A. Mishra

ALTERNATOR

A.C generators or alternators operate on the same fundamental laws of Faraday's law of electromagnetic induction. Here armature winding mounted on a stationary element called stator and field winding on a rotating element called Rotor.

The magnetic poles of the rotor are excited from a d.c current source at 125 to 600 volt. When the rotor rotates, the stator conductors being stationary are cut by the magnetic flux, hence they have induced emf produced in them, because the ~~mag~~ magnetic poles are alternatively north & south, they induce an emf and hence current in armature conductor which first flows in one direction & then in the other. Hence an a.c emf is produced in the stator conductors,

(i) Whose frequency depends on the no. of north and south poles.

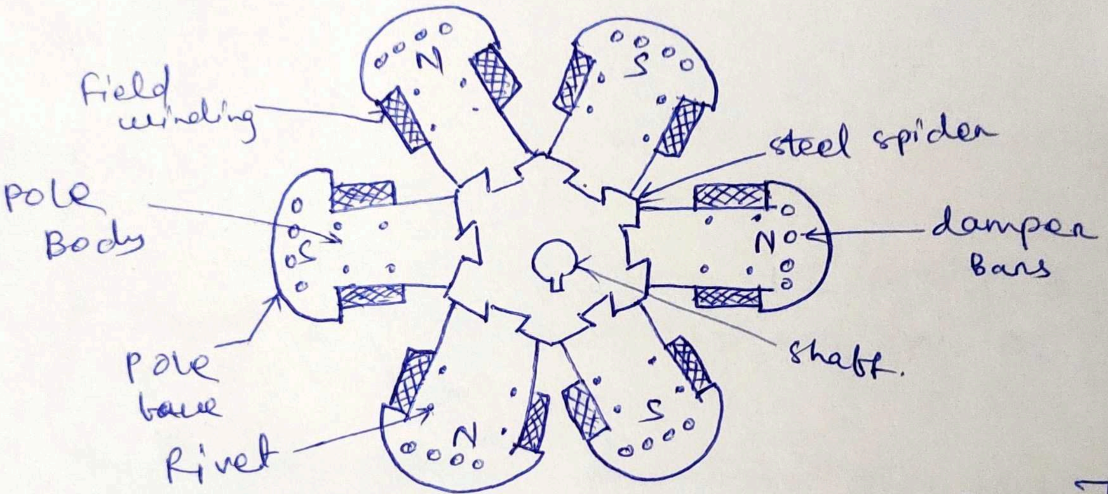
(ii) Whose direction is given by Flemming's Right hand rule.

Advantages of Stationary Armature:

(1) The O/P current can be led directly from the fixed terminals of the stator to the load ckt without having to pass it through the brush contacts.

mostly for '2' pole or '4' pole, the pole across are surrounded by the field windings placed in slots. To avoid excessive peripheral velocity such rotors have very small dia. Hence, turbogenerators are characterised by small dia, and very long axial length.

② Salient pole type Rotor:



[Six-pole salient-pole Rotor]

→ It is used in low, medium speed driven alternator. It has large no. of projecting salient poles. It is used in hydro-power plants. Such generators are characterised by their large diameter and short axial length.

Damper windings:

These are the copper bars short-ckt at both ends by heavy copper rings.

(2) Easier to insulate stationary ~~armation~~ armat winding from high A.C voltages.

The sliding contacts i.e, slip rings are transfer to the low voltage, low power d.c field circuit which can therefore easily insulated.

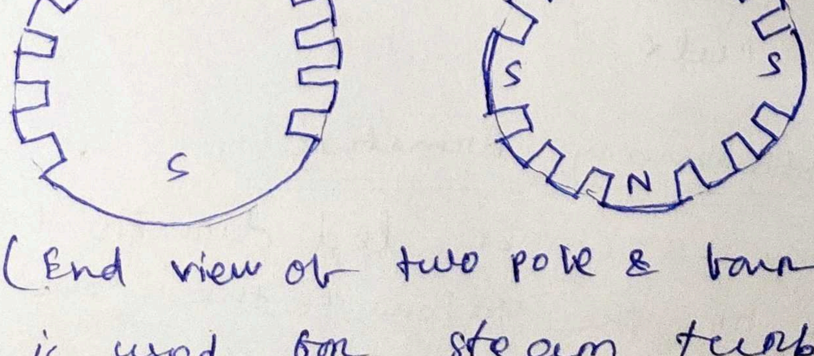
Construction!

