.
2. As there is no open loop zero, so root loci terminates at ∞.
3. Number of root locus branches $= N = P = 2$. $(\because P > Z)$
4. Root locus on real axis lies between $s=0$ to $s=-6$.
5. Break away point:

It is obtained by solving $\frac{dK}{ds} = 0$

Characteristic equation is

$$1 + G(s).H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{s(s+6)} = 0$$

$$\Rightarrow s(s+6) + K = 0$$

$$\Rightarrow s^2 + 6s + K = 0$$

$$\Rightarrow K = -s^2 - 6s$$

$$\frac{dK}{ds} = \frac{d}{ds}(-s^2 - 6s) = -2s - 6 = 0$$

$$\Rightarrow s = \frac{6}{-2} \Rightarrow s = -3$$

So, $s = -3$ is a break away point.

6. Angle of asymptotes are given by

$$\phi = \frac{(2n+1)180}{P-Z} \quad n = 0, 1.$$

$$= \frac{180}{2}, \frac{3 \times 180}{2}$$

$$= 90^\circ, 270^\circ \ (\text{or } -90^\circ)$$

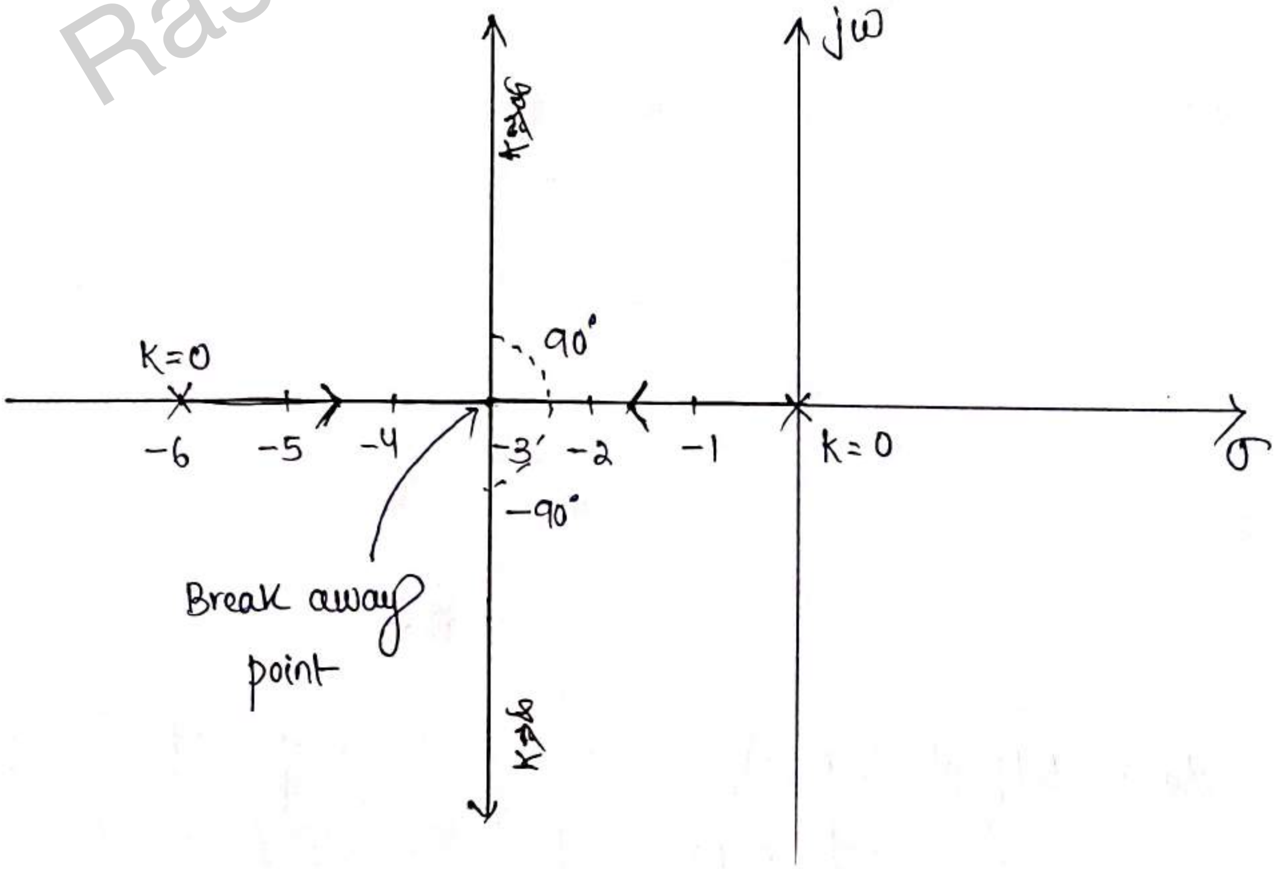
7. Centroid : $\sigma_A = \dfrac{\Sigma \text{ real part of poles} - \Sigma \text{ real part of zeros.}}{P-Z}$

