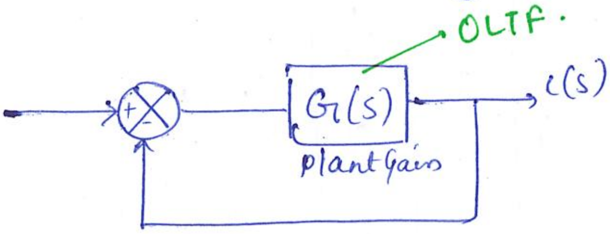
CHAPTER - 11 — 26/07/19.

COMPENSATOR.

* Compensator is a device which is used for improving the transient & steady state performance as per the requirement.



fig(a) w/o compensator

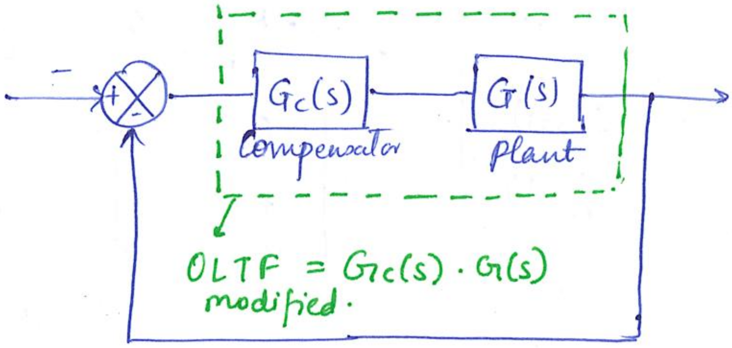
$$CLTF = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{N(s)}{D(s)}.$$

<u>C/s Eqⁿ</u> $1 + G(s) = 0$

C/s $= 1 + OLTF = 0$.

C/s $= f(OLTF)$.

Roots of CLTF.



fig(b) with compensater

$$\frac{C(s)}{R(s)} = \frac{OLTF_{mod}}{1 + OLTF_{mod}}$$

$$= \frac{G_c(s) \cdot G(s)}{1 + G_c(s) \cdot G(s)}.$$

C/s mod. $= 1 + G_c(s) . G(s) = 0$

Perfomance of closed s/s $= f[C/s]$.

& C/s $= f[OLTF]$.

∴ perf. of Closed s/s $= f(OLTF)$

CLASSIFICATION

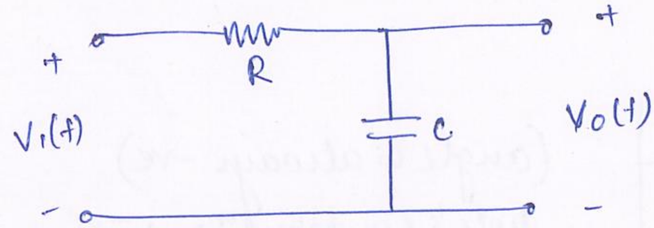
There are 4 types of Compensator

1. Lag Compensator
2. Lead Compensator.
3. Lag-Lead Compensator
4. Lead-Lag Compensator

(I) LAG COMPENSATOR

* Lag compensator provides lagging angle or -ve angle for all the values of frequency.
* Basic behaviour of Lag compensator is same as low pass filter.

(iii) Basic Circuit.



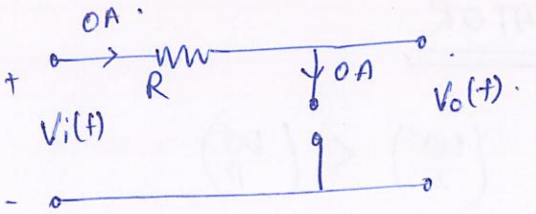
1st Order RC Lowpass filter.

∴ $V_o(t) = 0$ volts

∴ passed low freq.
& Blocked high freq.
C → open for low freq.
C → short for Low freq.

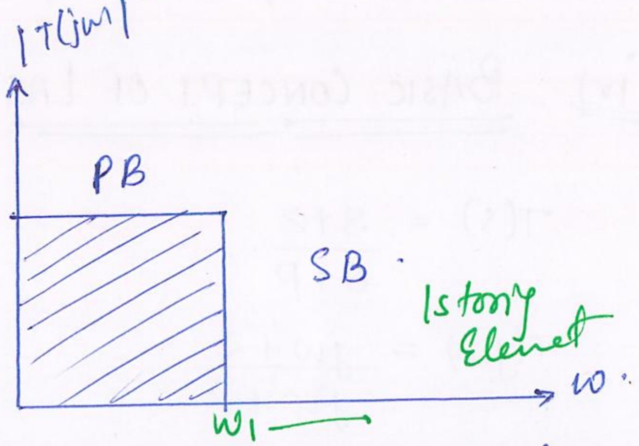
Case I Low freq ($\omega = 0$).

$X_C = \frac{1}{\omega C} = \infty$ (open ckt).



∴ $V_o(t) = V_i(t)$.

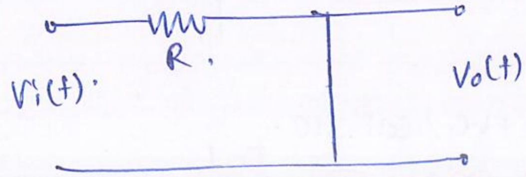
O/P follow I/P as current = 0.

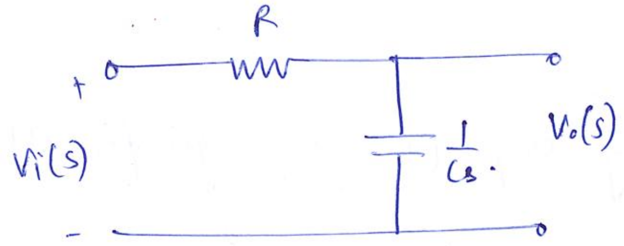


1st Order Low Pass filter.

Case II High freq ($\omega = \infty$)

$X_C = \frac{1}{\omega C} = \frac{1}{\infty} = 0$.





$$\frac{V_o(s)}{V_i(s)} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{1 + RCs}$$

$$\angle T(j\omega) = -\tan^{-1}\omega RC$$

$$T(s) = \frac{1}{1+RCs} \qquad T(j\omega) = \frac{1}{1 + j\omega RC}$$

ω	$\angle T(j\omega)$	Remarks
0	0°	$\angle T(j\omega) \leq 0$
$\frac{1}{RC}$	-45°	for $\omega \geq 0$ all +ve values of ω.
∞	-90°	

(angle is always -ve)
pole's contribuⁿ → -ve
max contri. = (-90°)

Key point.

* RC Low pass filter is referred as Lag Compensator n/w.

(iv) BASIC CONCEPT OF LAG COMPENSATOR

$$T(s) = \frac{s+z}{s+p}$$

$$T(j\omega) = \frac{j\omega + z}{j\omega + p}$$

$$\angle T(j\omega) = \tan^{-1}\left(\frac{\omega}{z}\right) - \tan^{-1}\left(\frac{\omega}{p}\right)$$

$\angle T(j\omega) < 0^\circ$ (-ve angle).

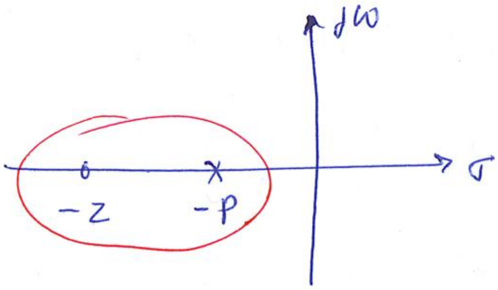
$$\tan^{-1}\left(\frac{\omega}{z}\right) - \tan^{-1}\left(\frac{\omega}{p}\right) < 0$$

$$\tan^{-1}\left(\frac{\omega}{z}\right) < \tan^{-1}\left(\frac{\omega}{p}\right)$$

$$\left(\frac{\omega}{z}\right) < \left(\frac{\omega}{p}\right)$$

$$\frac{1}{z} < \frac{1}{p}$$

$$\boxed{p < z}$$

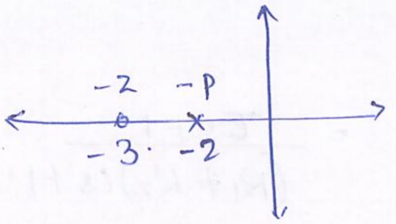


p & z are +ve real no.

pole nearer to origin ⇒ Pole dominating s/s

* Lag Compensator is referred as Pole Dominating s/s.

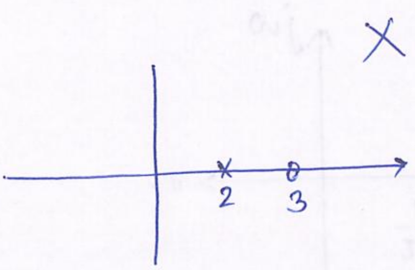
Ex $T(s) = \frac{s+3}{s+2}$



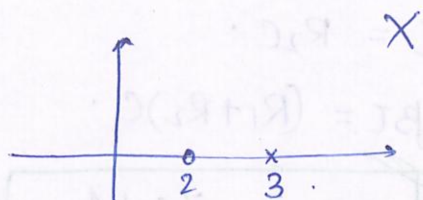
$P = 2$.
$Z = 3$.
$\boxed{P < Z}$.

Pole dominating s/s.

Ex $T(s) = \frac{s-3}{s-2}$.



Ex $T(s) = \frac{s-2}{s-3}$.

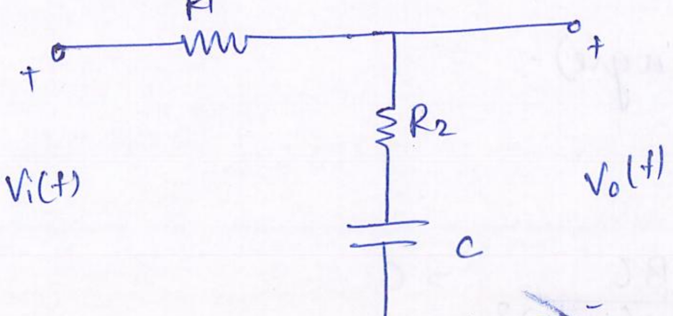


Ex $T(s) = \frac{s+3}{s-2}$.

* all are non-minimum phase s/s.

** Imp. $\boxed{\text{All Compensators are minimum phase s/s.}}$
(No sign change is possible).
R & C can't be -ve.

STANDARD LAG COMPENSATOR NETWORK



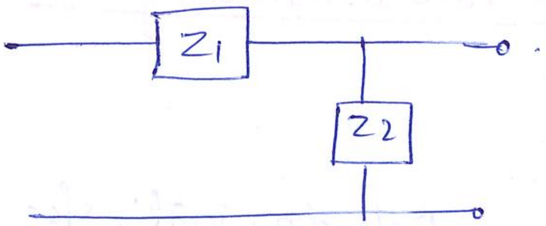
(i) $\tau = R_2 C$ = O/p time constant

(ii) $\beta = \frac{R_1 + R_2}{R_2} =$

Lag Comp. Coefficient
$(\beta > 1)$.

$\frac{V_o(s)}{V_i(s)} = T(s) = \frac{Z_1}{Z_1 + Z_2}$.

$Z_1[R_1] = R_1$ $\quad$ $Z_2\left[R_2 + \frac{1}{Cs}\right] = \frac{R_2(s+1)}{Cs}$.



$$T(s) = \frac{\frac{R_2Cs+1}{Cs}}{R_1 + \frac{R_2Cs+1}{Cs}} = \frac{R_2Cs+1}{R_1Cs + R_2Cs + 1} = \frac{\tau s+1}{(R_1+R_2)Cs+1}.$$

$$\beta = \frac{R_1+R_2}{R_2} \quad (\beta > 1).$$

$\tau = R_2C.$

$\therefore \beta\tau = (R_1+R_2)C.$

$$\therefore \boxed{T(s) = \frac{\tau s+1}{\beta\tau s+1}}$$

T.F of Lag compensator.

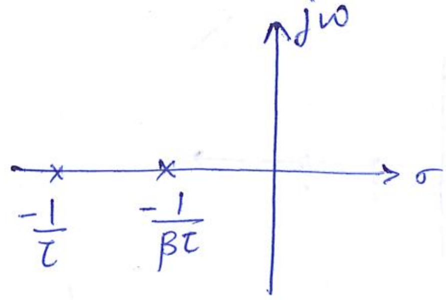


fig Lag Compensator
(Pole Dominating System).

$$T(j\omega) = \frac{1+j\omega\tau}{1+j\omega\beta\tau}.$$

$$\angle T(j\omega) = \tan^{-1}\omega\tau - \tan^{-1}\omega\beta\tau = \phi(\omega).$$

$\phi(\omega)_{max} = ?$ (max -ve angle).

$$\frac{d}{d\omega}\phi(\omega) = 0.$$

$$\frac{d}{d\omega}\phi(\omega) = \frac{\tau}{1+(\omega\tau)^2} - \frac{\beta\tau}{1+(\beta\omega\tau)^2} = 0.$$

$$\frac{1}{1+(\omega\tau)^2} = \frac{\beta}{1+\beta^2\omega^2\tau^2}$$

$$1+\beta^2\omega^2\tau^2 = \beta + \beta\omega^2\tau^2$$

$$\cancel{1-\beta} = \beta\omega^2\tau^2\cancel{(1-\beta)}$$

$$\therefore\ 1 = \beta \omega^2 \tau^2$$

$$\omega^2 = \frac{1}{\beta \tau^2}$$

$$\boxed{\omega_0 = \frac{1}{\tau\sqrt{\beta}}\ \text{r/sec.}}$$

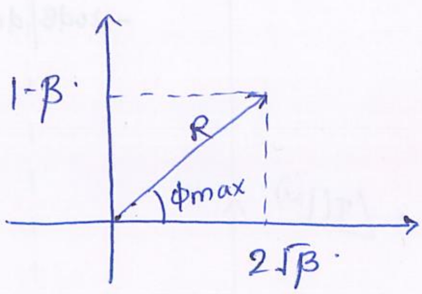
$$\therefore\ \phi_{max} = \tan^{-1}\omega\tau - \tan^{-1}\omega\beta\tau$$

$$\phi_{max} = \tan^{-1}\frac{1}{\sqrt{\beta}} - \tan^{-1}\beta \times \frac{1}{\sqrt{\beta}}.$$

$$\boxed{\phi_{max} = \tan^{-1}\frac{1}{\sqrt{\beta}} - \tan^{-1}\sqrt{\beta}}$$

$$\tan\phi_{max} = \frac{\frac{1}{\sqrt{\beta}} - \sqrt{\beta}}{1 + \frac{1}{\cancel{\sqrt{\beta}}} \times \cancel{\sqrt{\beta}}} \qquad (\tan(A - B)).$$

$$\boxed{\tan\phi_{max} = \frac{1-\beta.}{2\sqrt{\beta}.}}$$

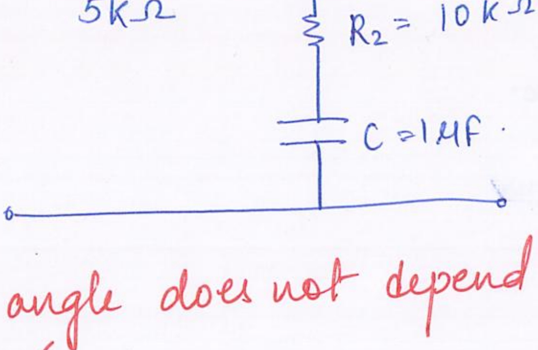


$$R = \sqrt{x^2+y^2}.$$

$$R = \sqrt{4\beta + (1-\beta)^2} = \sqrt{4\beta + 1 + \beta^2 - 2\beta} = \sqrt{(1+\beta)^2} = 1+\beta.$$

$$\therefore\ \boxed{\sin\phi_{max} = \frac{1-\beta}{1+\beta.}}$$

Ex.



$$\boxed{\beta = \frac{R_1+R_2}{R_2}} = \frac{5+10}{10} = \frac{3}{2}.$$

$$\sin\phi_{max} = \frac{1-\beta}{1+\beta} = \frac{1-3/2}{1+3/2}$$

$$= \frac{-1/2}{5/2} = \frac{-1}{5}$$

$$\phi_{max} = \sin^{-1}(-1/5) = -11.536^\circ$$

**. max angle does not depend upon C

$$T(s) = \left(\frac{1+\tau s}{1+\beta\tau s}\right)$$

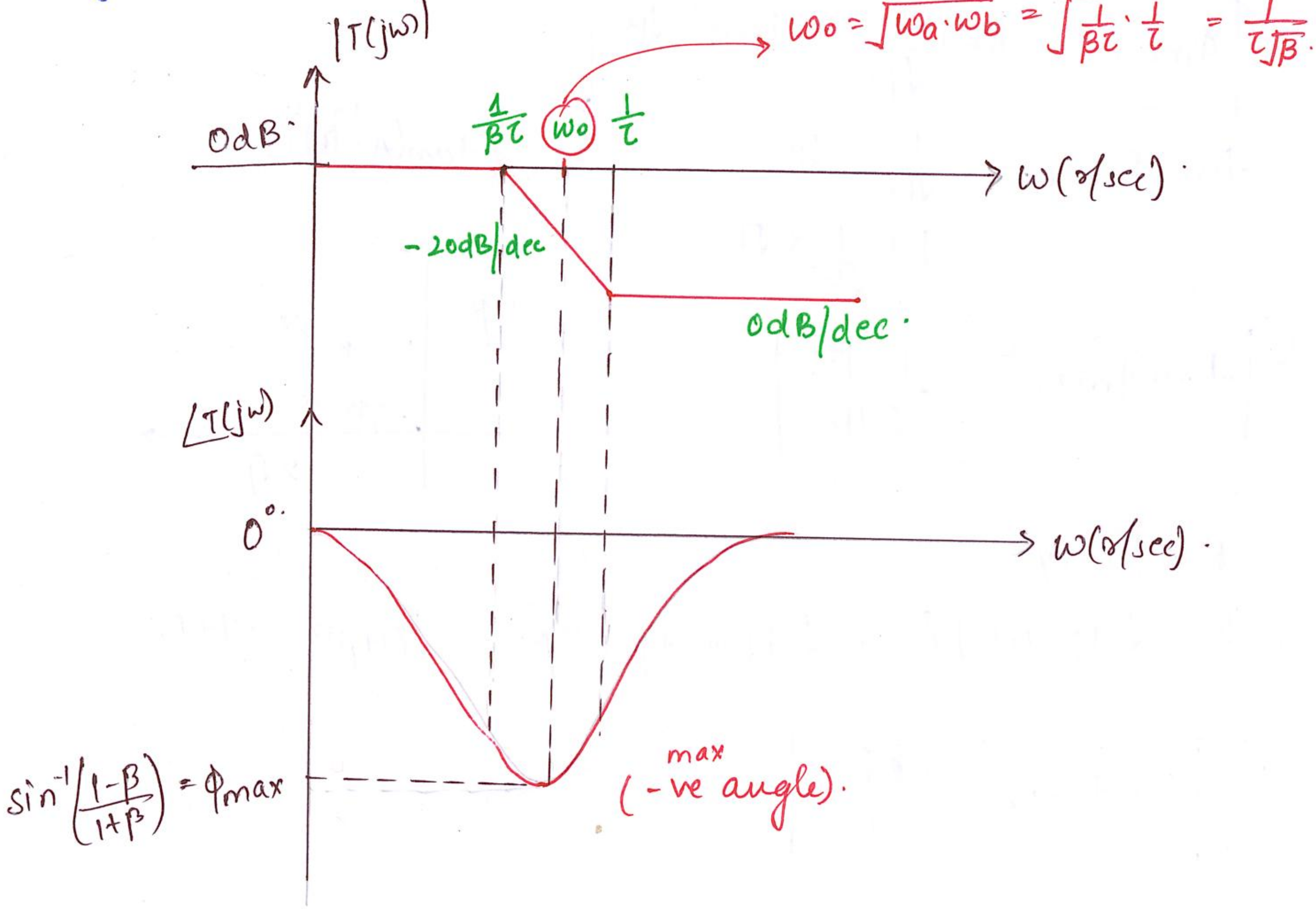
$K = 1.0$

$20\log K = 0\,dB$

$\angle T(j\omega) = \tan^{-1}\omega\tau - \tan^{-1}\beta\tau$.