Stator: The armature core is supported by the stator frame and is build up of lamination of special magnetic iron or steel alloy. The core is laminated to minimise loss due to eddy current. The lamination are insulated from each other and have spaces between them for allowing the cooling air to pass through. The slots are there for housing the armature conductors.

Rotor! Rotors are of two types.

(i) Cylindrical type
(ii) Salient pole type.

(1) Smooth cylindrical type Rotor!



(End view of two pole & four-pole cylindrical Rotors)

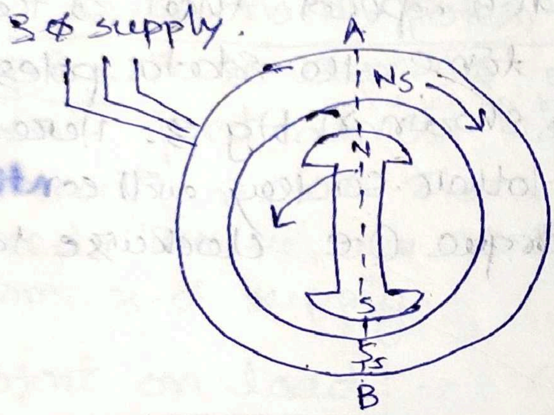
→ It is used for steam turbine driven alternator i.e, turboalternators, which run at high speed. Such rotors are designed

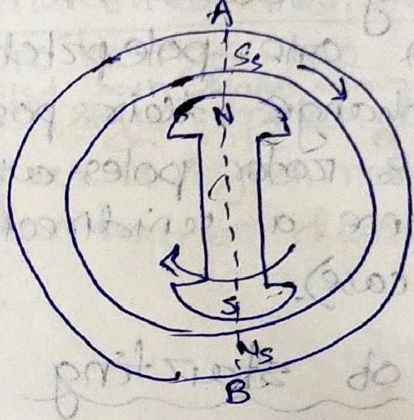
Synch. Motor.

A. Mishra.

- → It runs either at synch. speed or not at all.
- → It is not a self started motor. It has to be run upto synch. speed by some means before it can be synchronised to the supply.
- → It is capable of being operated under a wide range of pow. factors.

Principle of operation: →





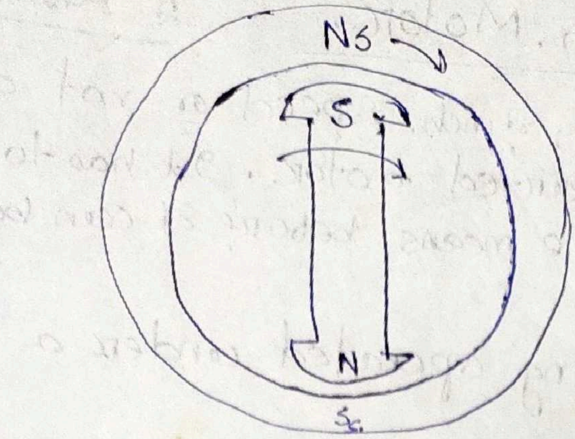
Principle: → Fig (A) (B)

When a 3-Φ wdg is fed by a 3Φ supply, a mag. flux of constant magnitude but rotating at synch. speed is produced.

→ Consider a 2 pole stator, two stator poles N_s & S_s rotate at synch. speed let in clockwise dir^n. with the rotor posit^n as shown. The two similar pole 'N' of rotor & 'N_s' of stator as well as S and S_s will repel each other with the result that rotor tends to rotate in anticlockwise direction.