$$= \frac{(0-6)-0}{2}$$

$$= \frac{-6}{2}$$

$$= -3$$

The required root locus is plotted as



Assignment - 7.

1. What do you mean by root locus?

2. What is Routh - Hurwitz stability criterion?

3. What is the need of root locus in control system?

4. Write down the rules for constructing root locus.

5. Determine the stability of a system with characteristic equation

$$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

6. Open loop transfer function of a control system is

$$G(s) \cdot H(s) = \frac{K}{s(s+6)}$$

Draw the root locus.

Chapter - 8

Frequency Response analyses and Bode plot

Syllabus : Frequency response, Relationship between time and frequency response, Methods of frequency response, polar plot and steps for polar plot, Bode plot and steps for bode plot, stability in frequency domain, Gain margin, Phase margin, Nyquist plots, Nyquist stability criterion, Simple problems as above.

Frequency Response :—

The frequency response of a system is defined as the steady state response of the system to a sinusoidal input signal.

Relationship between time and frequency response :—

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Putting $s = j\omega$,

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\delta\omega_n j\omega + \omega_n^2}$$

$$= \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\delta\omega_n\omega}$$

$$= \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j2\delta\left(\frac{\omega}{\omega_n}\right)}$$

Let $u = \frac{\omega}{\omega_n}$, then

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{(1-u^2)+j2\delta u}$$

Now, $M(j\omega) = |M(j\omega)| \angle M(j\omega)$

where, $|M(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2+(2\delta u)^2}}$.

$$\theta = -\tan^{-1}\frac{2\delta u}{1-u^2}$$

Methods of frequency Response :—

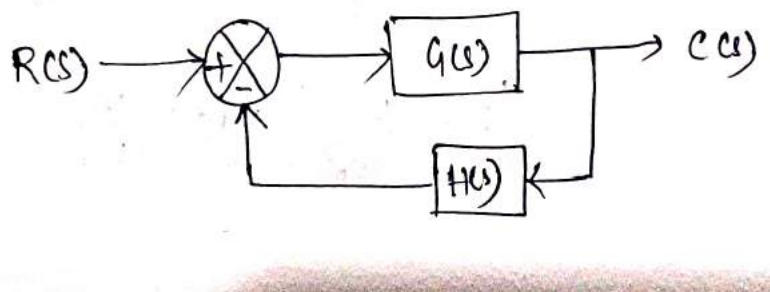
There are three types of methods in frequency domain. response. They are