$\angle T(j\omega=0) = 0°$

$\angle T(j\omega=\infty) = 0°$



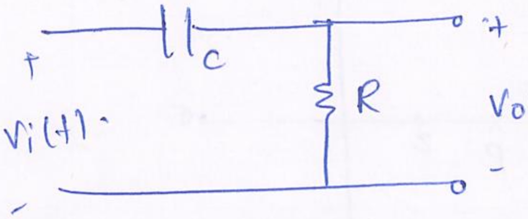
Slope at lowest freq = 0 dB/dec

Slope at highest freq = 0 dB/dec.

(II) LEAD COMPENSATOR

* Lead compensator provide +ve angle (leading angle) for all +ve value of frequencies.
* Basic behaviour of lead compensator is same as High Pass filter.

(iii) BASIC CIRCUIT.



$$\frac{V_o(s)}{V_i(s)} = \frac{R}{R + 1/Cs} = \frac{RCs}{1 + RCs}$$

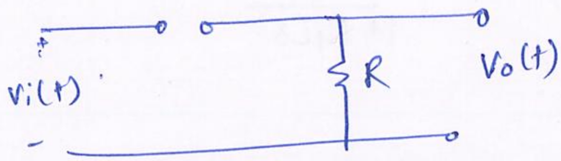
~~Ca~~ 1st Order High pass filter.

Case I. $X_C = \frac{1}{\omega_c}$

Low freq $\omega_c = 0$.

$X_C = \frac{1}{0} = \infty$. Open ckt

$i = 0$.



$\therefore V_o(t) = 0$ Volts.

Capacitor open ckt

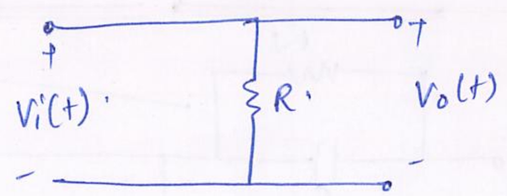
$$T(s) = \frac{RCs}{1 + RCs}.$$

$$T(j\omega) = \frac{j\omega RC}{1 + j\omega RC}.$$

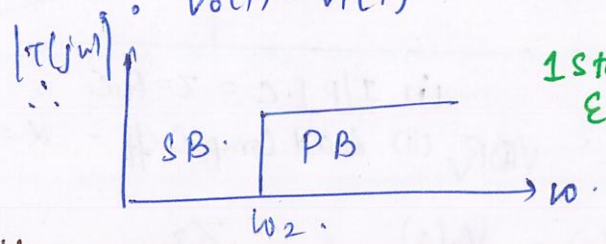
$$\angle T(j\omega) = 90^\circ - \tan^{-1}\omega RC$$

Case 2 High freq. $\omega_c = \infty$

$X_C = \frac{1}{\infty} = 0$ short ckt.



$\therefore V_o(t) = V_i(t)$.



1 story Elem

1st order HP filter.

ω	$\angle T(j\omega)$	Remarks
0°	90°	$\angle T(j\omega) \geq 0$
$1/RC$	45°	for $\omega \geq 0$
∞	0°	

Always +ve angle

* HPF is refered as Lead compensator n/w.

(iv) BASIC CONCEPT

$$T(s) = \frac{s+\tau}{s+\beta}$$

$$T(j\omega) = \frac{j\omega+\tau}{j\omega+\beta}. \qquad \angle T(j\omega) = \tan^{-1}\left(\frac{\omega}{\tau}\right) - \tan^{-1}\left(\frac{\omega}{\beta}\right)$$

$$\angle T(j\omega) > 0^\circ.$$

$$\therefore \tan^{-1}\left(\frac{\omega}{\tau}\right) - \tan^{-1}\left(\frac{\omega}{\beta}\right) > 0.$$

$$\cancel{\tan^{-1}}\left(\frac{\omega}{\tau}\right) > \cancel{\tan^{-1}}\left(\frac{\omega}{\beta}\right)$$

$$\frac{\cancel{\omega}}{\tau} > \frac{\cancel{\omega}}{\beta}$$

$$\boxed{P > Z}$$

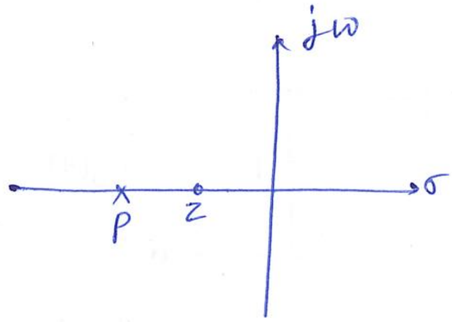
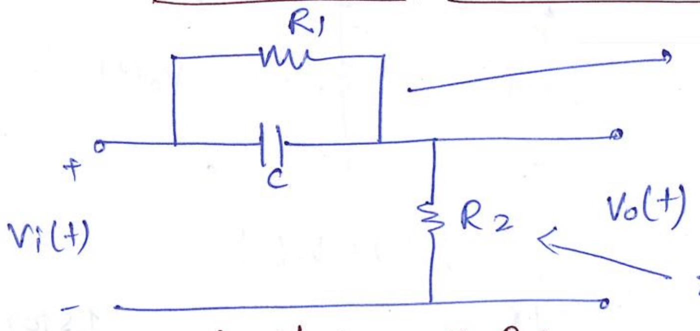


fig Zero Dominating s/s

(v) STANDARD LEAD COMPENSATOR N/W.



$Z_2 = R_2.$

$$Z_1 = R_1 \,//\, \frac{1}{Cs} = \frac{R_1 \times 1/Cs}{R_1 + 1/Cs}. = \frac{R_1}{1+R_1Cs}.$$

(i) I/P T.C $= \tau = R_1C$

$\therefore$ VDR, (ii) Lead Comp Coeff $= \alpha = \frac{R_2}{R_1+R_2} \qquad \alpha < 1$

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{R_2}{\frac{R_1}{1+R_1Cs}+R_2} = \frac{R_2(1+R_1Cs)}{R_1+R_2+R_1R_2Cs}$$

$$T(s) = \frac{R_2(1+\tau s)}{(R_1+R_2)\left(1+\frac{R_1R_2Cs}{R_1+R_2}\right)} \qquad \alpha\tau = \left(\frac{R_1R_2}{R_1+R_2}\right)C$$

$$\boxed{T(s) = \alpha\left(\frac{1+\tau s}{1+\alpha\tau s}\right)}$$

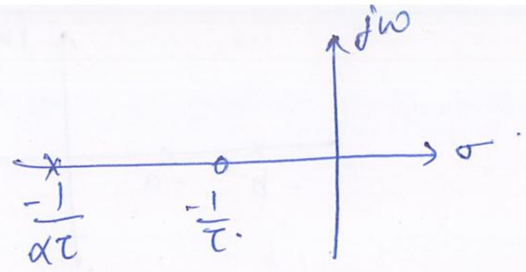


fig Zero Dominating system (Leading Compensator)

Lag Compensator.	Lead Compensator
* $T(s) = \dfrac{1+\tau s}{1+\beta\tau s}$	* $T(s) = \dfrac{\alpha(1+\tau s)}{(1+\alpha\tau s)}$
* $\omega_0 = \dfrac{1}{\tau\sqrt{\beta}}$ r/sec.	* $\omega_0 = \dfrac{1}{\tau\sqrt{\alpha}}$ r/sec.
* $\sin\phi_{max} = \dfrac{(1-\beta)}{(1+\beta)}$ (+ve angle).	* $\sin\phi_{max} = \left(\dfrac{1-\alpha}{1+\alpha}\right)$ +ve angle.

Ex. $T(s) = \dfrac{s+a}{s+b}$.

$$T(s) = \frac{a(1+s/a)}{b(1+s/b)}$$

$\tau = 1/a$, $\beta\tau = \frac{1}{b}$. on compairig with above T.F we get

$$\cancel{\omega_0 = \frac{1}{\tau\sqrt{\beta}} = \frac{1}{\frac{1}{a}\sqrt{\frac{1}{\tau}\times\frac{1}{\beta\tau}}}}$$

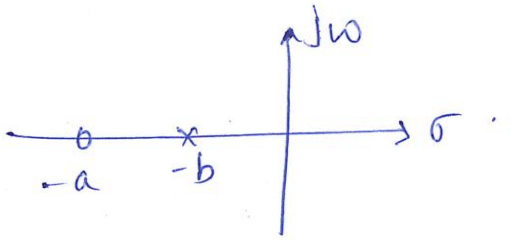
$$\omega_0 = \sqrt{\frac{1}{\tau}\times\frac{1}{\beta\tau}}$$

$$\omega_0 = \sqrt{a\times b}$$

$$\boxed{\omega_0 = \sqrt{ab}\ r/sec}$$

depending upon value of a+b it can be lag / lead comp.

* ωo value जिसपे max +ve या -ve angle होगा।



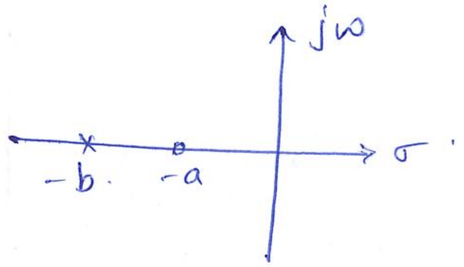
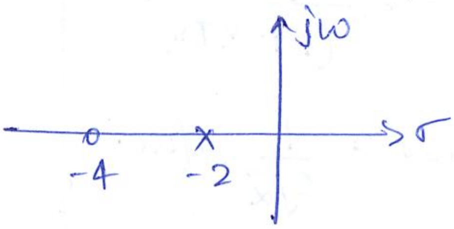


fig $b<a$.
pole dominating s/s
→ lag comp.

fig $a<b$.
Zero dominating s/s
Lead compensator.

Ex. $T(s) = \frac{s+4}{s+2}$



pole dominating s/s
Lag compensator.

$$T(s) = \frac{4(1+s/4)}{2(1+s/2)}$$

$\omega_1 = 2 \quad \omega_2 = 4$.

$\omega_0 = \sqrt{2\times 4} = \sqrt{8}$ r/sec

$$\sin\phi_{max} = \frac{1-\beta}{1+\beta}$$

$$T(s) = \frac{2(1+s/4)}{(1+s/2)} \qquad T(s) = \frac{1+\tau s}{1+\beta\tau s}$$

$\tau = 1/4 \qquad \beta\tau = \frac{1}{2}$.

$\therefore \beta = \frac{1}{2\tau} = \frac{1}{2\times\frac{1}{4}} = 2$.

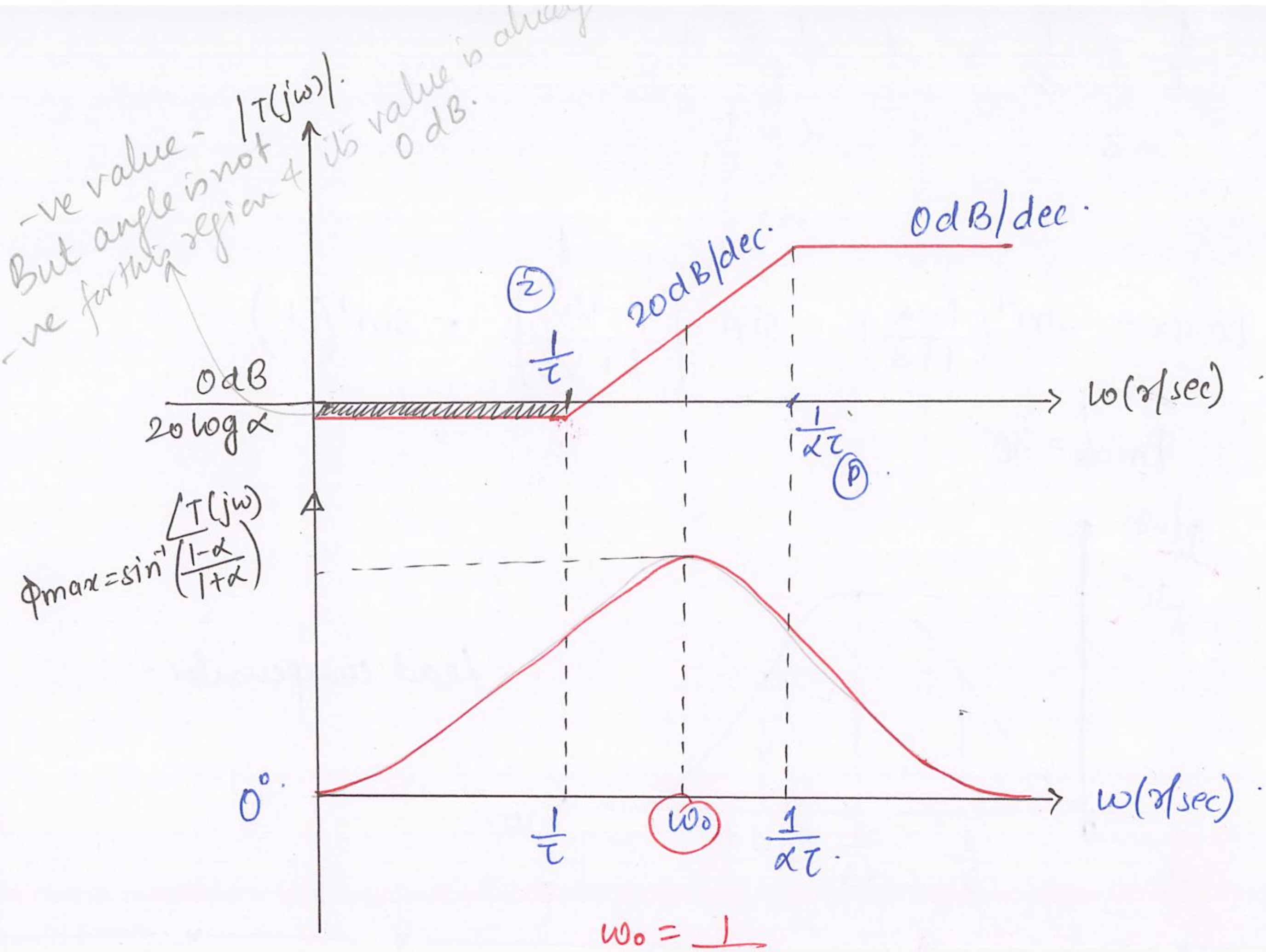
$\therefore \sin\phi_{max} = \frac{1-\beta}{1+\beta} = \frac{1-2}{1+2} = -\frac{1}{3} =$

$$\boxed{\phi_{max} = -19.47^\circ} \quad \text{-ve angle.}$$

∴ Lag compensator, pole dominating s/s.