0-01 2/28 → 50t

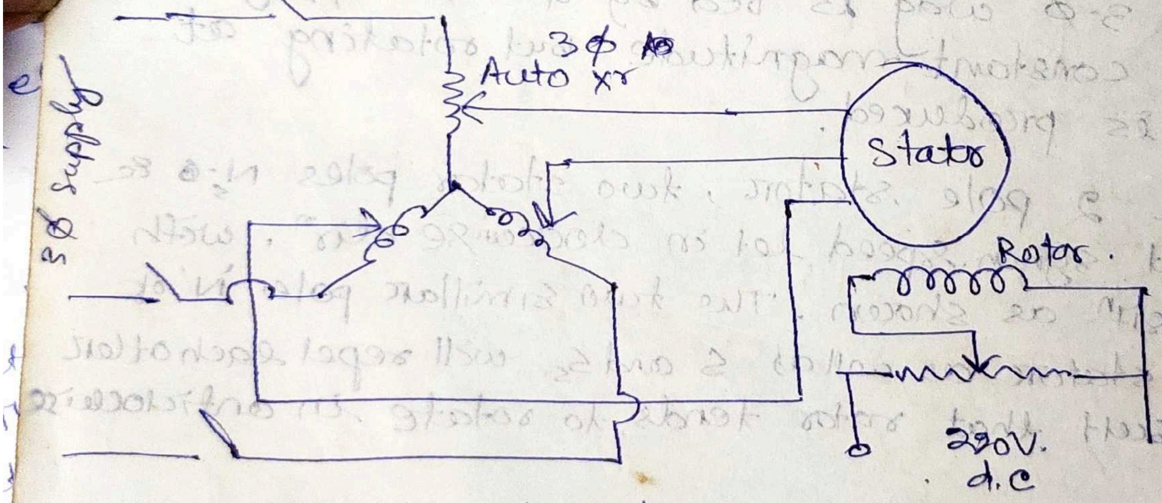
Half a period later stator pole interchange their position (fig B). At this position N_s and S and S_s & N attract each other with the result that rotor tends to rotate in the clockwise direction. So the rotor is subjected to a torque which is rapidly reversing. Owing to its large inertia the rotor cannot instantly response to such quickly reversing torque with the result that it remain stationary.



Suppose that the rotor is not stationary but is rotating clockwise with such a speed that it turns through one pole pitch by the time the stator poles interchange their position as shown in fig 3. Here the stator & rotor poles attract each other. So they will continually experience a unidirectional torque (i.e clockwise torque in this case).

Method of starting.

Starting by Damper wdg (Induction motor starting)



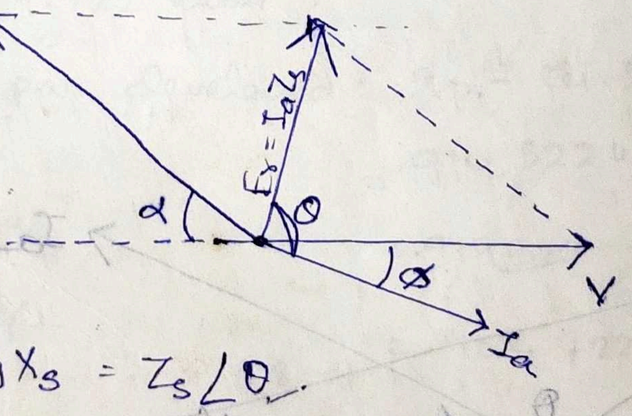
- → main field wdg shortckted.
- → Reduced volt. with the help of autox^r is applied across the stator terminals. The motor starts up like an ind^n motor.
- → when it reaches a steady speed, a weak dc excitat^n is applied by removing the shortckt on the main field wdg. If excitat^n is sufficient, then the m/c will be put into synchronism.
- → Full supplied volt. is applied across stator terminals ~~afto~~ by cuttingout the autox^r

→ The motor may be operated at any desired p.f by changing the d.c excitatⁿ.