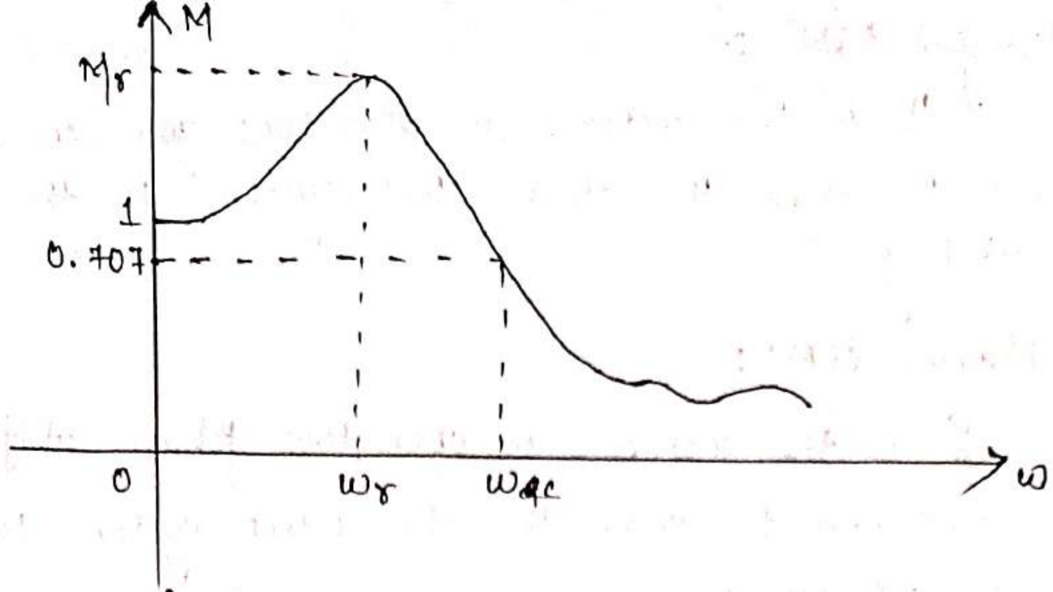
1. Bode Plot
2. Polar plot
3. Nyquist plot.

These methods are used in frequency domain to find out the stability of the system.

Defination of frequency domain specifications :—

The frequency domain refers to the analysis of mathematical functions or signals with respect to frequency rather than time.





Resonant peak (M_r) :—

Maximum value of $M(j\omega)$ when ω is varied from 0 to ∞.

Where,
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)} = M(s)$$

$$M(j\omega) = \frac{G(j\omega)}{1 + G(j\omega) \cdot H(j\omega)}$$

Response frequency (ω_r) :—

The frequency at which the peak resonance (M_r) is attained is known as response frequency.

cut-off frequency (ω_c) :—

The frequency at which $M(j\omega)$ has a value $\frac{1}{\sqrt{2}}$. It is the frequency at which the magnitude is 3dB below its zero frequency value.

Band width (ω_b) :—

It is the range of frequency in which the magnitude of a closed loop system is $\frac{1}{\sqrt{2}}$ times of M_r.

Phase cross-over frequency :—

The frequency at which phase plot crosses -180°.

Gain cross over frequency :—

The frequency at which gain or magnitude plot crosses 0dB line.

Gain Margin (GM) :—

It is the increase in open loop gain in dB required to drive the closed loop system to the verge of instability.

Phase Margin (PM) :—

It is the increase in open loop phase shift in degree required to drive the closed loop system to the verge of instability.

Polar Plot and steps for polar plot :—

Polar Plot :—

It is a graphical method of determining stability of feedback control systems by using the polar plot of their open loop transfer functions.

It is a sinusoidal transfer function $G(j\omega)$ is a plot of magnitude of $G(j\omega)$ versus the phase angle of $G(j\omega)$ on polar plot coordinates as 'ω' is varied from zero to infinity.

Therefore, the locus of vectors $|G(j\omega)| \angle G(j\omega)$ as ω varied from zero to infinity.

Steps for polar plot :—

Step-1 : Determine the transfer function $G(s)$ of the system.

Step-2 : Put $s = j\omega$ in the transfer function to obtain $G(j\omega)$.

Step-3 : At $\omega = \infty$ calculate $|G(j\omega)|$ by $\lim_{\omega \to 0} |G(j\omega)|$ and

$$\lim_{\omega \to \infty} |G(j\omega)|.$$

Step-4: Calculate the phase angle of $G(j\omega)$ at $\omega = 0$ and $\omega = \infty$ by $\lim_{\omega \to 0} \angle G(j\omega)$ and $\lim_{\omega \to \infty} \angle G(j\omega)$.

Step-5: Rationalize the function $G(j\omega)$ and separate the real and imaginary parts.