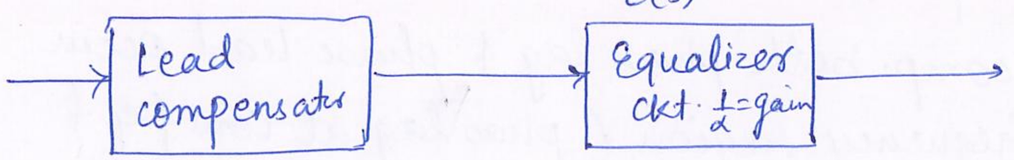
$$T(s) = \frac{s+a}{s+b} \qquad T(s) = \frac{1+\tau s}{1+\beta\tau s}$$

$$\omega_0 = \sqrt{ab}\ \text{r/sec} \qquad \omega_0 = \frac{1}{\tau\sqrt{\beta}}\ \text{r/sec}$$



$$\omega_0 = \frac{1}{\tau\sqrt{\alpha}}$$

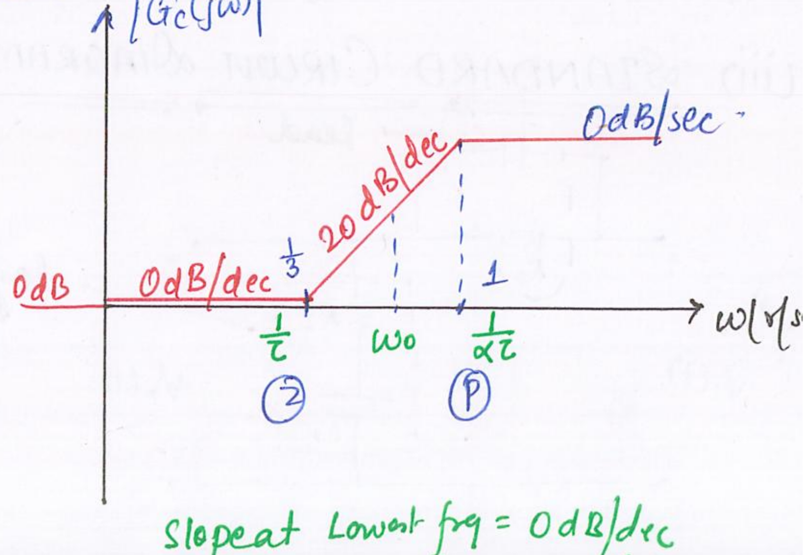
fig Bode plot for lead compensater.



$$G_c(s) = \frac{\alpha(1+\tau s)}{(1+\alpha\tau s)}$$

$G_c'(s) =$

$$G_c'(s) = \frac{(1+\tau s)}{(1+\alpha\tau s)}$$



$$\omega_0 = \sqrt{\frac{1}{\tau} \times \frac{1}{\alpha\tau}}$$

$$\omega_0 = \sqrt{\frac{1}{3} \times 1}$$

$$\omega_0 = \frac{1}{\sqrt{3}}$$

Slope at Lowest frq = 0dB/dec

Slope at highest frq = 0dB/dec

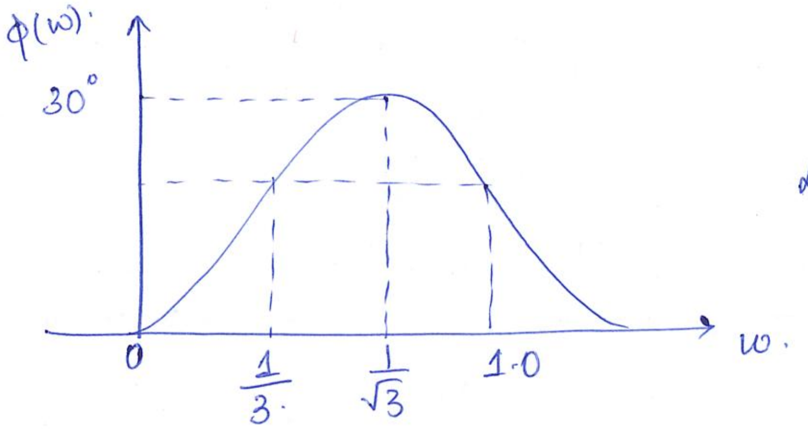
$$\frac{1}{\tau} = \frac{1}{3} \qquad \& \qquad \frac{1}{\tau\alpha} = 1$$

$$\therefore \tau = 3 \qquad \alpha = \frac{1}{\tau}$$

$$\therefore \alpha = \frac{1}{3}$$

$$\phi_{max} = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) = \sin^{-1}\left[\frac{1-1/3}{1+1/3}\right] = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\therefore \phi_{max} = 30^\circ$$

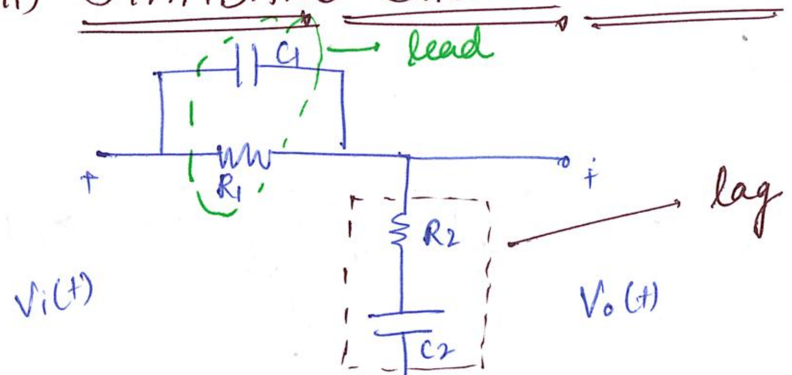


Lead compensator.

(III) LAG-LEAD COMPENSATOR

* Lag-Lead comp. improve both Transient & steady state perfomance of ~~comp~~ s/s.
* In Lag-lead comp. both phase lag & phase lead occur but in diff. frequency region (phas lag at low freq & phase lead at high freq).

(iii) STANDARD CIRCUIT DIAGRAM



$$T(s) = \frac{(s+\tau_1)(s+\tau_2)}{(s+P_1)(s+P_2)}$$

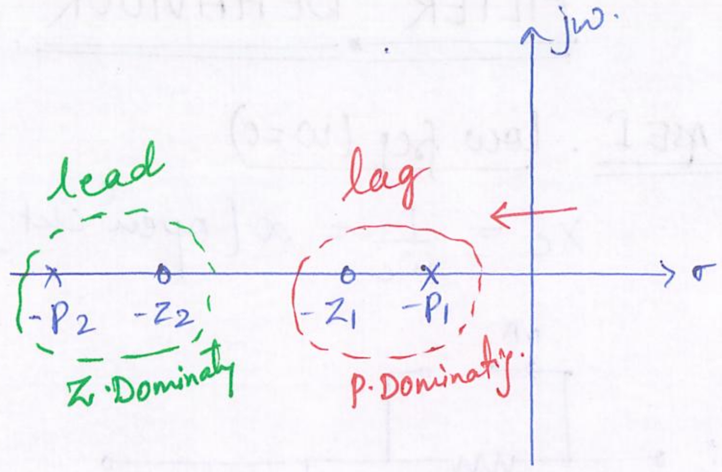


fig pole zero diagram of Lag-Lead compensator

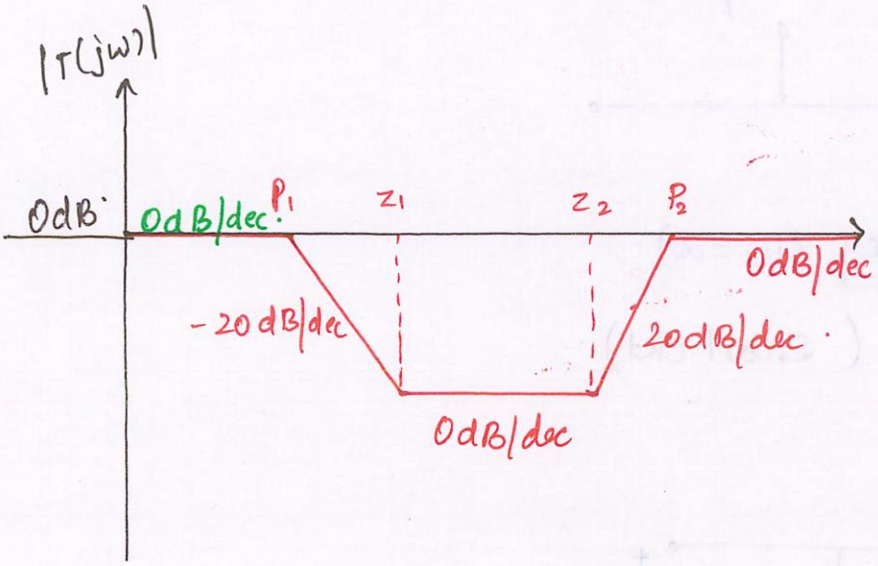


fig Asymptotic Bode magnitude plot for lag-lead comp. (Band stop).

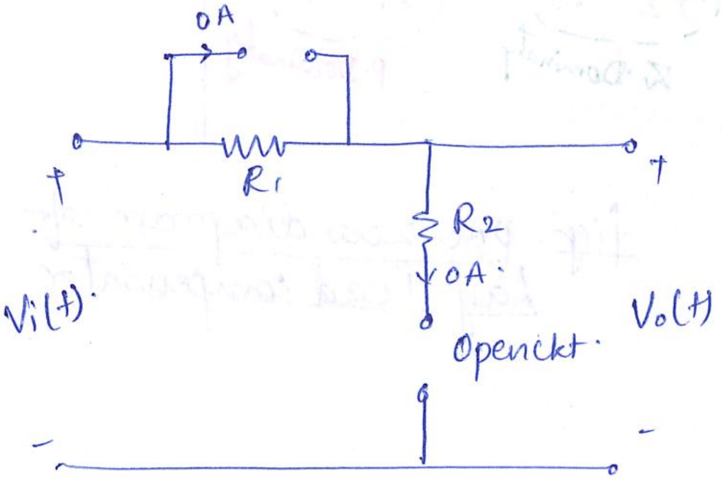
Slope at highest frequency = 0dB/dec
Slope at Low freq = 0dB/dec

FILTER BEHAVIOUR

LPF → Lag comp
HPF → Lead cap

Case I. Low freq ($\omega = 0$)

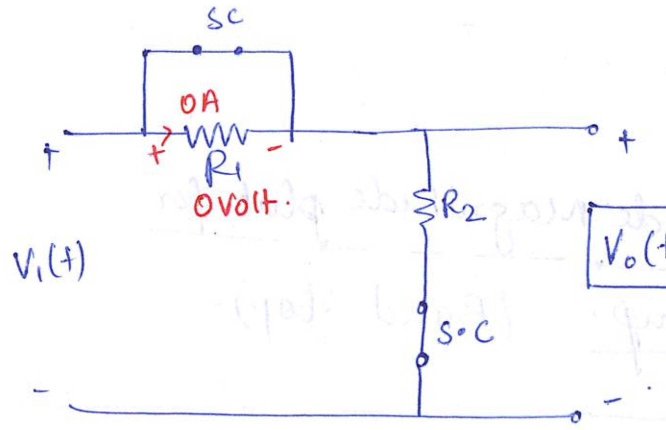
$$X_c = \frac{1}{\omega c} = \infty \text{ [open ckt]}.$$



$\boxed{V_o(t) = V_i(t)}$ at low frequency.

Case 2 High freq ($\omega = \infty$)

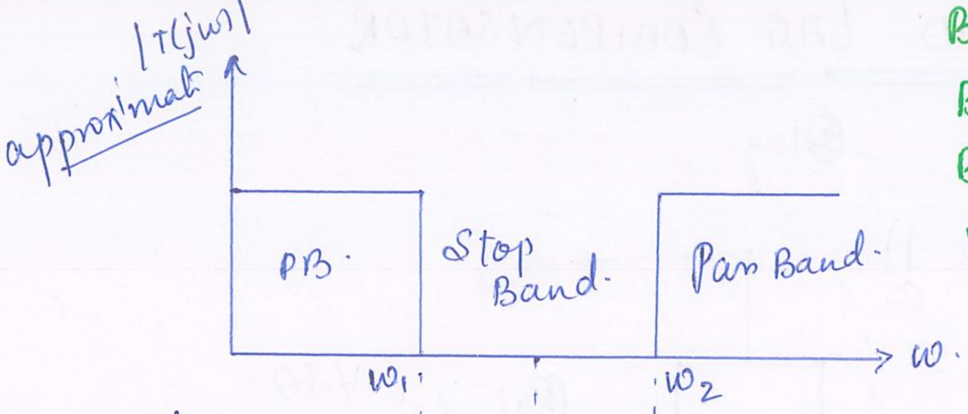
$$X_c = \frac{1}{\infty} = 0 \text{ (short ckt)}.$$

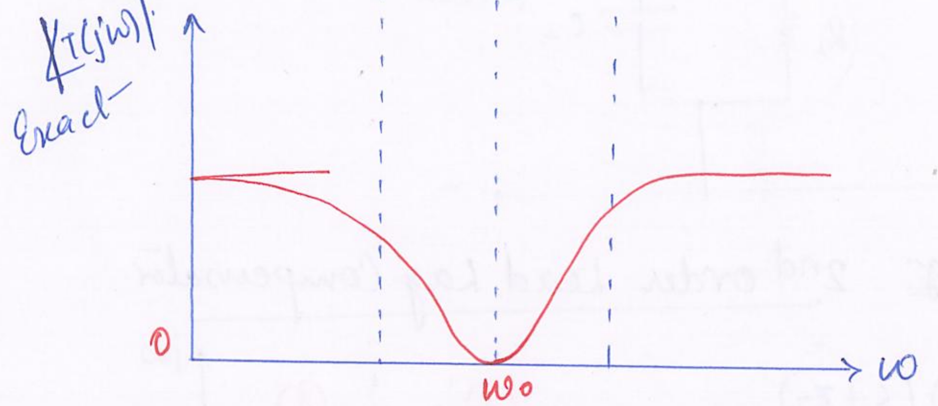


$\boxed{V_o(t) = V_i(t)}$.

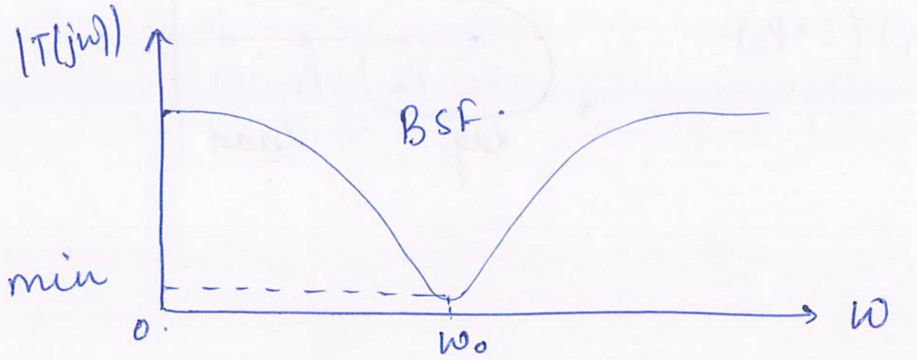
** Lag-Lead Compensator behaves as 2nd order Band stop filter.

Band Stop Filter/
Band Reject Filter/
Band Elimination Filter
Notch Filter





$|T(jw)|_{min} = 0.$



* Minimum Order of Band Stop filter = 2. = 2 minimum. Storage Element req. to design B.S.F. (No Existence of 1st order ~~Cut~~ BSF).

IV LEAD LAG COMPENSATOR.

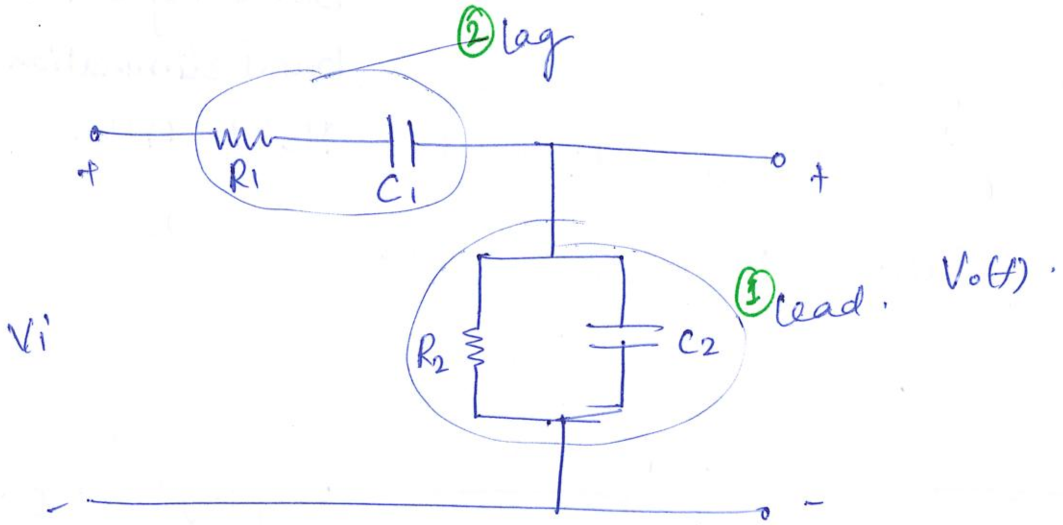
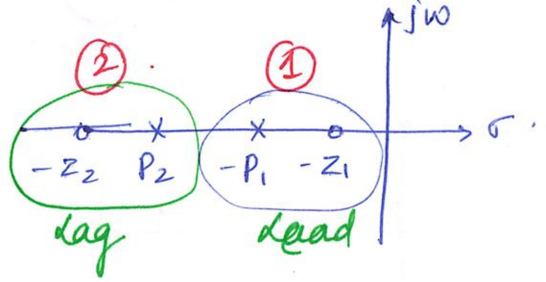


fig 2nd order Lead Lag Compensator

~~T(s) =~~

$$T(s) = \frac{(s+z_1)(s+z_2)}{(s+P_1)(s+P_2)}.$$



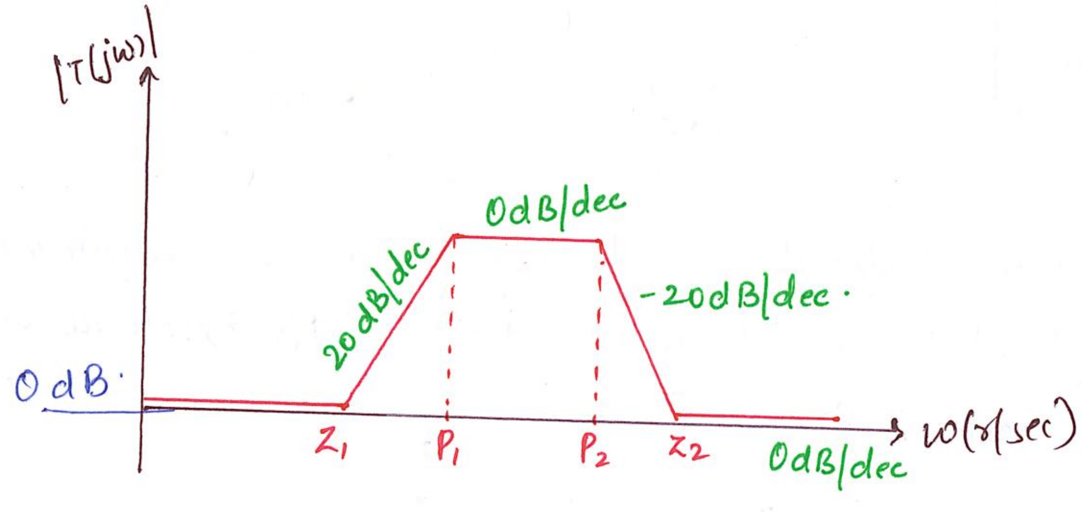


fig Asymptotic Bode plot of Lead Lag Comp

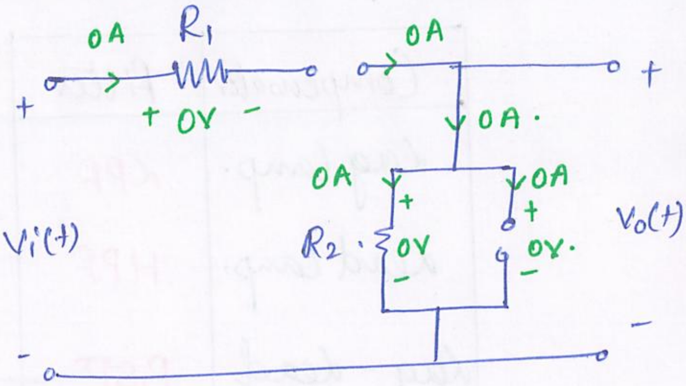
Slope at lowest freq = 0dB/dec →

Slope at highest freq = 0dB/dec.

BEHAVIOUR OF FILTER

Case I Low freq. $\omega_c = 0$.