Auxilliary motor Starting :→

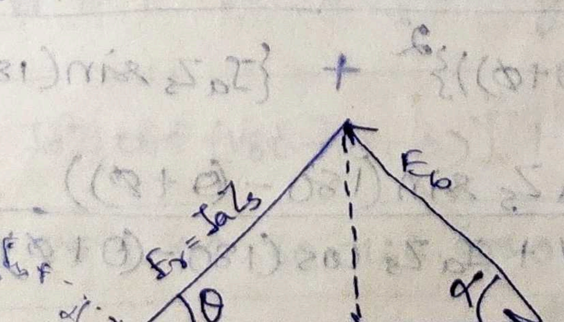
The purpose of the auxillary motor is to bring the synch. motor speed near to its synch. speed. The aux. motor may be an indⁿ motor or a d.c motor. when 3-φ alternindⁿ motor is used as an aux. motor, then it is mechanically coupled with synch. motor. Both the motors have same no. of poles & are energized from the same 3φ supply. Those aux. 3φ indⁿ motor brings the main motor speed almost equal to its synch. speed. when the field wdg. is excited both starts running at synch speed. As a result the main motor start running as a synch. motor at sych. speed. The the aux. indⁿ motor can now be disconnected from 3-φ supply.

Motor on load :→



$Z_s = R_a + jX_s = Z_s\angle\theta$

① unity p.f :→



$$E_b = \sqrt{(V - I_aZ_s\cos\theta)^2 + (I_aZ_s\sin\theta)^2}$$

Load angle $\tan\alpha = \dfrac{I_aZ_s\sin\theta}{V - I_aZ_s\cos\theta}$.

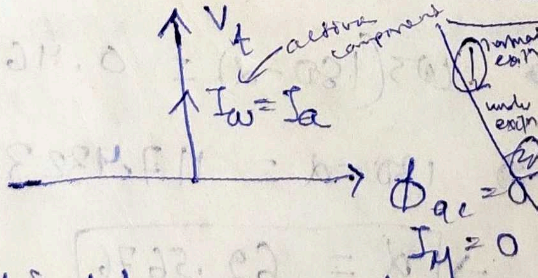
Power factor control of synch. motor :→

An ac ele. magnetic device must draw a magnetising current from the ac source in order to establish the working flux. This magnetising current laggs the applied volt. by almost 90°. ~~A synchron~~

A synch. motor is a doubly excited m/c. Its arm. wdg is excited from an ac source & the field wdg from a dc source. The resultant airgap flux is established by the cooperation of both ac of arm. wdg. & dc in the field wdg, i.e $\phi_{resultant} = \phi_{ac} + \phi_{dc}$

i) If the field current is sufficient enough to setup the airgap flux, then magnetising current required for the ac source is zero and therefore the motor operates at unity p.f. This field current is known as normal field current & the excitat^{n} known as normal excitat^{n}.

①



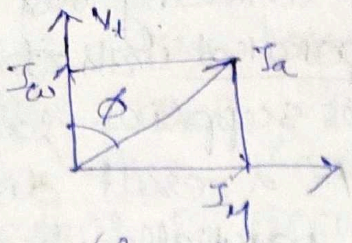
① normal exc^{n} (n.e) field flux = working flux (unity)

under exc^{n} (u.e) f. " < " (lagging p.f)

② (o.e) field flux > working flux (leading)

over excitat. ③

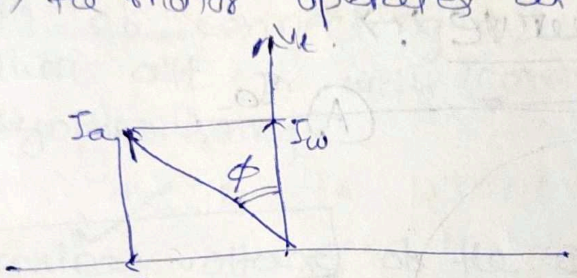
ii) If the field current is made less than the normal excitation, the motor is underexcited. Then the deficiency in the flux must be made up by the arm. wdg. In order to do ~~so~~ this, the arm. wdg draws a magnetising current from the ac source, and as a result of this, the motor operates at a lagging powerfactor.