Step-6: Equate the imaginary part $Im\,|G(j\omega)|$ to zero and determine the frequencies at which plots intersects the imaginary axis and calculate the value of $G(j\omega)$ at the point of intersection by substituting the determined value of frequency in the ~~realti~~ rationalized expression of $G(j\omega)$.

Step-7: Equate the real part $Re\,|G(j\omega)|$ to zero and determine the frequencies at which plots intersects the imaginary axis and calculate the value of $G(j\omega)$ at the point of intersection by substituting the determined value of frequency in the rationalized expression of $G(j\omega)$.

Step-8: Sketch the polar plot with the help of above information.

Bode Plot and steps for Bode plot :—

Bode plot :—

It consists of (i) The plot of the logarithm of the magnitude in dB of a sinusoidal transfer function vs. frequency in logarithmic scale.

(ii) The plot of phase angle vs. frequency in logarithmic scale.

Steps for Bode plot :—

Step-1 : To identify the corner frequency.

Step-2 : Draw the asymptotic magnitude plot. The slope will change at each corner frequency by ~~±20dB~~ +20 dB/dec. for zero and −20 dB/dec for pole. For complex conjugate pole and zero the slope will change by ∓ 0dB/decade.

Step-3 : (a) For type 0 system draw a line having upto 1st (lowest) corner frequency having 0dB/dec slope.

(b). For type 1 system draw a line having slope −20dB/dec upto ω = K. Mark 1st (lowest) corner frequency.

(c) For type 2 system draw the line having slope −40 dB/dec upto $\omega = \sqrt{K}$ and so on. Mark 1st corner frequency.

Step-4 : Draw a line upto 2nd corner frequency by adding the slope of next pole or zero to the previous slope and so on.

Step-5 : Calculate phase angle for different values of ω from the equation and join all points.

Stability in frequency domain :—

The Nyquist criterion is a semigraphical method that determines the stability of closed loop system by investigating the properties of the frequency domain plot of the loop transfer function G(s). H(s).

Gain Margin and Phase Margin :—

Gain Margin :— The greater the gain margin, the greater the stability of the system. The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable. It is usually expressed as a magnitude in dB.

We can usually read the gain margin directly from the Bode plot. This is done by calculating the vertical distance between the magnitude curve and the x-axis at the frequency where the Bode phase plot = 180°. This point is known as the phase crossover frequency.

Gain Margin formula :—

The formula for gain margin can be expressed as

$$GM = 0 - G \text{ dB}$$

Where, G is the gain. This is the magnitude (in dB) as read from the vertical axis of the magnitude plot at the phase cross over frequency.

Eg: If the gain (G) is 20.

Hence, using our formula for gain margin,

The gain margin $= 0 - 20\,dB$
$= -20\,dB$

It is an unstable system.

Phase Margin :—

The greater the phase margin, the greater will be the stability of the system. The phase margin refers to the amount of phase, which can be increased or decreased without making the system unstable. It is usually expressed as a phase in degrees.

We can usually read the phase margin directly from the Bode plot. This is done by calculating the vertical distance between the phase curve and the x-axis at the frequency where the Bode magnitude plot = 0 dB. This point is known as the gain cross over frequency.

Phase margin formula :—

The formula for phase margin (PM) can be expressed as

$$PM = \phi - (-180^\circ)$$

where, ϕ is the phase lag. This is the phase as read from the vertical axis of the phase plot at the gain cross over frequency.

Eg: If the phase lag is $-189°$.

Hence using the formula for phase margin

$$PM = -189° - (-180°)$$

$$= -9° \quad \text{(unstable)}$$

If the phase lag is $-120°$.

Hence, using the formula for phase margin

$$PM = -120° - (-180°)$$

$$= 60° \quad \text{(stable)}$$

Nyquist Plots and Nyquist stability criteria :—

A closed loop system is stable if the contour Γ_{GH} of open-loop transfer function $G(s)\,H(s)$. corresponding to Nyquist contour in the s-plane encircles the point $(-1+j0)$ in counter clockwise direction as many times as the number of open loop transfer function poles on right half of s-plane.