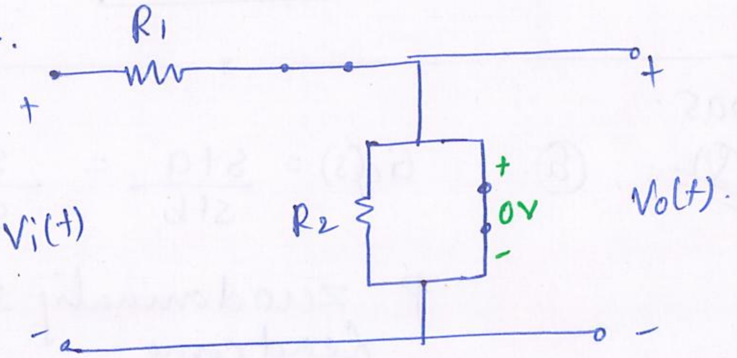
$X_C = \frac{1}{0} = \infty$ = Cap. open ckt



$V_o(t) = 0$ Volt.

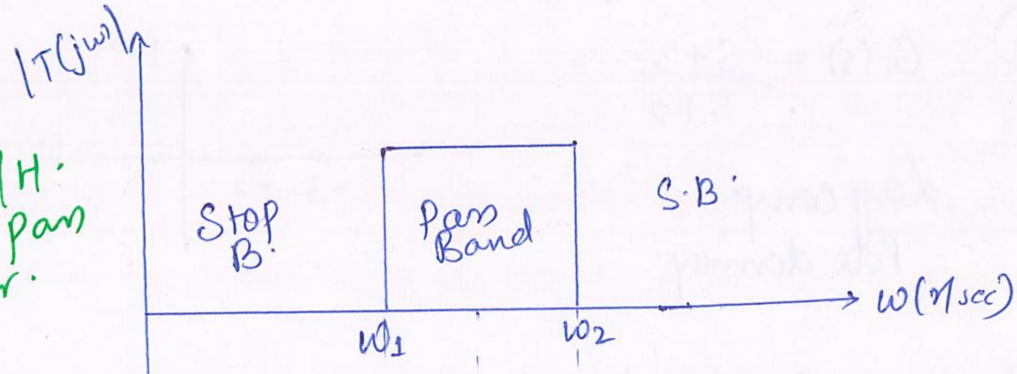
Case 2 High frequency $\omega_c = \infty$

$X_C = \frac{1}{\infty} = 0$ = cap. short ckt.

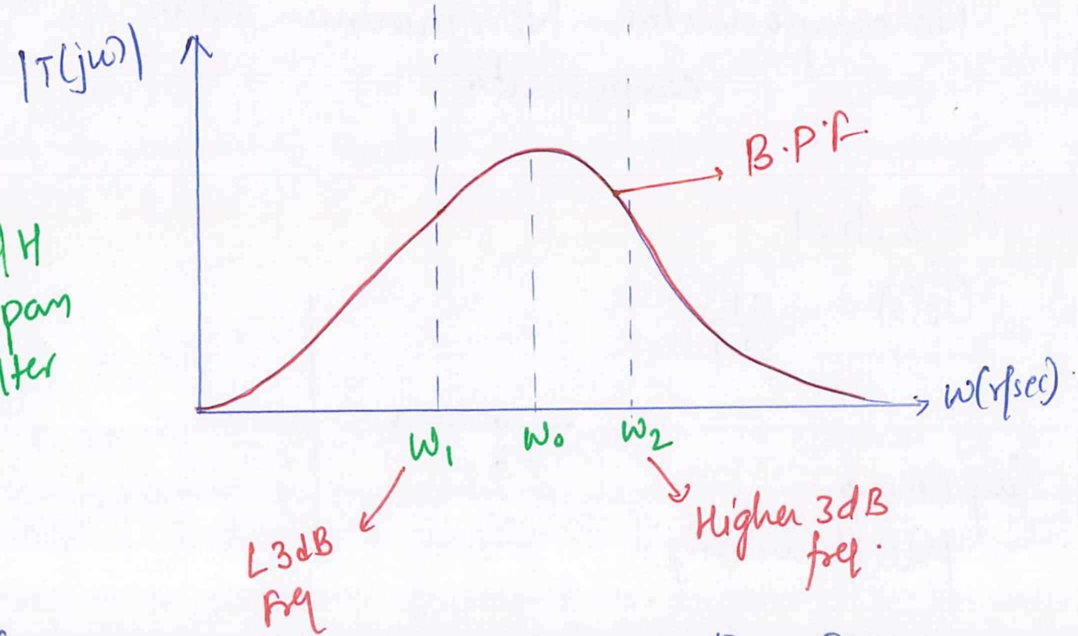


$\boxed{V_o(t) = 0 \text{ Volts}}$

Ideal C/H of Band pass filter.



Exact C/H of Band pass filter



frequency Response of Band Pass Filter.

CONCLUSION.

Compensator	Filter
Lag Comp.	LPF
Lead Comp.	HPF
Lag - Lead	BSF $\longrightarrow P_2 Z_2 Z_1 P_1$
Lead Lag	BPF. $\longrightarrow Z_2 P_2 P_1 Z_1$.

$\frac{s+1}{s+2}$

$\frac{1+s}{2(1+s/2)}$

$\tau = 1$

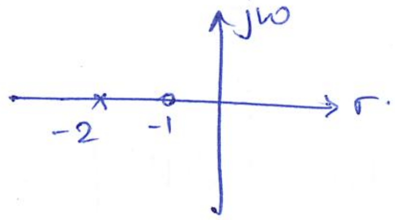
$\alpha\tau = \frac{1}{2}$

$\alpha = \frac{1}{2}$

---

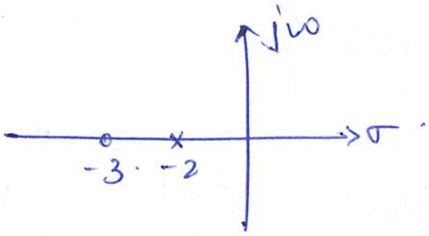
pas.

Q1. (a). $G_c(s) = \frac{s+a}{s+b} = \frac{s+1}{s+2}$.



P zero dominating s/s.
Lead comp.

(b) $G_c(s) = \frac{s+3}{s+2} =$

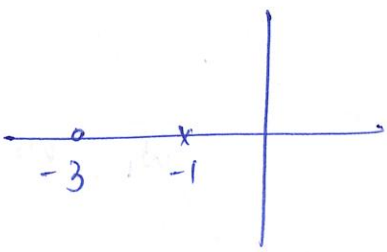


Lag comp
Pole dominy.

(c). $a = -3,\ b = -1$
No compensator. Non minimum phase s/s are all compensator.

(d). $a = 3,\ b = 1$

$G_c(s) = \frac{s+3}{s+1}$



Lag Comp.
pole dominanly

Q2. phase of above lead comp is max at

$$\omega_0 = \sqrt{ab} = \sqrt{1 \times 2} = \sqrt{2} \text{ rad/sec.}$$

$$G_c(s) = \cancel{\alpha(1+s)}\ \frac{s+1}{s+2} = \frac{(s+1)}{2(1+s/2)} = \alpha\left(\frac{1+\tau s}{1+\alpha\tau s}\right)$$

$$= 0.5\left(\frac{1+s}{1+s/2}\right)$$

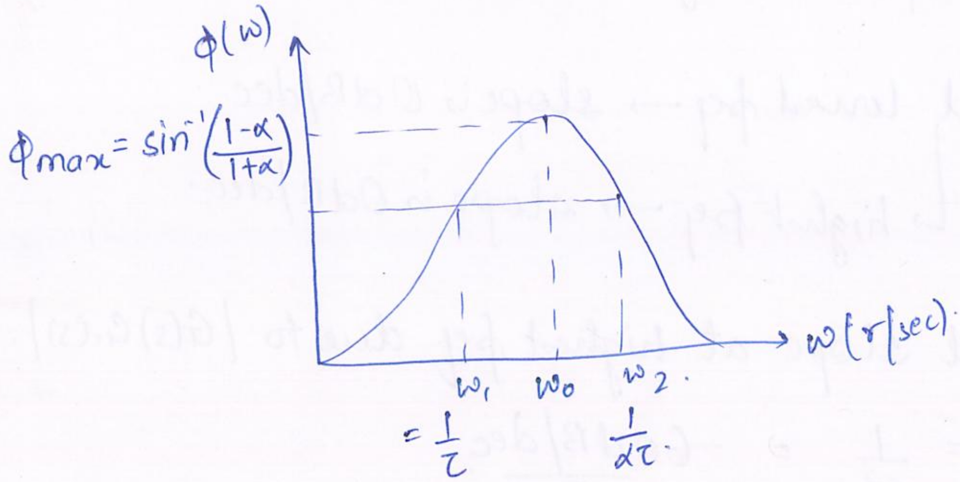
$\tau = 1$, $\alpha\tau = \frac{1}{2}$.

$\therefore \alpha = \frac{1}{2}$.

$$\phi_{max} = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right) = \sin^{-1}\left(\frac{1-1/2}{1+1/2}\right) = \sin^{-1}\left(\frac{1/2}{3/2}\right)$$

$$= 19.47^\circ.$$

(max +ve angle)



We know that. $\tau = R_1 C$.

$$\alpha = \frac{R_2}{R_1 + R_2}.$$

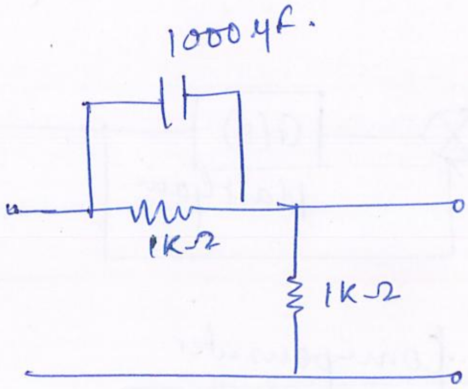
($\alpha = \frac{1}{2}$ from above) $\quad \frac{1}{2} = \frac{R_2}{R_1 + R_2}$.

Let $R_1 = R_2 = R$.

$\therefore R = 1K\Omega$, $C = 1mF$.

$\therefore \tau = 1000 \times 10^{-3} F = R_1 C$.

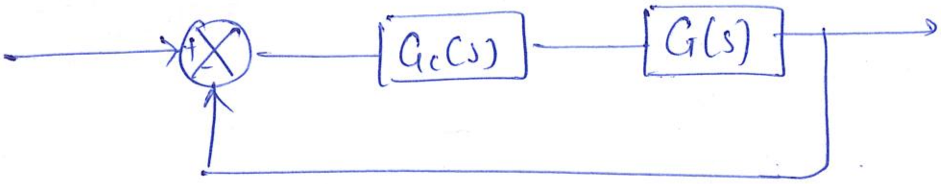
$\tau = 1 sec$.



Lead Compensator.

Q3.

$$G_c(s) = \left[\frac{1+\tau_1 s}{1+\left(\frac{\tau_1 s}{\beta}\right)}\right]\left[\frac{1+\tau_2 s}{1+\beta\tau_2 s}\right]. \qquad \beta<1.$$



given lag-lead comp.

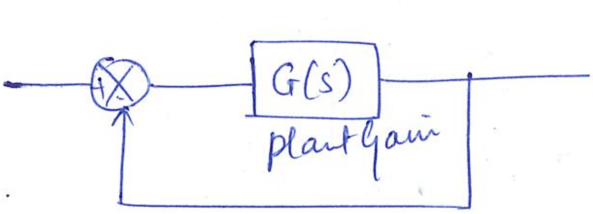
& $G(s) \longrightarrow$ 3 poles in left hand.

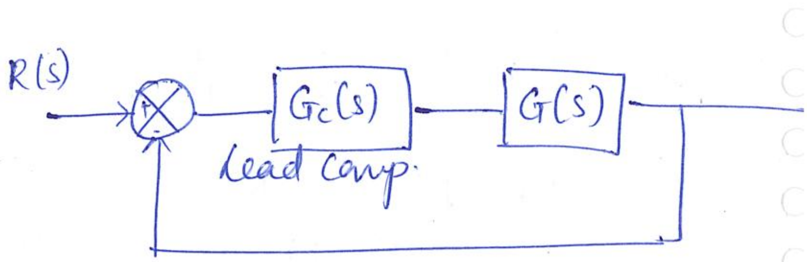
For all comp. at lowest freq → slope is 0 dB/dec.
↳ highest freq → slope is 0 dB/dec.

∴ the Resultant slope at highest freq due to $|G(s)G_c(s)|$.

3 poles of $G(s) = \frac{1}{s^3} = -60\,dB/dec$

Q4. OLTF . $G(s) = \frac{1}{(s^2-1)}$





w/o compensator
Block diagram.

Block diagram with compensator.

Take lead comp.

$G(s) = \dfrac{1}{s^2-1}$

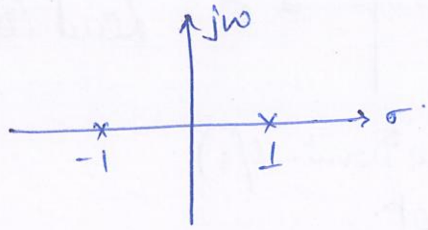


fig Unstable OLTF.

* This OLTF can never be stable. $\dfrac{1}{(s+1)(s-1)}$ for this to be stable $\left(\dfrac{1}{s+1}\right) = G(s)$.

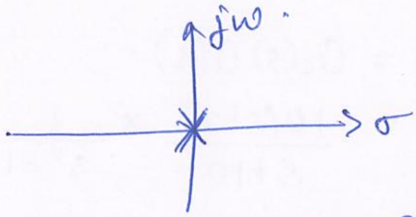
$G_c(s) = \dfrac{(s-1)}{x} \rightarrow$ Non-mini phase s/s.

No comp. can do this Bcoz all comps are M.P.S.

~~Req~~ ~~fig~~ find $G_c(s)$ lead, so that CLTF is stablized.

$$CLTF = \frac{G(s)}{1+G(s)}$$

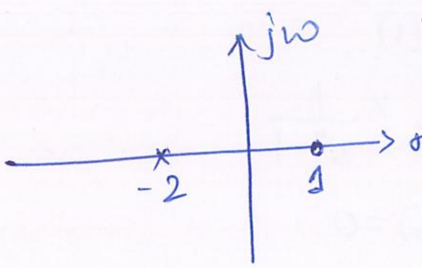
$$CLTF = \frac{1}{s^2-1+1} = \frac{1}{s^2}$$



pole zero of CLTF of fig (a) (w/o compensator)

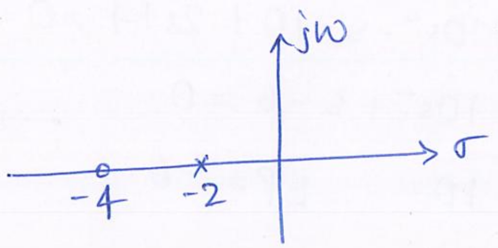
→ Unstable due to multiple pole on imag axis ~~dte~~ including origin.

a). $\dfrac{10(s-1)}{s+2} = G_c(s)$



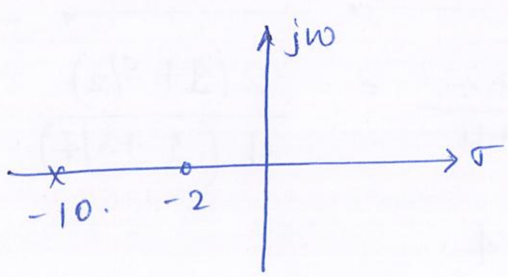
X NMPS
No Compensator.

b) $G_c(s) = \dfrac{10(s+4)}{(s+2)}$



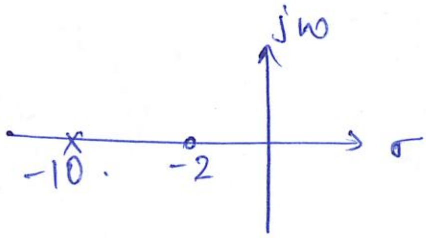
X pole dominating
Lag Comp.

c) $G_c(s) = \dfrac{10(s+2)}{s+10}$



Zero dominating
Lead comp

d) $G_c(s) = \frac{2(s+2)}{s+10}$



Zero daninty

Lead Comp.

option ⓒ & ⓓ are lead comp. (Zero Domi. s/s).
1 will give stability & other will not.

ⓒ $G'(s) = G_c(s)\,G(s)$

$= \frac{10(s+2)}{s+10} \times \frac{1}{s^2-1}$

C/E $= 1 + G'(s) = 0$

$(s+10)(s^2-1) + 10s + 20 = 0$

$= s^3 + 10s^2 - s - 10 + 10s + 20 =$

$s^3 + 10s^2 + 9s + 10 = 0$

$IP = 90 > EP = 10$ Stable.

ⓓ $G'(s) = G_c(s)\,G(s)$

$= \frac{2(s+2)}{s+10} \times \frac{1}{s^2-1}$

C/E $= 1 + G'(s) = 0$

$(s+10)(s^2-1) + 2s + 4 = 0$

$s^3 + 10s^2 - s - 10 + 2s + 4 = 0$

$s^3 + 10s^2 + s - 6 = 0$ ⟶ unstable. sign change.

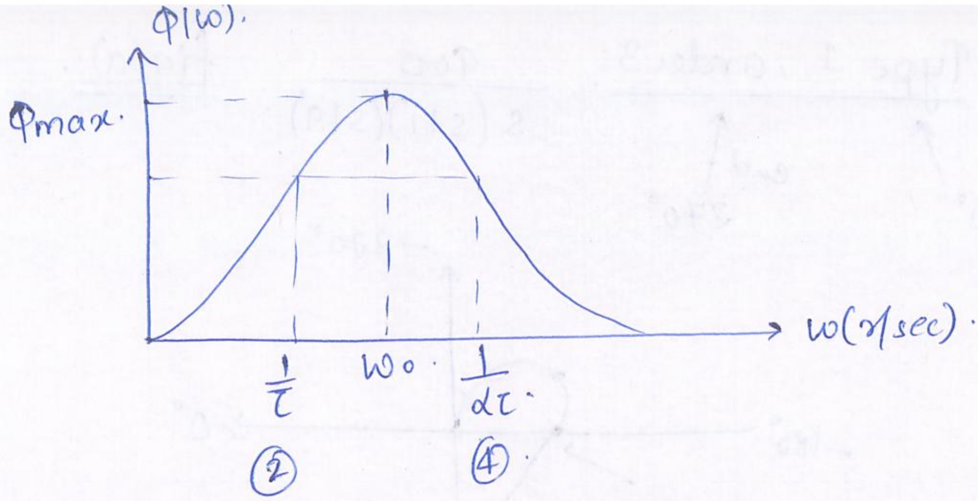
$IP = 10$. $EP = -6$.

∴ Unstable.