②

(Lagging p.f)

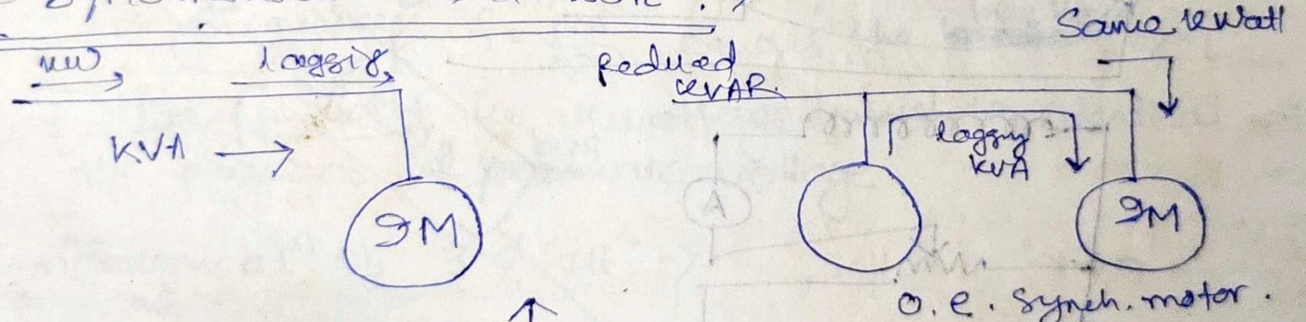
In case the field current is more than its normal excitat^n, i.e motor is overexcited, then the excess flux must be neutralised by the armature wdg. The arm. can do so, only if it draws a demagnetising component of current from the ac source. As magnetising component is lagging behind V by 90°, the demagnetising component must lead V by 90° as a result, the motor operates at a leading p.f.

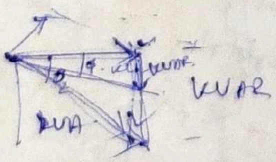


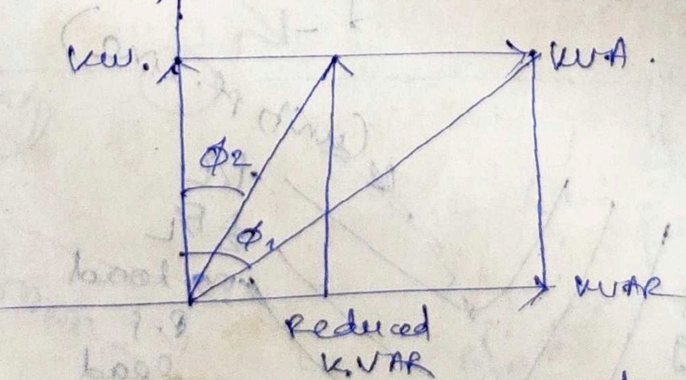
③

(leading p.f)