Mathematically ;

In order to apply the Nyquist criterion a special contour called the Nyquist contour is chosen so, that it encloses the entire right half side of the $j\omega$ axis of the s plane. When the s-plane contour C is the Nyquist contour, the Nyquist stability criterion is given by

$$Z = N + P$$

where, P = Number of poles of $G(s).H(s)$ or $F(s)$ in the right-half of the s plane

N = Number of clockwise encirclements of $(-1, j0)$ point in the GH plane or of the origin in $F(s)$ plane by the Nyquist path.

Z = number of zeros of $F(s) = 1 + G(s).H(s)$ in the right half of the s.plane.

To make a closed loop system stable, there must not be any zero in the right half of s-plane.

Therefore $Z = 0$

that is $N = -P$

Assignment - 8

1. What do you mean by polar plot?
2. What do you mean by Bode plot?
3. What do you mean by Nyquist stability criterion?
4. State the rules for plotting a polar plot?
5. Write down the procedure to draw a bode plot?
6. What is the need of a Nyquist plot, explain with a suitable example.

———— o ————

Chapter-9

State Variable Anaysis

Syllabus : Concept of state, State variable, state model, State models for linear continuous time functions (simple)

State space analysis is an excellent method for the design and analysis of control systems. The conventional and old method for the design and analysis of control systems is the transfer function method. The transfer function method for design and analysis had many drawbacks.

Advantages of state variable analysis :

- It can be applied to non-linear system.
- It can be applied to time invarient systems.
- It can be applied to multiple input multiple output systems.
- Its gives idea about the internal state of the system.

Concept of state :—

The state of a control system at time $t = t_0$ is the smallest set of variables called state variables such that the knowledge of the variables along with the knowlede of the inputs at $t = t_0$ is sufficient to determine the output dynamics of the system at any time $t \geq t_0$.

Concept of state variable :—

The state variables of a dynamic system are the smallest set of variables that determine the state of the dynamic system, that is, the state variables are the minimal set of variables such that the knowlede of these variables at any initial time $t = t_0$ together with the knowledge of the inputs for $t \geq t_0$ is sufficient to completely determine the behaviour of the system for any time $t \geq t_0$.

State Model :—

lets consider a multi input and multi output system having 'r' inputs $u_1(t), u_2(t) \ldots \ldots u_r(t)$

'm' number of outputs $y_1(t), y_2(t) \ldots y_m(t)$

'n' number of state variables $x_1(t), x_2(t) \ldots x_n(t)$

Then the state model is given by state and output equation

$$\dot{X}(t) = A\,x(t) + B\,U(t) \quad \text{———— (state equation)}$$

$$Y(t) = C\,X(t) + D\,U(t) \quad \text{———— (output equation)}$$

Where, A is state matrix of size $(n \times n)$

B is the input matrix of size $(n \times r)$

C is the output matrix of size $(m \times n)$

D is the direct transmission matrix of size $(m \times r)$

$X(t)$ is the state vector of size $(n \times 1)$

$Y(t)$ is the output vector of size $(m \times 1)$

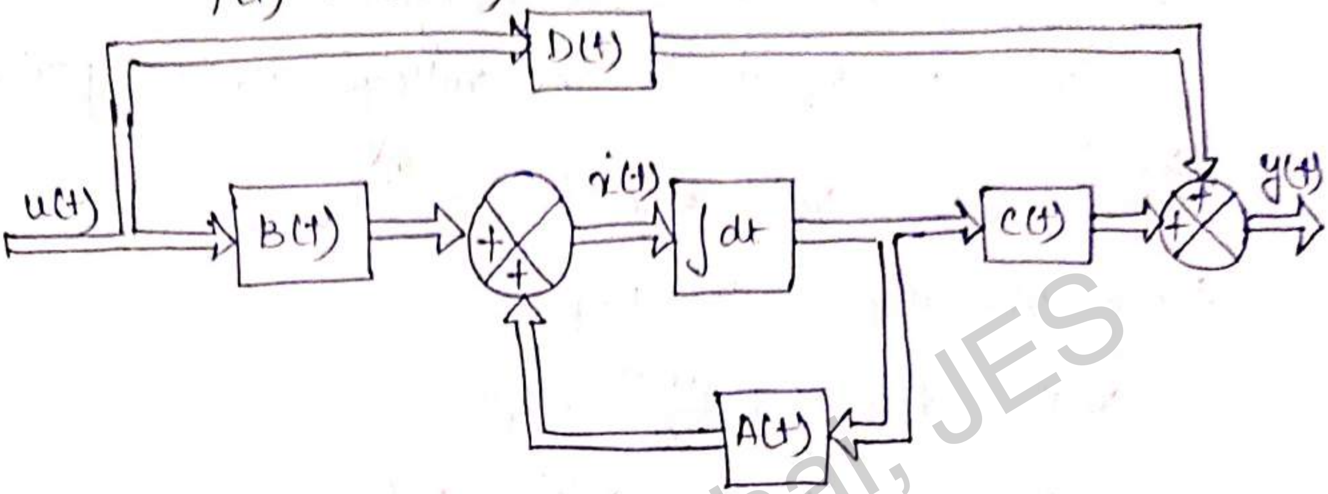
$U(t)$ is the input vector of size $(r \times 1)$