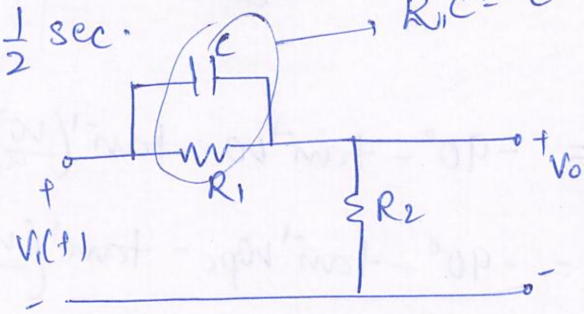
Q24 $G_c(s) = \frac{s+2}{s+4} = \frac{2(1+s/2)}{4(1+s/4)} = \alpha\left(\frac{1+\tau s}{1+\alpha\tau s}\right)$

$\omega_1 = 2 \quad \omega_2 = 4$

$\therefore \omega_0 = \sqrt{ab} = \sqrt{2 \times 4} = \sqrt{8}$



$\frac{1}{\tau} = 2.$ $\quad \therefore \tau = \frac{1}{2}$ sec.



$\frac{1}{\alpha\tau} = 4.$

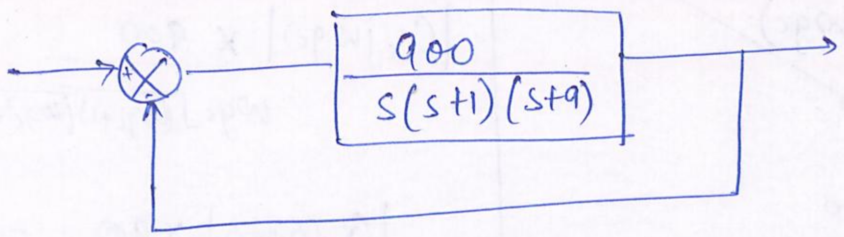
$\alpha = \frac{1}{4\tau} = \frac{1}{2}$

$\underline{\tau = R_1C = 0.5\ \text{sec}}$

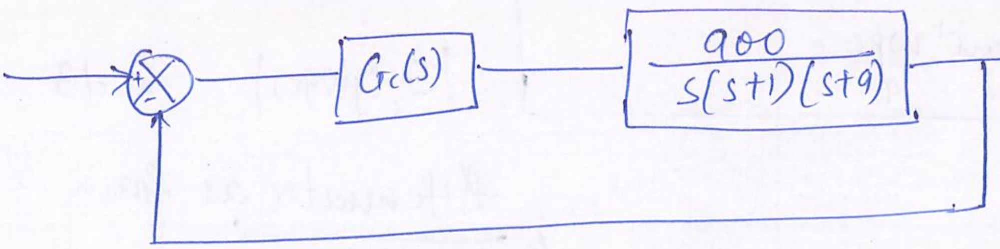
Q22. $G(s) = \frac{900}{s(s+1)(s+9)}$ is to be compensated.

$PM = 45°$.

Gain C.O freq = Uncompensated ω_{pc}



fig(a) w/o comp.



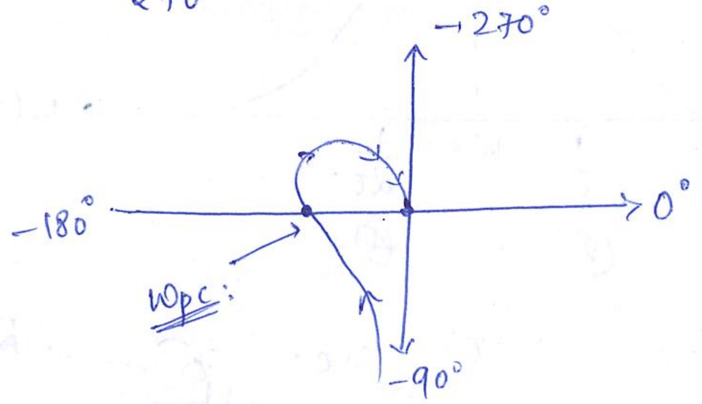
fig(b) with comp.

(1) $\omega_{gc}[\text{fig}(b)] = \omega_{pc}[\text{fig}(a)]$

(2) $PM[\text{fig}(b)] = 45°$

ω_{pc}. Type 1, order 3: $\dfrac{900}{s(s+1)(s+9)}$. fig(a).

-90° (Type 1) end -270° (order 3)



$$\angle G(s) = -90° - \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega}{9}\right)$$

$$-180° = -90° - \tan^{-1}\omega_{pc} - \tan^{-1}\left(\frac{\omega_{pc}}{9}\right)$$

$$+90° = \tan^{-1}\omega_{pc} + \tan^{-1}\left(\frac{\omega_{pc}}{9}\right).$$ (Trial & Error).

$\therefore$ $\boxed{\omega_{pc} = 3\,\text{rad/sec}}$. = ω_{gc} of fig(b).

$$PM = 180° + \angle GH(j\omega_{gc}).$$

$$45° = 180° + \angle GH(j\omega_{gc}).$$

$$\angle GH(j\omega_{gc}) = -135°.$$

$$\angle GH(j\omega_{pc}) = -135°.$$

$$-90° - \tan^{-1}\omega_{pc} - \tan^{-1}\frac{\omega_{pc}}{9} = -135°$$

$$\tan^{-1}\omega_{pc} + \tan^{-1}\frac{\omega_{pc}}{9} = .$$

$$|G'(j\omega_{gc})| = 1.0$$

$$|G_c j\omega_{gc}| \times \frac{900}{\omega_{gc}\sqrt{(\omega_{gc}^2+1)(\omega_{gc}^2+81)}} = 1.0.$$

$$|G_c(j\omega_{gc})| \times \frac{900}{3\times\sqrt{900}} = 1.0.$$

$$|G_c(j\omega_{gc})| = 0.1$$

$$|G_c(j\omega_{gc})| = -20\,dB.$$

Attenuator as gain is less than 1.

* $PM = 180° + \angle G'(j\omega_{gc}) = 45°$

$\therefore \angle G'(j\omega_{gc}) = -135°.$

$\omega_{gc} = $ from fig (b) $= 3 rad/sec.$

$\angle G(j\omega_{gc}) = -90° - \tan^{-1} 3 - \tan^{-1} \frac{1}{3}.$

$\angle G'(j\omega_{gc}) = \angle G_c(j\omega_{gc}) + \angle G(j\omega_{gc}).$

$-135° = \angle G_c(j\omega_{gc}) + \underbrace{\left[-90° - \tan^{-1} 3 - \tan^{-1} \frac{1}{3}\right]}_{-180°}.$

$\therefore \angle G_c(j\omega_{gc}) = -135° + 180°$

$= 45° \Rightarrow$ H.P

CONTROLLERS.

* Controller is a device which is used to improve the transient & steady state performance of the s/s.

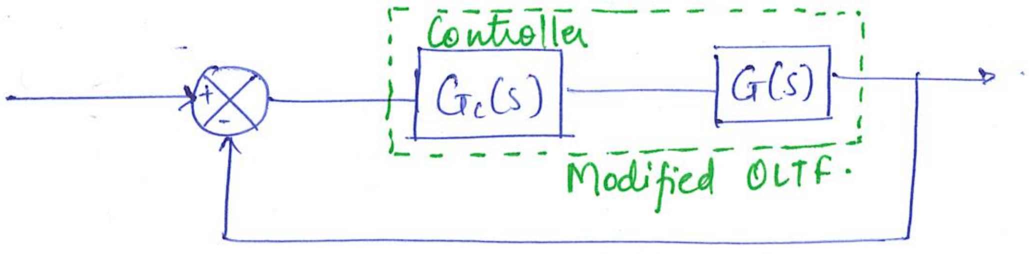


fig. Block diagram with Controller.

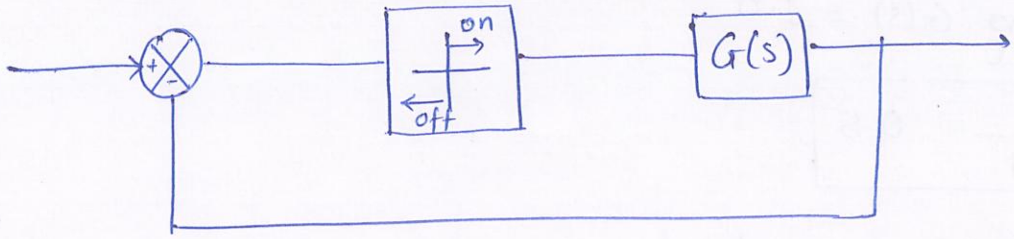
$$OLTF_{modified} = G(s) \cdot G_c(s) = G'(s).$$

$$C/s_{modified} = 1 + G'(s) = 0.$$

CLASSIFICATION OF CONTROLLER.

1. On-off Controller
2. Proportional Controller
3. Proportional derivative Controller (PD)
4. Integral Controller
5. Derivative Controller
6. Proportional Integral Controller (PI)
7. PID Controller.

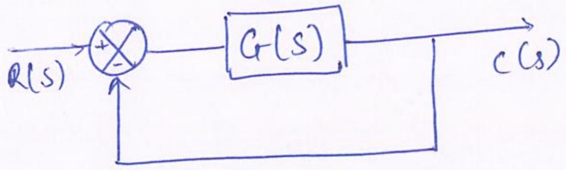
(I) ON-OFF CONTROLLER

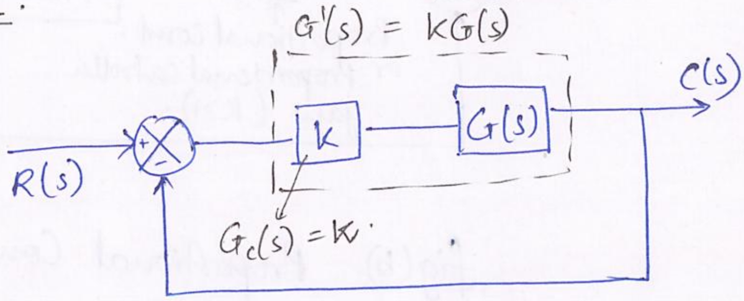


** On-off controller is refered as Non-Linear Controller.

27/07/19

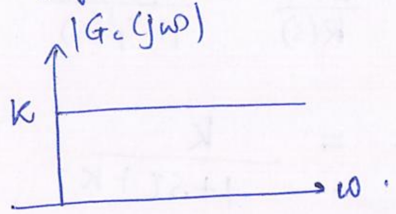
(II) PROPORTIONAL CONTROLLER.





fig(a) w/o Controller

fig(b) With controller.



Case I. Type '0' Order '1' System

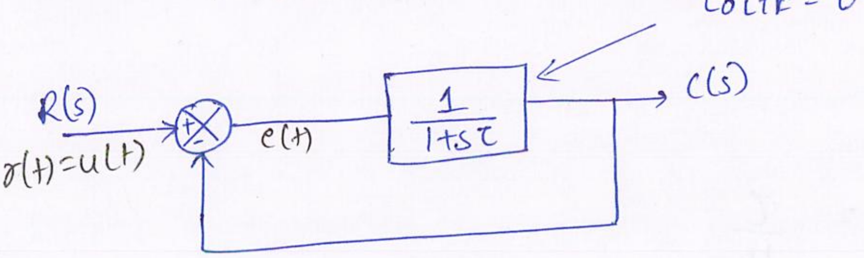
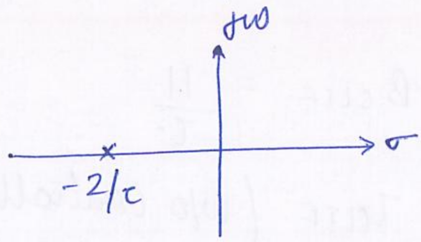


fig (a).

$$CLTF = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{1}{2+s\tau}$$



$$\boxed{\text{BW of 1}^{st}\text{ Order s/s} = \frac{1}{\text{Time Const}}}$$

Pole Zero diagram of CLTF.

$T = \tau/2$.

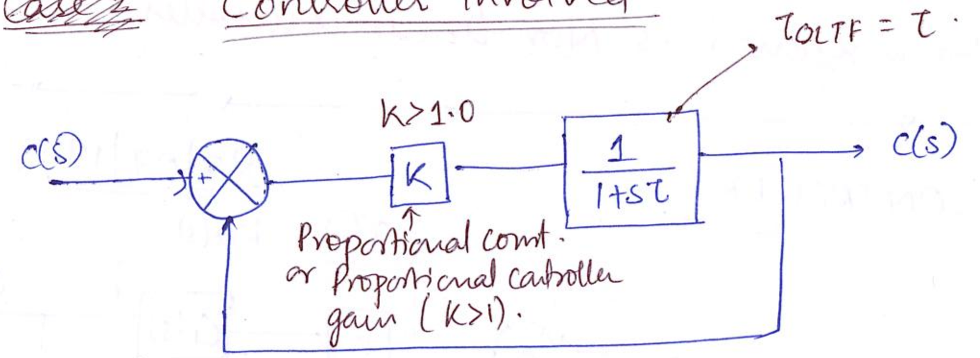
$BW = 2/\tau$.

$$e_{ss} = \frac{1}{1+K_p}$$

$$K_p = \lim_{s \to 0} G(s) = 1.0$$

$$\therefore \boxed{e_{ss} = \frac{1}{1+1} = 0.5}$$

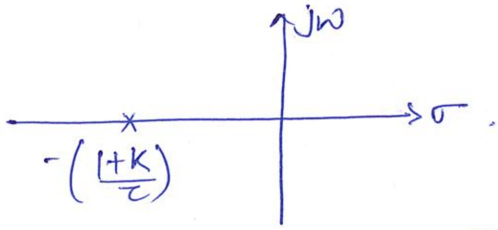
Case 2 Controller Involved



fig(b) Proportional Controller

$$CLTF = \frac{C(s)}{R(s)} = \frac{G'(s)}{1+G'(s)} \cdot \quad \left[G'(s) = \frac{K}{1+s\tau}\right]$$

$$\therefore CLTF = \frac{K}{1+s\tau+K}$$



Pole zero diagram of ~~G'(s)~~ -- CLTF mod.

$$\therefore \boxed{\begin{aligned} T_{CLTF} &= \frac{\tau}{1+K} \\ BW_{CLTF} &= \frac{1+K}{\tau} \end{aligned}}$$

* If $K = 10$

$$T_{CLTF} = \tau_{CLTF} = \frac{\tau}{1+K} = \frac{\tau}{11}$$

then, $B_{CLTF} = \frac{11}{\tau}$

& τ_{CLTF} (w/o controller) $= \frac{\tau}{2}$

$$\therefore \boxed{\tau_{CLTF}\text{ (with Controller)} < \tau_{CLTF}\text{ [w/o Controller]}}$$

* Proportional Controller decreases the value of Time Constant.
* Proportional Controller increases the value of Bandwidth.

* Let I/P $r(t) = u(t)$

$$e_{ss}' = \frac{1}{1+K_p'}$$

$$K_p' = \lim_{s \to 0} G'(s) = \lim_{s \to 0} \frac{K}{1+s\tau}$$

$$\therefore K_p' = K$$

$$\therefore e_{ss}' = \frac{1}{1+K} = \frac{1}{11}$$

* Proportional Controller decreases the value of Steady state Error in Same domain

* Feedback decrease the value of time constant.

$$\tau[OLTF] = \tau$$

$$\tau[CLTF] = \tau/2$$

$$\tau[CLTF]_{K=10} = \frac{\tau}{11}$$

(Response will be fast, t_{sett} will be less). (Error is also decreased).

CASE 2 TYPE '1' ORDER '2'

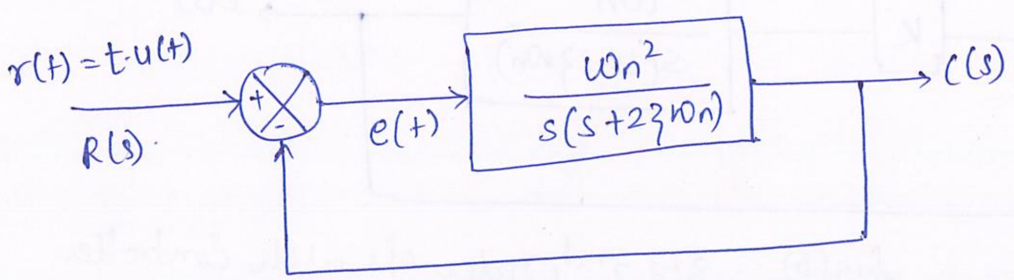


fig 2nd Order std s/s w/o Controller

$$\frac{C(s)}{G(s)} = \frac{G(s)}{1+G(s)}$$

$$CLTF = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(A) $e_{ss} = \frac{1}{K_v}$

$K_v = \lim_{s \to 0} sG(s)$.

$K_v = \frac{\omega_n}{2\zeta}$.

$\therefore e_{ss} = \frac{2\zeta}{\omega_n}$ [finite Error].

(B) $\omega_d = \omega_n\sqrt{1-\zeta^2}$

(C) $\tau = \frac{1}{\alpha} = \frac{1}{\zeta\omega_n}$

(d) $t_{sett} = 4\tau = \frac{4}{\zeta\omega_n}$

(e) $t_p = \frac{\pi}{\omega_d}$

(f) $t_r = \frac{\pi - \theta}{\omega_d}$.

(g) $t_d = \frac{1 + 0.7\zeta}{\omega_n}$ [$t_d \propto t_r$]

(h) $MPO = e^{-\pi\cot\theta}$.

$MPO \propto \theta$.

(i) $\cos\theta = \zeta$.

$\theta \propto \frac{1}{\zeta}$.

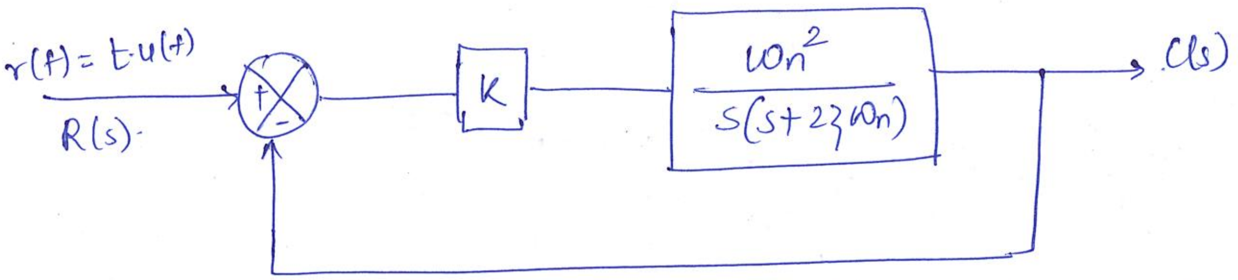
$MPO \propto \frac{1}{\zeta}$.

(j) 2nd order Under Damped s/s.

$s = -\alpha \pm j\omega_d$.

$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

Now Controller is Involved



fig(b). Std 2nd Order s/s with Controller

ⓐ $e_{ss}' = \frac{1}{K_v'}$

$K_v' = \lim_{s \to 0} sG'(s)$.

$K_v' = \frac{K\omega_n}{2\zeta}$.

$\therefore e_{ss}' = \frac{2\zeta}{K\omega_n}$

$\therefore \boxed{e_{ss}' = \frac{e_{ss}}{K}}$

$$\boxed{e_{ss}'[\text{with } K] = \frac{e_{ss}[\text{w/o } C]}{K}}$$

* Proportional Controller dec. the steady State Error by factor of K in same domain

Closed Loop T.f (mod)

$$\frac{C(s)}{R(s)} = \frac{G'(s)}{1+G'(s)} = CLTF = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+K\omega_n^2}$$

$$s^2+2\zeta'\omega_n' s+\omega_n'^2 = s^2+2\zeta\omega_n s+K\omega_n^2$$

On comparing these 2 we get,

* $\omega_n'^2 = K\omega_n^2$

$\boxed{\omega_n' = \omega_n\sqrt{K}}$

$\boxed{\omega_n' > \omega_n}$

* also $\cancel{2}\zeta'\omega_n'\cancel{s} = \cancel{2}\zeta\omega_n s$

$\zeta'\omega_n\sqrt{K} = \zeta\omega_n$

$\boxed{\zeta' = \zeta/\sqrt{K}}$

* $\boxed{\zeta'\omega_n' = \alpha' = \zeta\omega_n = \alpha}$

1. $\omega_d' = \omega_n'\sqrt{1-\zeta'^2}$

   $\omega_d \propto \omega_n$

   $\omega_d \propto \frac{1}{\zeta}$

   $\boxed{\omega_d' \gg \omega_d}$

2. $\tau' = \frac{1}{\alpha'}$ $[\alpha' = \alpha]$

   $\therefore \boxed{\tau' = \tau}$

3. $\boxed{t_{sett}' = t_{sett}}$

4. $\boxed{t_p' < t_p}$

5. $\boxed{t_d' < t_d}$

6. $\boxed{\zeta' < \zeta}$

7. $\boxed{\theta' > \theta}$

8. $\boxed{MPO' \cancel{\gg} MPO}$

9. $t_r' = \frac{\pi - \theta'}{\omega_d'}$

   $\therefore \boxed{t_r' < t_r}$

10. $\longrightarrow$

$\alpha' = \alpha$

$\tau' = \tau$

$t_{sett}' = t_{sett}$

$t_p' < t_p$

$\omega_d' > \omega_d$

$\theta' > \theta$

$\zeta' < \zeta$

$MPO' > MPO$

$t_r' < t_r$

$t_d' < t_d$

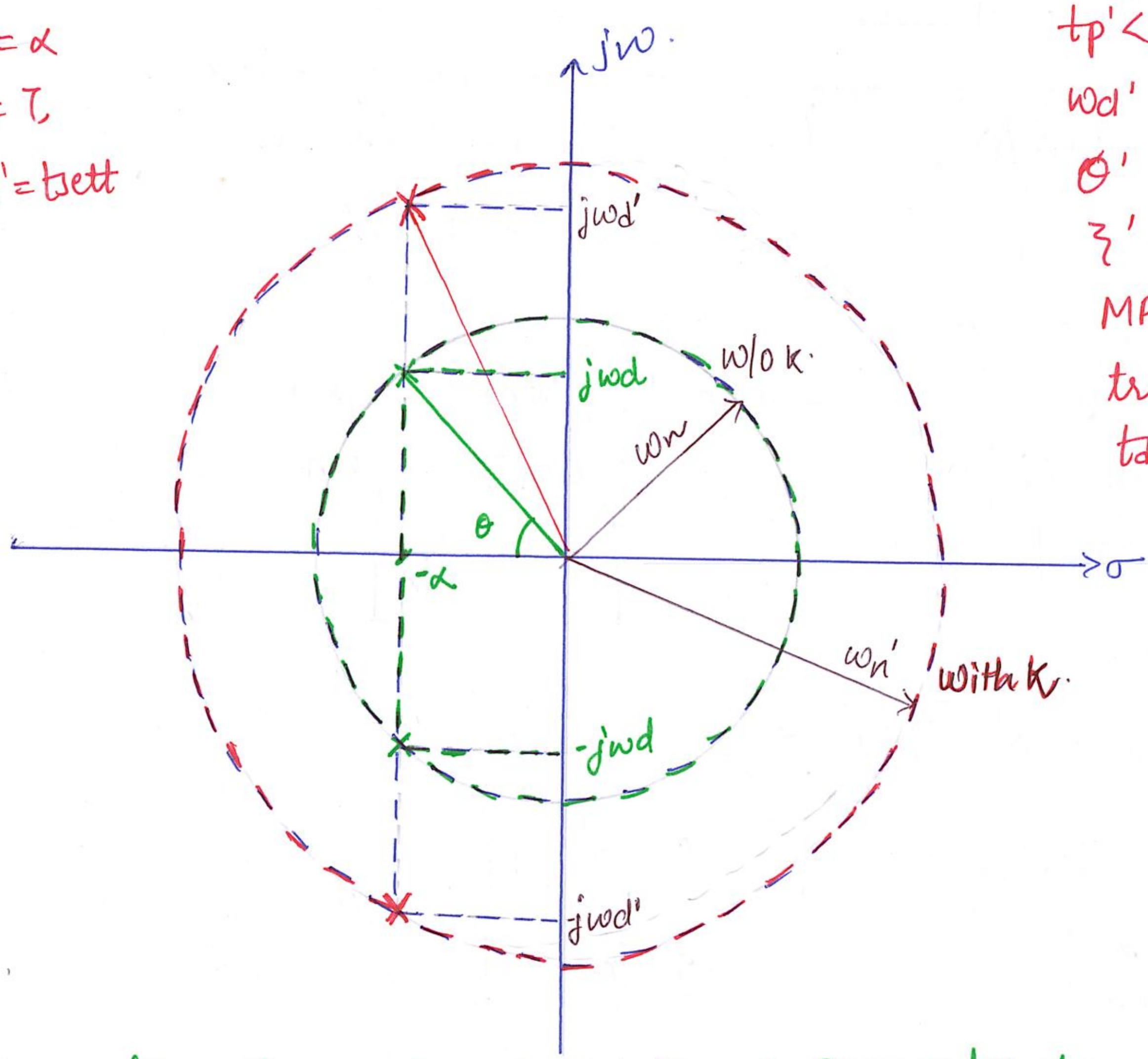
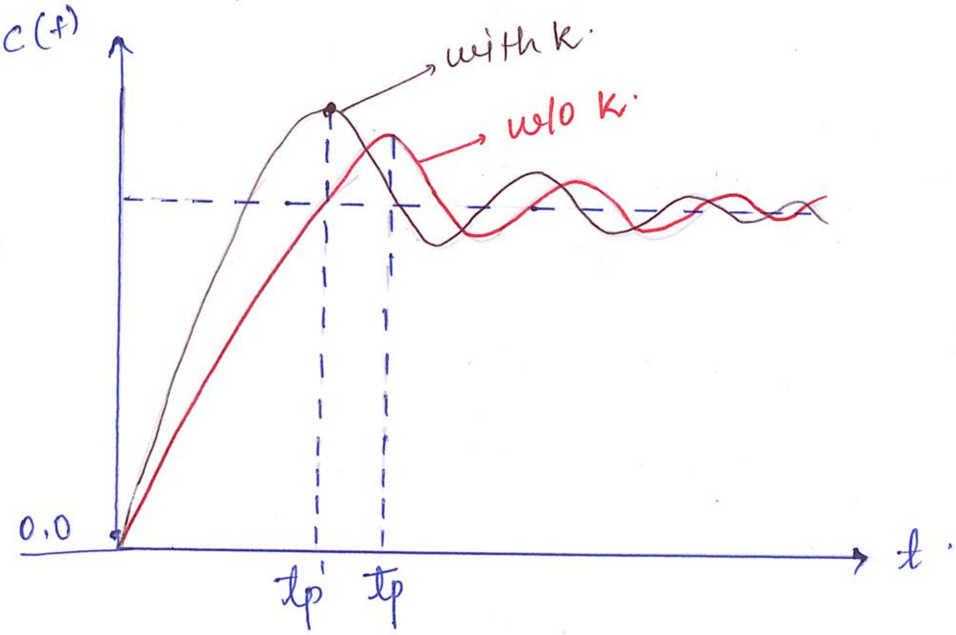


fig.- Comparison of Time Domain Parameters of with & w/o Controller.



Comparison of 2 step Responses of Std 2nd Order s/s with & w/o controller.

* One disadvantage with Proportional Controller, it Increases Maximum Peak Overshoot. (which is not desirab

(III) DERIVATIVE CONTROLLER

$G_c(t) \propto \frac{d}{dt}$

$G_c(s) \propto s$

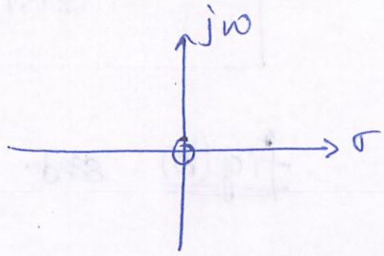
$G_c(s) = K_D s$.

K_D = Derivative Constant

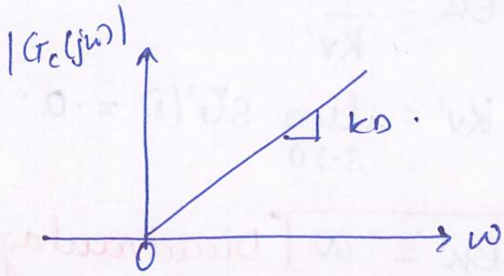
K_D = Derivative Controller gain

$G(j\omega) = j\omega K_D$.

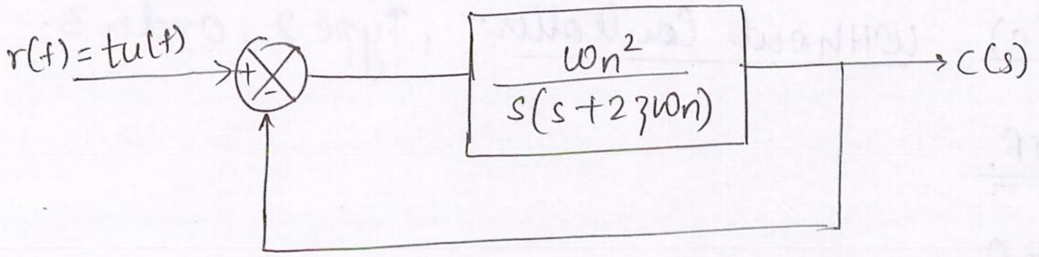
$|G_c(j\omega)| = \omega K_D$



Pole zero diagram of Derivative Controller



* In Derivative Controller, we added one zero at origin.
* Derivative Controller increases the stability of s/s. (advantage)
* Derivative Controller ~~decreases~~ increases the steady state Error of the s/s. (disadvantage)



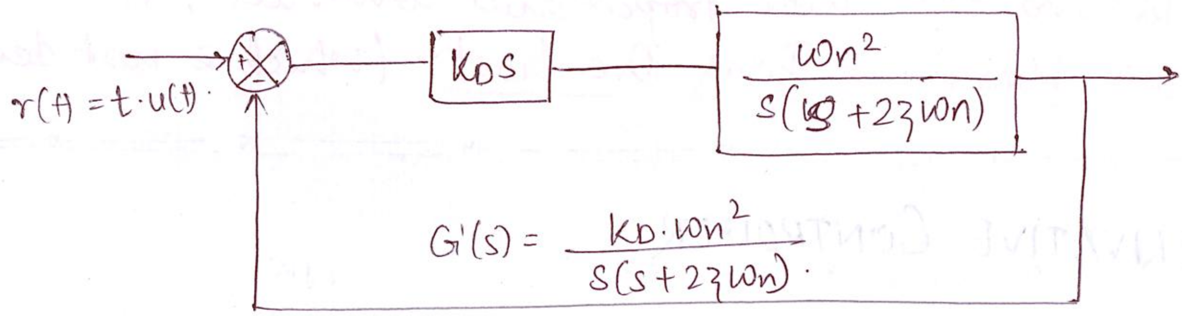
fig(a) w/o Controller std. 2nd order s/s

Type 1, Order 2

* $e_{ss} = \frac{1}{K_V}$

$K_V = \lim_{s \to 0} s\,G(s) = \frac{\omega_n}{2\zeta}$.

$$\boxed{e_{ss} = \frac{2\zeta}{\omega_n} \text{ [finite error]}}$$



fig(b) ~~std.~~ Type 0, order 1 s/s. with Controller.

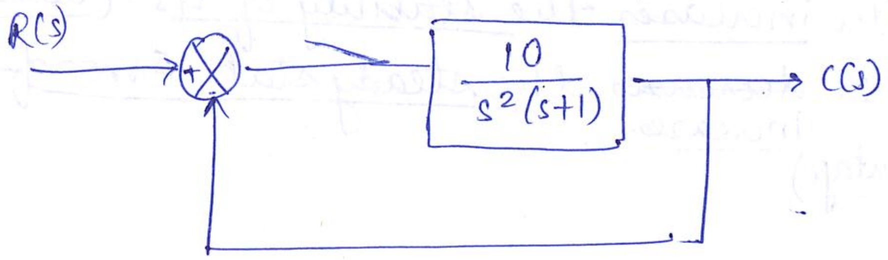
* Type 1 order 2 s/s was converted into type 0, order 1 s/s.

* $e_{ss}' = \frac{1}{K_v'}$

$K_v' = \lim_{s \to 0} sG'(s) = 0.$

$\boxed{e_{ss}' = \infty}$ [Disadvantage]

Ex.



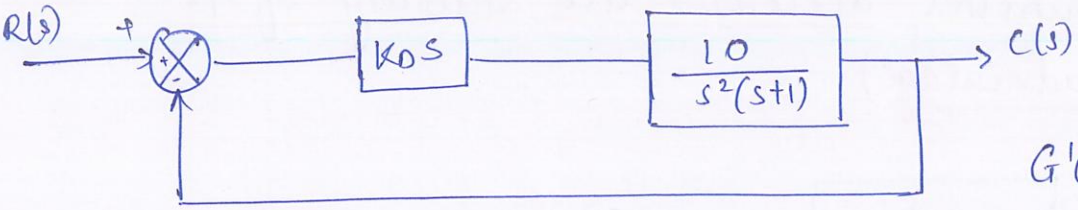
fig(a) Without Controller: , Type 2, order 3.

Stability of CLTF.

$1 + G(s) = 0.$

$s^3 + s^2 + 0s' + 10 = 0.$

[Unstable CLTF].



$$G'(s) = \frac{10\,K_D}{s(s+1)}.$$

fig b. with Controller.

Stability of CLTF.

$1 + G'(s) = 0.$

$s^2 + s + 10K_D = 0$

s^2	1	$10K_D$
s^1	1	0
s^0	$10K_D$	0

∴ for the s/s to be stable.

$10K_D > 0.$

$K_D > 0.$

& Normally $K_D > 0$. as it is gain.

** [Derivative is good for Stability, not for error]

---

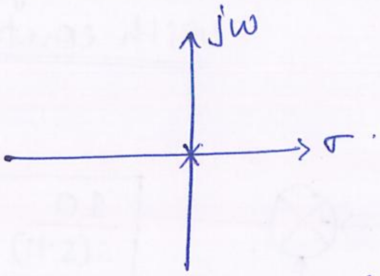
(4) INTEGRAL CONTROLLER

* $G_c(t) \propto \int$

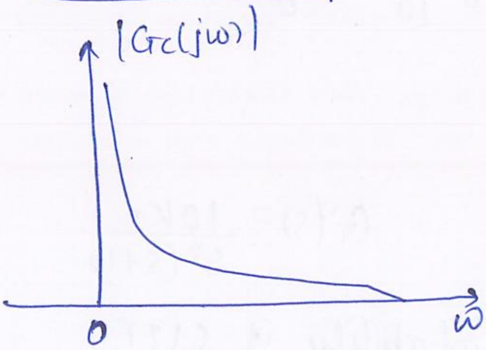
$G_c(s) \propto \frac{1}{s}$

$G_c(s) = \frac{K_I}{s}$

$$\boxed{|G_c(j\omega)| = \frac{K_I}{\omega}.}$$



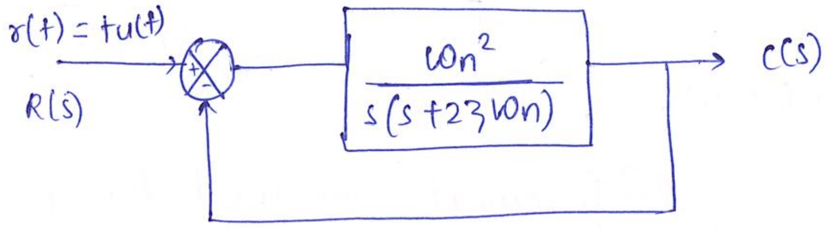
P-Z of Integral Controller
(Added one pole at origin)



Hyperbola.

* Integral controller is referred as Reset Controller
* In Integral controller we added one pole at Origin.
* Integral controller decreases the steady state Error. (advantage)

* Integral controller decreases the stability of s/s.
(Disadvantage).

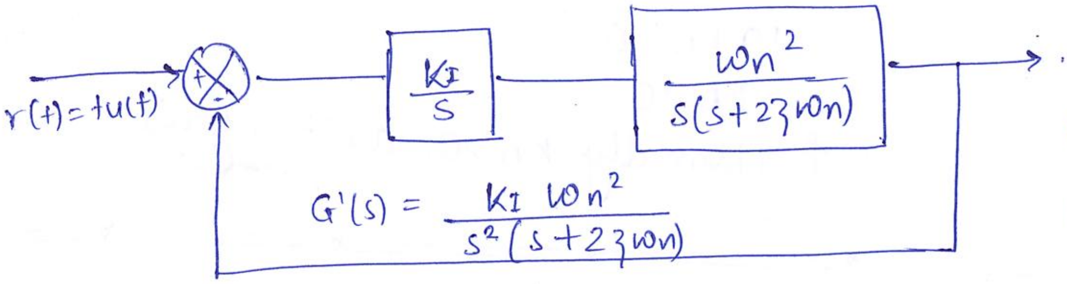


fig(a) w/o Controller.

$e_{ss} = \frac{1}{K_V}$.

$K_V = \lim_{s \to 0} s \cdot G(s) = \frac{\omega_n}{2\zeta}$

$e_{ss} = \frac{2\zeta}{\omega_n}$ [finite error].