State Model for linear continuous Time functions :—

The block diagram of the linear, continuous time functions is represented by following equations.

$$\dot{x}(t) = A x(t) + B u(t)$$

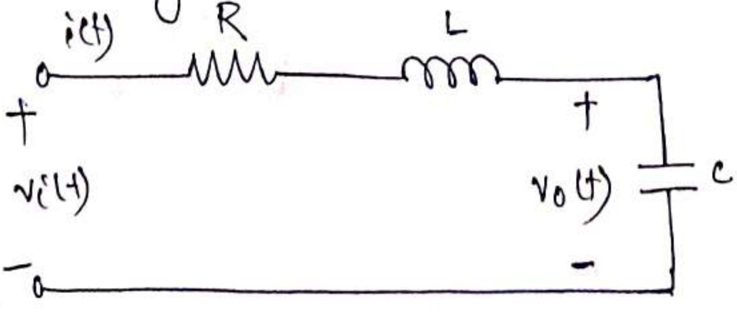
$$y(t) = C x(t) + D u(t)$$



(Block diagram of the linear, continuous time functions)

It has multiple number of input, output and state variables, this has been represented by thick arrow. Therefore n, parallel integrators must be present as there are n state variables where the output of each integrator is a separate variable.

Consider the following series RLC circuit. It is having an input voltage $v_i(t)$ and the current flowing through the circuit is $i(t)$.



There are two storage elements (inductor and capacitor) in this circuit. So, the number of the state variables is equal to two and these state variables are the current flowing through the inductor $i(t)$ and the voltage across capacitor $v_c(t)$.

From the circuit the output voltage,

$v_o(t)$ is equal to the voltage across capacitor, $v_c(t)$.

$$v_o(t) = v_c(t)$$

Apply KVL around the loop,

$$v_i(t) = R\,i(t) + L\frac{di(t)}{dt} + v_c(t)$$

$$\Rightarrow \frac{di(t)}{dt} = -\frac{R\,i(t)}{L} - \frac{v_c(t)}{L} + \frac{v_i(t)}{L}$$

The voltage across the capacitor is -

$$v_c(t) = \frac{1}{C}\int i(t)\,dt$$

Differentiate the above equation with respect to time

$$\frac{dv_c(t)}{dt} = \frac{i(t)}{C}$$

State vector, $X = \begin{bmatrix} i(t) \\ v_c(t) \end{bmatrix}$

Differential state vector, $\dot{X} = \begin{bmatrix} \frac{di(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix}$.

We can arrange the differential equations and output equation into the standard form of state space model as

$$\dot{X} = \begin{bmatrix} \frac{di(t)}{dt} \\ \frac{dv_c(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} [v_i(t)]$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i(t) \\ v_c(t) \end{bmatrix}$$

Where,

$$A = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

Assignment - 9

1. What do you mean by state variable ?

2. How the state variable electrical circuit is represented ?

3. Write down the state space representation of a LTI system ?

4. Derive the state space representation of a series RLC circuit .