$G'(s) = \frac{K_I \, \omega_n^2}{s^2(s+2\zeta\omega_n)}$

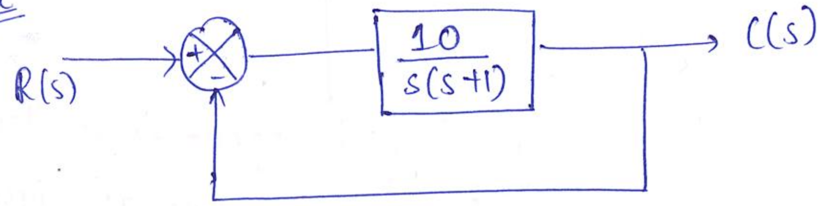
fig(b) Type 2, Order 3.
With controller.

$e_{ss}' = \frac{1}{K_V'}$

$K_V' = \lim_{s \to 0} s G'(s) = \infty$

$\boxed{e_{ss}' = 0 \text{ (Advantage)}}$

Ex



w/o Controller

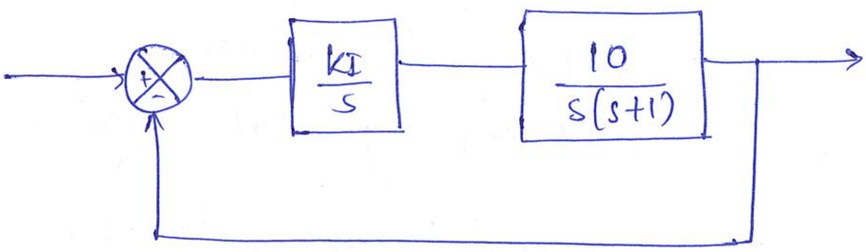
Stable system.

Stability of CLTF

$1 + G(s) = 0$

$s^2 + s + 10 = 0$.

s^2	1	10
s^1	1	0
s^0	10	0.



fig(b) with controller.

$G'(s) = \frac{10K_I}{s^2(s+1)}$

Stability of CLTF

$1 + G'(s) = 0$.

$s^3 + s^2 + 0s + 10K_I = 0$.

$\boxed{\text{Unstable s/s (disadvantage)}}$

* Stable s/s was converted into Unstable s/s.

(5) PROPORTIONAL DERIVATIVE CONTROLLER.

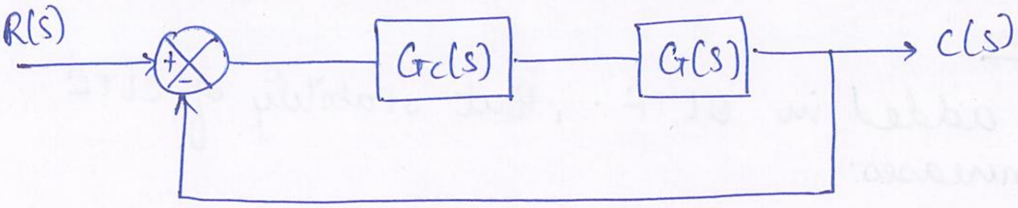


fig (a) $G_c(s) = \underbrace{K_P}_{\text{Proportional}} + \underbrace{K_D s}_{\text{Derivative Controller}}$

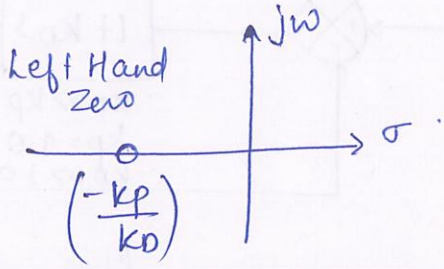


fig P-Z of $G_c(s)$.

W/O PD Controller.

* $r(t) = t\,u(t)$
* $G(s) = \dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}$
* $e_{ss} = \dfrac{1}{K_v}$

$K_v = \dfrac{\omega_n}{2\zeta}$

$\therefore e_{ss} = \dfrac{2\zeta}{\omega_n}$.

* PD Controller decreases the steady state Error By factor of 'K_P' is same domain like proportional controller

With PD Controller.

* $G'(s) = (K_P + K_D s)\dfrac{\omega_n^2}{s(s+2\zeta\omega_n)}$
* $r(t) = t u(t)$
* $e_{ss}' = \dfrac{1}{K_v'}$

$K_v' = \lim_{s\to 0} s\,G'(s) = \dfrac{K_P\,\omega_n}{2\zeta}$

$\therefore e_{ss}' = \dfrac{2\zeta}{K_P\,\omega_n}$.

$$\boxed{\begin{gathered} e_{ss}' = \dfrac{e_{ss}\,[\text{W/O PD}]}{K_P}. \\ e_{ss}' < e_{ss} \end{gathered}}$$

→

In case of PD controller, we added Left Hand zero. that means PD Controller, increases the stability of Closed system.

* zero added in OLTF., But stability of CLTF increases.

TIME DOMAIN ANALYSIS OF P-D CONTROLLER

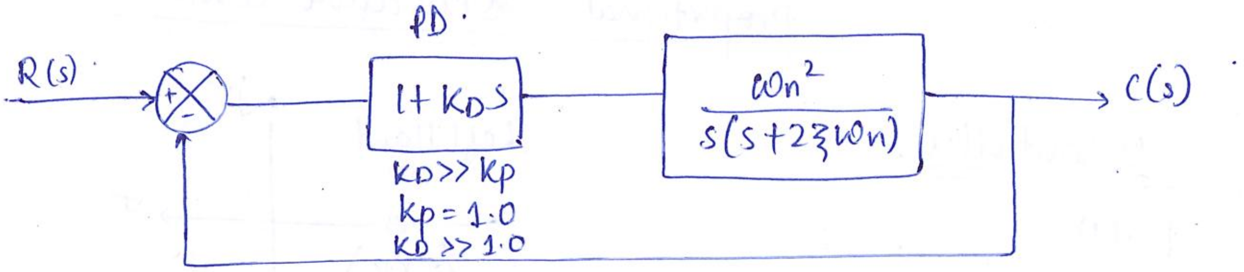


fig P-D Controller:

$$G'(s) = \frac{(1+K_D s)\omega_n^2}{s^2+2\zeta\omega_n s}$$

$$CLTF = \frac{G'(s)}{1+G'(s)} = \cancel{s^2+2\zeta\omega_n} \frac{\omega_n^2 + K_D\omega_n^2 s}{s^2+(2\zeta\omega_n + K_D\omega_n^2)s + \omega_n^2}$$

$$s^2 + 2\zeta'\omega_n' s + \omega_n'^2 = s^2 + (2\zeta\omega_n + K_D\omega_n^2)s + \omega_n^2$$

On comparison we get,

* $\omega_n'^2 = \omega_n^2$

$$\boxed{\omega_n' = \omega_n}$$

* $2\zeta'\omega_n' = 2\zeta\omega_n + K_D\omega_n^2$

Imp $\boxed{\zeta' = \zeta + \frac{K_D\omega_n}{2}}$ Effective Damping Ratio **

* $\boxed{\zeta' > \zeta}$ $\boxed{\theta' < \theta}$ $\boxed{MPO' < MPO}$ Advantage.

* $\omega_d = \omega_n\sqrt{1-\zeta^2} \Rightarrow \omega_d \propto \omega_n \Rightarrow \omega_d \propto \frac{1}{\zeta}$.

$\therefore \boxed{\omega_d' < \omega_d}$

$\alpha' = \zeta' \omega_n'$

$$\boxed{\begin{matrix} \alpha' > \alpha \\ \tau' < \tau \end{matrix}}$$ Advantage

$\boxed{t_{sett}' < t_{sett}}$

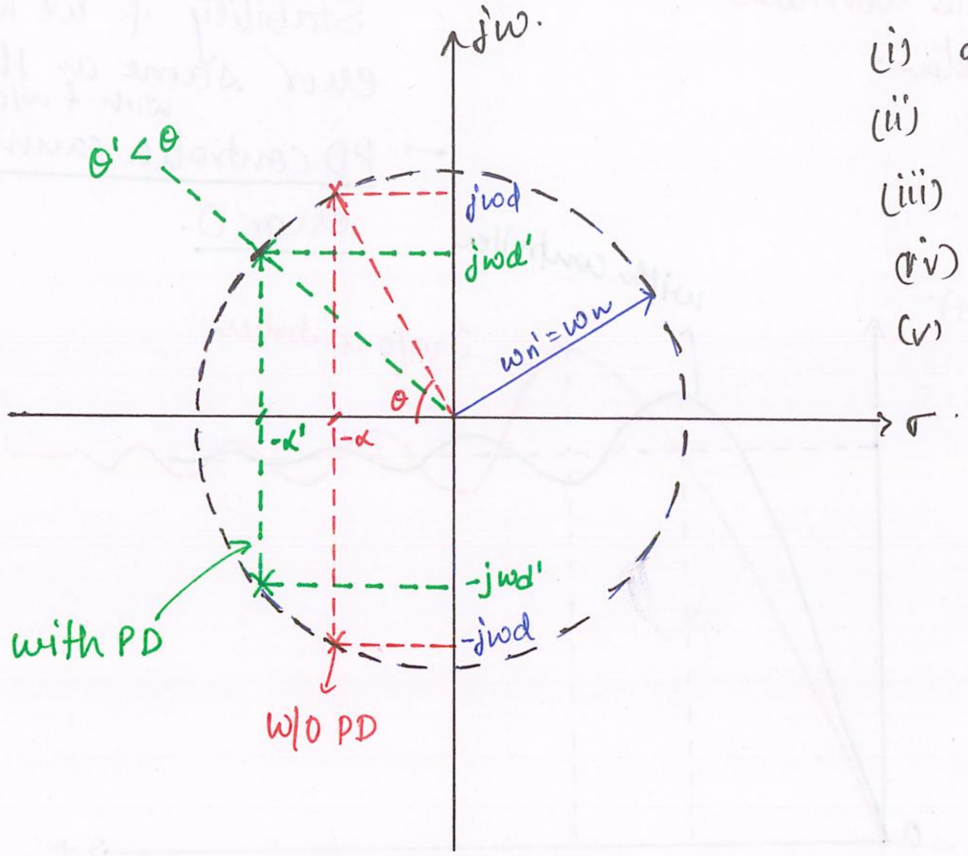
Disadvantages.

X * $t_p' > t_p$.

* $t_r = \dfrac{\pi - \theta}{\omega_d}$. X wrong → P-7.0

X $t_r' > t_r$

$t_d' > t_d$



(i) $\alpha' > \alpha$

(ii) $\theta' < \theta$

(iii) $MPO' < MPO$

(iv) $\zeta' > \zeta$

(v) $\omega_d' < \omega_d$.

Graphical Representation of Time Domain parameters of P-D Controller.

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n'^2}{s^2 + 2\zeta'\omega_n' s + \omega_n'^2} + \frac{K_D s \omega_n'^2}{s^2 + 2\zeta'\omega_n' s + \omega_n'^2}$$

If $r(t) = u(t)$.

$$c(t) = \underbrace{1 - \frac{e^{-\zeta'\omega_n' t}}{\sqrt{1-\zeta'^2}} \sin(\omega_d' t + \theta')}_{\text{Step Response.}} + \underbrace{\frac{K_D \omega_n'}{\sqrt{1-\zeta'^2}} e^{-\zeta'\omega_n' t} \sin \omega_d' t}_{\text{Impulse Response.}}$$

CORRECT.

$$\boxed{\begin{array}{l} t_r' < t_r \\ t_d' < t_d \\ t_p' < t_p \end{array}}$$

mathematically proved for PD Controllers.

(Advantage).

* PD controller has many advantages

$$\left\{ \begin{array}{ll} t_r' < t_r & M_{PO}' < M_{PO} \\ t_d' < t_d & t_{sett}' < t_{sett} \\ t_p' < t_p & \theta' < \theta \end{array} \right\}.$$

All for ~~Ess~~ $K_p = 1$.

All parameters are good for Stability & we kept the error same as that of previous with & w/o

→ PD controller cannot make error 0.

* PD controller is very good for time domain parameters.

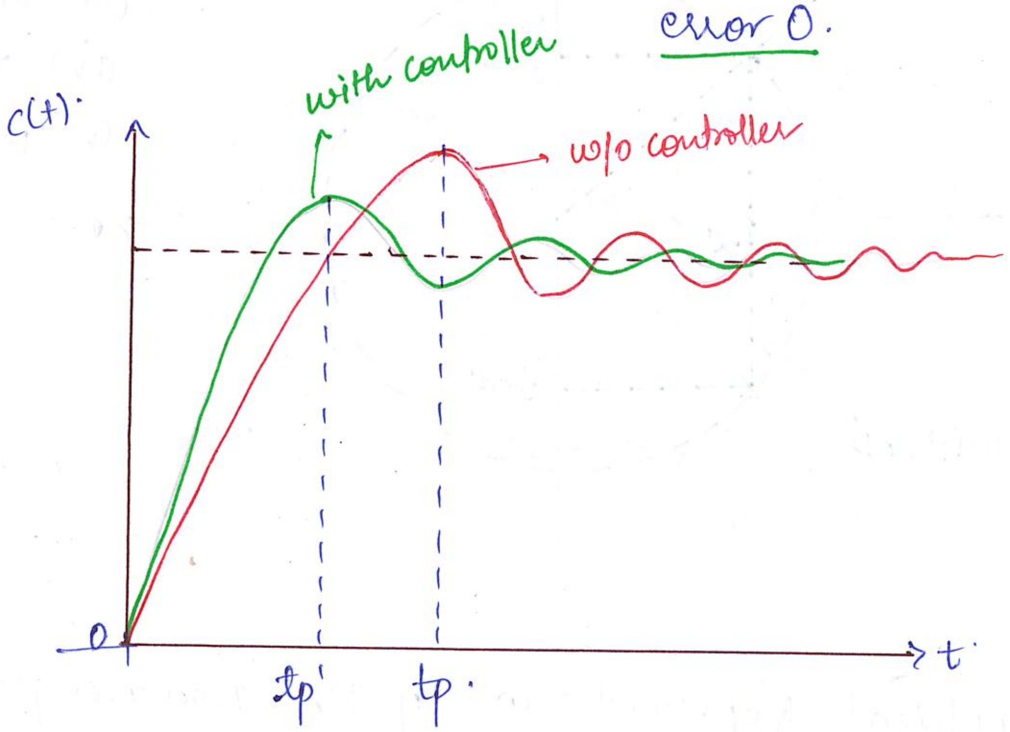


fig Comparison of step Response of with & w/o PD Controller (2nd Order)

⇒ ∵ PD C. cant change the type. Hence PD con. cannot make error 0.

⑥ PROPORTIONAL INTEGRAL CONTROLLER

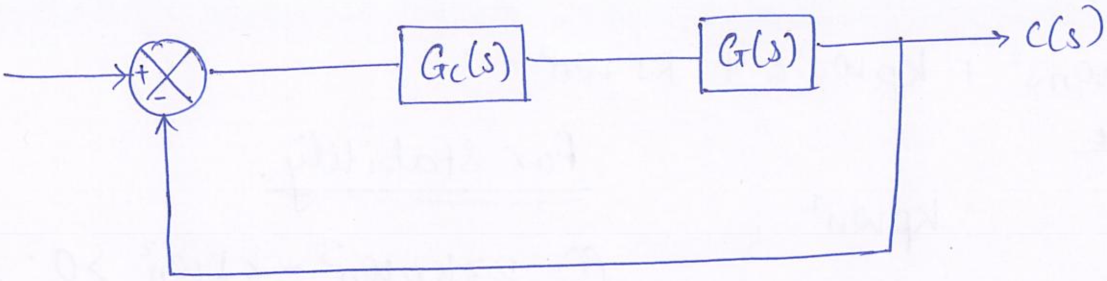


fig P-I Controller $= K_p + \frac{K_I}{s} = \frac{K_p s + K_I}{s}$.

PARAMETERS.

1) Proportional Band

$PB\% = \frac{100}{K_p}$.

2) Reset time $= \frac{K_p}{K_I}$.

** याद रखने के लिए।

Reciprocal.

time $t = \frac{1}{f}$

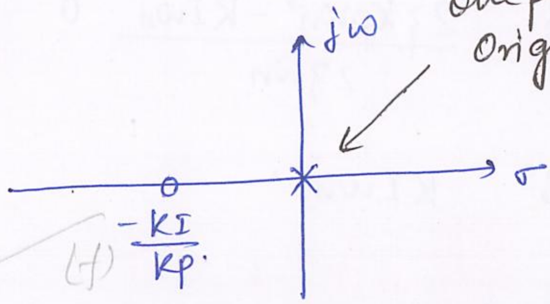


fig P-Z of PI Controller

WITHOUT PI CONTROLLER

* $G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$
* $r(t) = t \cdot u(t)$
* $e_{ss} = \frac{1}{K_v}$

$K_v = \frac{\omega_n}{2\zeta}$

* $e_{ss} = \frac{2\zeta}{\omega_n}$ [finite error]

WITH PI CONTROLLER

* $G'(s) = \left(\frac{k_p s + k_I}{s}\right) \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$

$T = 2, \; O = 3$.

* $r(t) = t u(t)$
* $e_{ss}' = \frac{1}{K_v'}$

$K_v' = \lim_{s \to 0} s \cdot G(s) = \infty$.

$\therefore \boxed{e_{ss}' = 0 \quad \text{Advantage **}}$

PI Controller.

With PI controller - Stability Analysis (CLTF)

Stability of CLTF.

$1 + G'(s) = 0.$

$s^3 + 2\zeta\omega_n s^2 + k_p \omega_n^2 s + K_I \omega_n^2 = 0.$

RH Table

s^3	1	$k_p \omega_n^2$
s^2	$2\zeta\omega_n$	$K_I \omega_n^2$
s^1	$\dfrac{2\zeta k_p \omega_n^3 - K_I \omega_n^2}{2\zeta\omega_n}$	0
s^0	$K_I \omega_n^2$.	

For stability.

(1) $2\zeta k_p \omega_n^3 - K_I \omega_n^2 > 0.$

$\boxed{K_I < 2\zeta k_p \omega_n}$

(2) $K_I \omega_n^2 > 0.$

$\boxed{K_I > 0}$

$\therefore$ Range of K_I

$= \boxed{0 < K_I < 2\zeta k_p \omega_n}$

for PI Controller.

$\boxed{\begin{array}{l} K_p = 1.0 \\ K_I >> k_p \\ K_I >> 1.0 \end{array}}$ for Stability $\Rightarrow$ Conditional Stable.

---

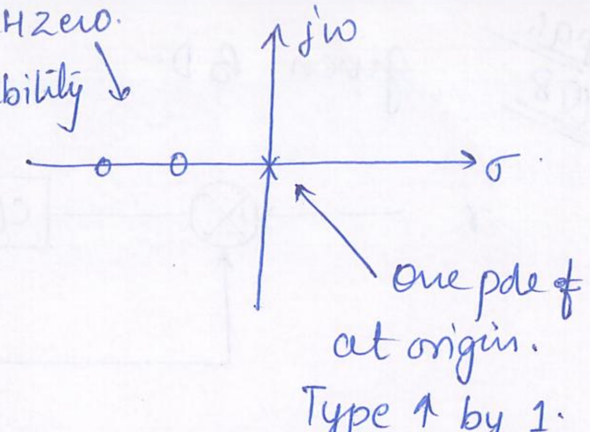
(7) PID CONTROLLER

$G_C(s) = K_p + \dfrac{K_I}{s} + K_D s.$

$K_p \rightarrow$ P. ; $\dfrac{K_I}{s} \rightarrow$ I $\xrightarrow{\text{good}}$ Error ; $K_D s \rightarrow$ D $\xrightarrow{\text{good}}$ stability.

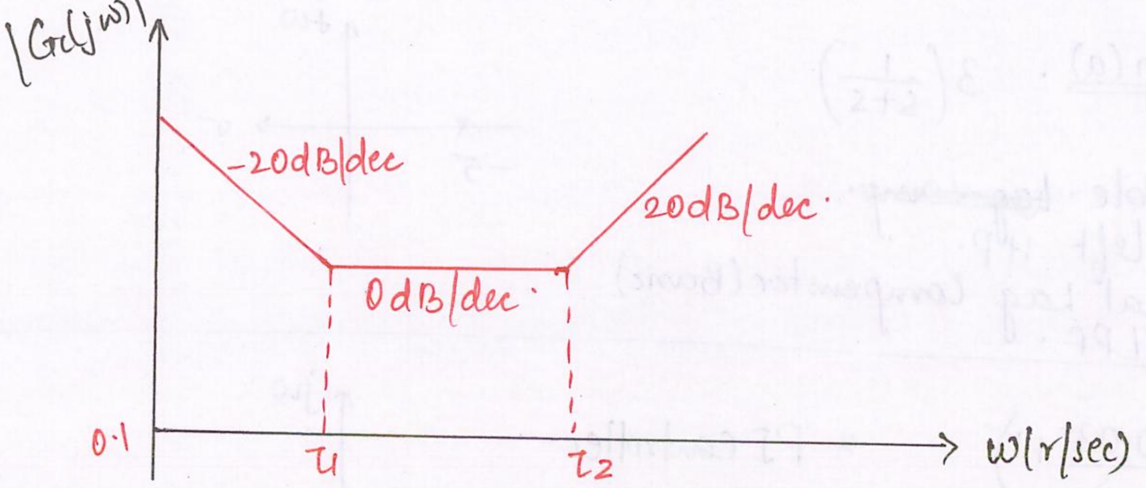
$G_c(s) = \dfrac{K_D s^2 + k_p s + K_I}{s}.$

* We ~~had~~ add 2LH zeros which increase the stability.
* We also add ~~one~~ one pole at origin, which increas the type of s/s by one, ~~hence~~ hence decreasing the error.



P-Z diagram of PID Controller

fig: Asymptotic Bode magnitude plot for PID Controller.



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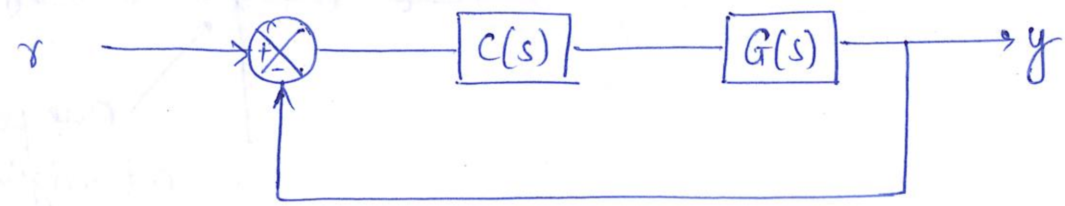
*** $\boxed{\text{Addition of zero in OLTF, increases the stability of CLTF.}}$

* Compensator cannot increase the type of s/s. it cannot make the error of the s/s '0'.
  No. of poles = No. of zeros.
* Addition of zeros in OLTF. ↑↑ the stability of CLTF.
* But addition of poles in CLTF, increases the risk of its stability.

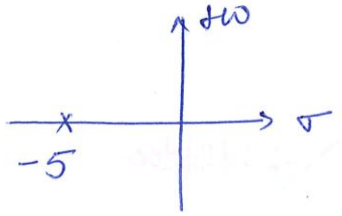
pq6
Q8 given B.D.



given, $G(s) = \frac{1}{(s+1)(s+2)}$ The 2% tsett of step Response is req. to be less than 2 sec.

Soln.

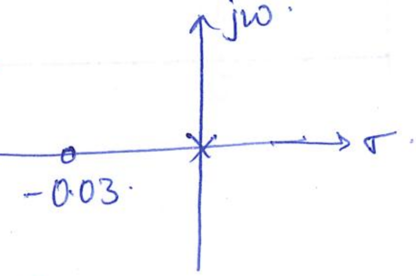
Option (a). $3\left(\frac{1}{s+5}\right)$.



* Stable. ~~Lag comp~~.
* One Left H.P.
* Ideal Lag Compensator (Basic)
* RC LPF.

(b) $5\left(\frac{0.03}{s}+1\right)$

* PI Controller.
* One left Hand zero. One pole at origin



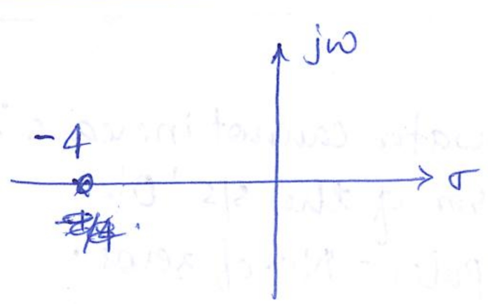
$= 5\left(\frac{0.03+s}{s}\right)$

$= 5\times 0.03\left(\frac{1+s/0.03}{s}\right)$

$PB\% = 20\% = \frac{100}{5} = \frac{100}{kp}$ $kp = 5$ / Reset time $= \frac{kp}{ki} = \frac{5}{0.15} = 5$

(c) $2(s+4)$.

$2\times 4(1+s/4)$.



* One left hand zero.

~~Lead compensator~~.

PD controller, $Kp = 8$, $K_D = 2$

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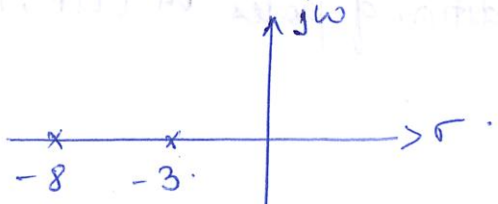
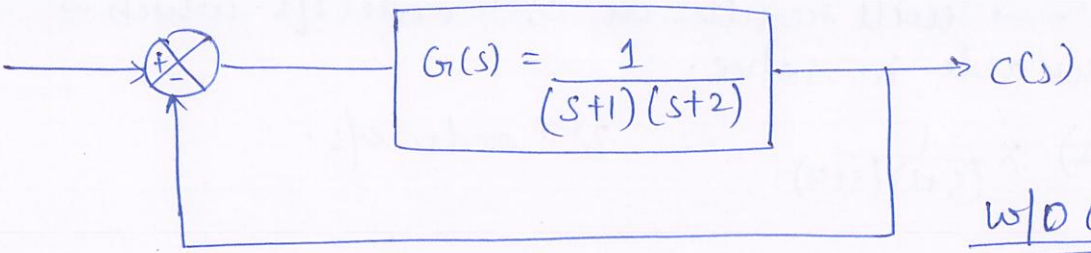
d) $4\left(\frac{s+8}{s+3}\right)$

$\frac{4\times 8}{3}\left(\frac{1+s/8}{1+s/3}\right)$



Lag compensater

** ∵ tsett has to be less than 2 sec · which is increased by using PD Controller · As it decreases the value of tsett · that means in questⁿ previously w/o comp. tsett > 2 sec ·



fig(a)
w/o compensator

$s = -1, -2 \longrightarrow$ Overdamped OLTF.

* $1 + G(s) = 0$.

$s^2 + 3s + 2 + 1 = 0$.

$s^2 + 3s + 3 = 0$.

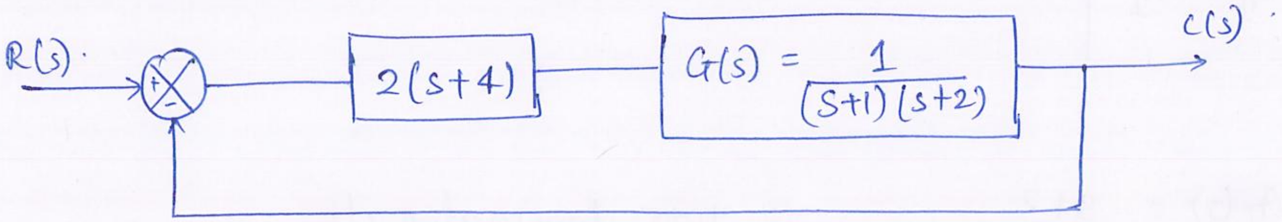
$\therefore 2\zeta\omega_n = 3$.

$\zeta = \frac{3}{2 \times \sqrt{3}} = \frac{\sqrt{3}}{2} = 0.866$. (Underdamped CLTF).

* $t_{sett} = 4\tau = \frac{4}{\alpha} = \frac{4}{\zeta\omega_n} = \frac{4}{3/2} = \frac{8}{3} = 2.67$ sec.

i.e their ques is correct, w/o comp. the tsett was 2.67 sec which is to be reduced to less than 2 sec by the use of compensator.

FROM option c. - Introductⁿ of PD Controller



$G'(s) = \frac{2s+8}{s^2 + 3s + 2}$.

$1 + G'(s) = 0$.

$s^2 + 3s + 2 + 2s + 8 = 0$

$s^2 + 5s + 10 = 0$.

$\therefore 2\zeta\omega_n = 5$

$\zeta\omega_n = 2.5 = 5/2$

$\longrightarrow$

$t_{sett} = 4\tau = \frac{4}{\zeta\omega_n} = \frac{4}{5/2} = 8/5 = 1.6$ sec.

* PD controller dec. the value of time constant.

a) $3/(s+5)$. $\longrightarrow$ will result in 3rd order s/s. which is unable to solve.

$$\frac{3\cancel{(s+5)}}{(s+5)} \times \frac{1}{(s+1)(s+2)} = \text{3rd order s/s.}$$

b) same with option (b.)

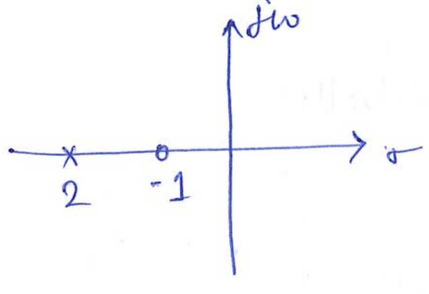
c) Only c option ~~was~~ resulted in 2nd order system.

Q7. $G(s) = \frac{1}{s^2+2s+2}$

a controller is added $G_c(s)$. For a unit step I/P, the x fer of the controller that gives minimum steady state error.

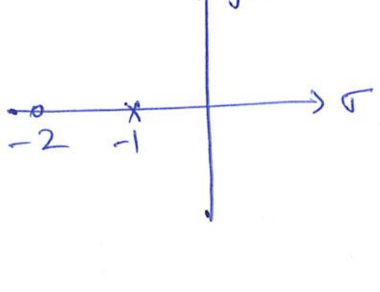
a) $G_c(s) = \frac{s+1}{s+2}$.

→ Zero dominating s/s
→ Lead comp.

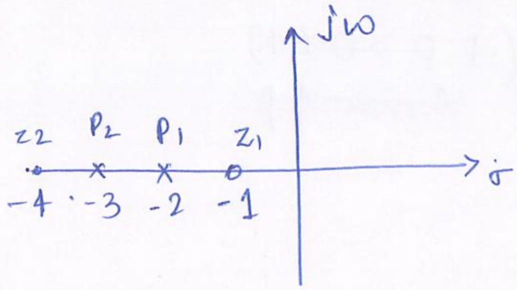


b) $G_c(s) = \frac{s+2}{s+1}$

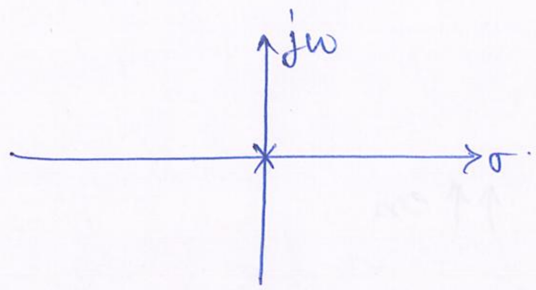
→ pole dominating s/s
→ lag compensator.



c) $G_c(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)}$ → Lead lag comp. $(z_1 P_1 P_2 z_2)$.
→ B.P. filter.



d) $G_c(s) = 1 + \frac{2}{s} + 3s$. → PID $\boxed{\text{controller}}$.
→ $K_p = 1$, $K_D = 3$, $K_I = 2$.
→ one pole at origin.
→ PID controller gives min. error & make s/s stable.



* W/O Controller.

* unit step I/P

$e_{ss} = \frac{1}{1+K_p}$ $K_p = \lim_{s \to 0} G(s)$.

* $G(s) = \frac{1}{s^2+2s+2}$. $K_p = \lim_{s \to 0} \frac{1}{s^2+2s+2} = 0.5$

$\therefore e_{ss} = \frac{1}{1+0.5} = \frac{1}{1.5} = \frac{2}{3} = 0.677$.

Error Calculat^n

(a). $G_c(s) = \frac{s+1}{s+2}$.

$G'_c(s) = G(s) \cdot G_c(s) = \left(\frac{s+1}{s+2}\right) * \frac{1}{s^2+2s+2}$.

$K_p = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

$e_{ss} = \frac{1}{1+0.25} = \frac{1}{1.25} = 0.8$ error increased. $(0.8 > 0.677)$

b) $G_c(s) = \frac{s+2}{s+1}$

$G'(s) = G_c(s)\cdot G(s)$

$K_p = \lim_{s\to 0} G'(s) = 2\times\frac{1}{2} = 1.0$ $\quad$ (~~1.0 > 0.67~~) ~~Error ↑↑~~

$\therefore e_{ss} = \frac{1}{1+1} = 0.5$ $\quad$ $(0.5 < 0.67)$ Error ↓↓.

(c) $G_c(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)}$

$G'(s) = G_c(s)\cdot G(s)$

$K_p = \lim_{s\to 0} G'(s) = \frac{4}{6}\times\frac{1}{2} = \frac{1}{3}$

$e_{ss} = \frac{1}{1+1/3} = \frac{3}{4} = 0.75 > 0.67$ ↑↑ e_{ss}

(d) $G_c(s) = 1 + \frac{2}{s} + 3s$

$G'(s) = G_c(s)\, G(s)$

$K_p = \lim_{s\to 0} G'(s) = \infty$

$e_{ss} = \frac{1}{1+\infty} = \underline{\underline{0}}$ < 0.67 $\quad$ ↓↓ Error Reduced.

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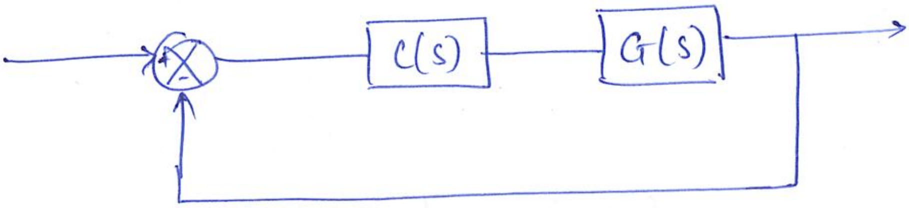
pg3.

Q5. $G(s) = \frac{1}{(s+1)^2}$. the s/s cannot be stabilized with.



Soln. w/o Comp.

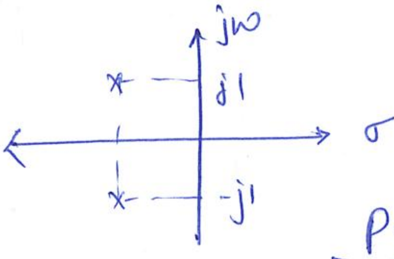
$1 + G(s) = 0$

$s^2 + 2s + 2 = 0$

$2\zeta\omega_n = 2$

$2\zeta\times\sqrt{2} = 2$

$\zeta = 0.707$



P-Z OLTF.

(a). $G'(s) = G(s) \cdot C(s)$

$$= \frac{1}{(s+1)^2} \times \left(1 + \frac{3}{s}\right).$$

$1 + G'(s) = 0.$

$\cancel{s(s+1)^2 +}$ $1 + \frac{s+3}{s(s+1)^2} = s(s+1)^2 + s + 3 = 0.$

$s(s^2+2s+1) + s + 3 = 0$

$s^3 + 2s^2 + 2s + 3 = 0.$

IP > EP.
$4 > 3.$ ✓ stable.

(b) $G'(s) = G(s) \cdot c(s)$

$$= \frac{1}{(s+1)^2} \times \left(3 + \frac{7}{s}\right).$$

$1 + G'(s) = 0.$

$1 + \frac{3s+7}{s(s+1)^2} = s^3 + 2s^2 + s + 3s + 7 = 0.$

$s^3 + 2s^2 + 4s + 7.$

IP > EP
$8 > 7$ ✓ stable.

(c) $G'(s) = G(s) \cdot C(s).$

$$= \frac{1}{(s+1)^2} \times \left(\frac{3s+9}{s}\right).$$

$1 + G'(s) = 0.$ $\quad s(s+1)^2 + 3s + 9 = 0$

$s^3 + 2s^2 + 4s + 9.$

IP < EP **Unstable**.
$8 < 9.$

Control Combat
22 video
Concept of Comp.
Do not miss
2019.
(2) videos.

(d). $G'(s) = G(s) \cdot C(s)$

$$= \frac{1}{(s+1)^2} \times \frac{1}{s}$$

$1 + G'(s) = 0.$

$s^3 + 2s^2 + s + 1 = 0.$

IP > EP

Stable?

Q26
p.54.

Type 1, Order 3, | $N = 2$.
Distance > 1
Gm < 1. $\longrightarrow$ unstable s/s.

* $\omega_{gc} \rightarrow$ 0dB cut mark point
* $\omega_{pc} \rightarrow -180°$ line cut.

(a) ω_{gc} occurs earlier than ω_{pc} $\omega_{gc} < \omega_{pc}$
s/s $\rightarrow$ stable.

(b) ω_{pc} occurs earlier than ω_{gc}. $\omega_{pc} < \omega_{gc}$.
s/s $\rightarrow$ unstable.

(c) $\omega_{pc} = \omega_{gc}$. $\rightarrow$ marginal stable.

(d) $\omega_{gc} < \omega_{pc} \rightarrow$ stable.