Synchronous Condensor :→





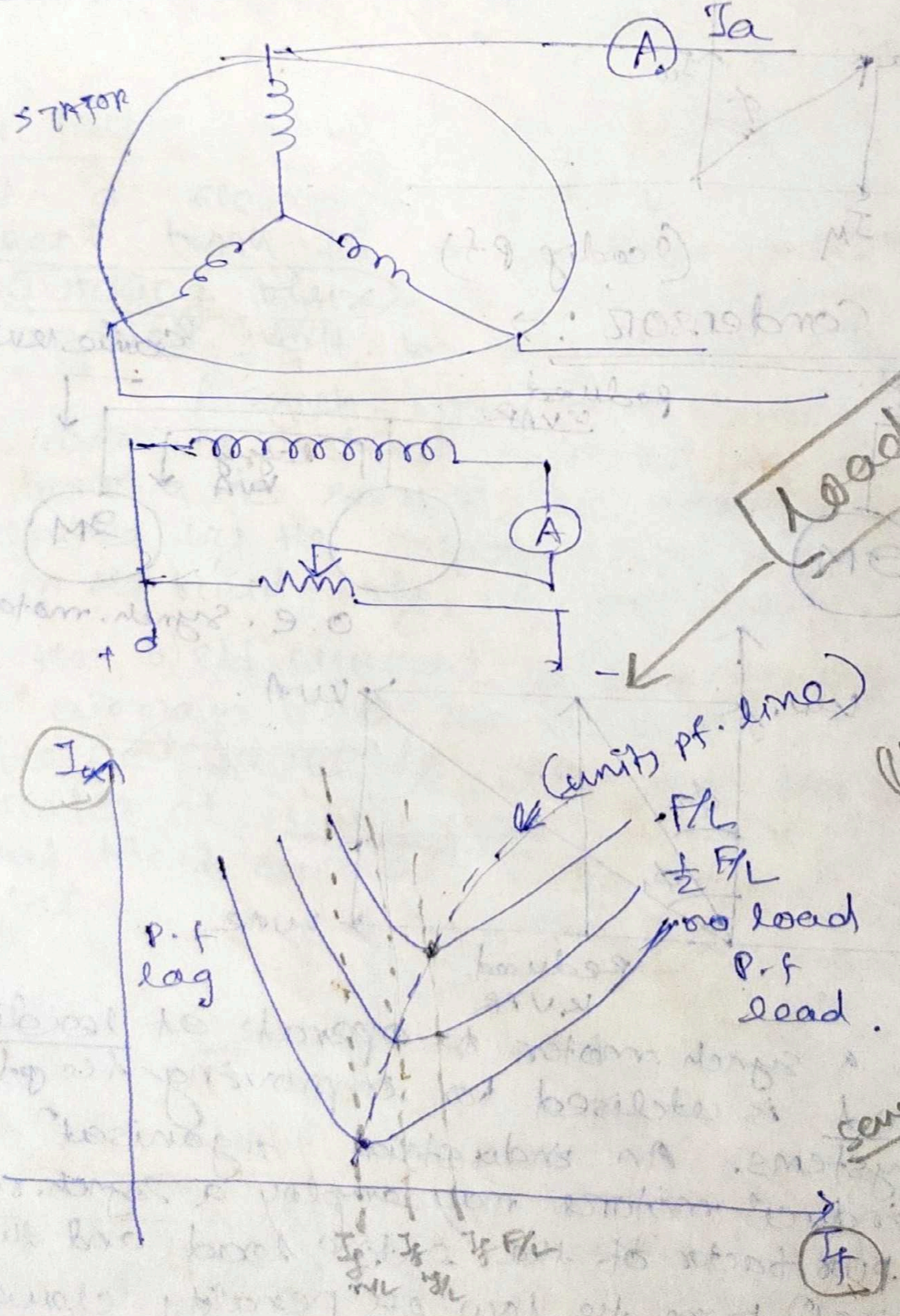


The ability of a synch. motor to operate at leading pf when over excited is utilised for improving the p.f. of many supply systems. An industrial organisat^n having a large no. of induct^n motors may employ a synch. motor to improve the pwr. factor of the entire load and thus save the organisat^n from the low p.f penalty charge.

An indn motor must take lagging KVAR from the supply to system to setup its working flux. When a synch. motor is connected in parallel with it, and if this motor is over excited, it supplies lagging KVAR to the indn motor locally. As a result distributn lines are relieved completely or partially from supplying the lagging KVAR needed by the induction motor. As a result p.f. improves.

So when an overexcited unloaded synch. motor is used to improve the p.f of a system it is known as synch. condensor.

Construction of V-curve :→



Load, If, p.f, Ia Constant

(unity p.f ରେ Ia minm ହୁଏ, Ia↑, If↑ lead. lead ରୁ [illegible] If [illegible] lag ହୁଏ)

same If F/L ରେ 1/2 F/L [illegible] (lag → lead ହୁଏ) a 1/2 F/L ରୁ F/L [illegible] lag ହୁଏ

It is the curve bet^n field current If & arm. current Ia. In order to draw these curves experimentally the motor is run from the constant volt. & cont. frq. bus bars. pwr. i/p to motor is kept constant at a definite value. Connection is made like the ckt digram. Field current is incresed in small steps and correspdy arm. currents are noted. when plotted Ia vs. I_f, we get a curve like 'V' thats why known as 'V' curve.

Parallel Operation of alternators

The operat^n of connecting an alt. in parallel with another alt. or with common bus bar is known as synchronising.

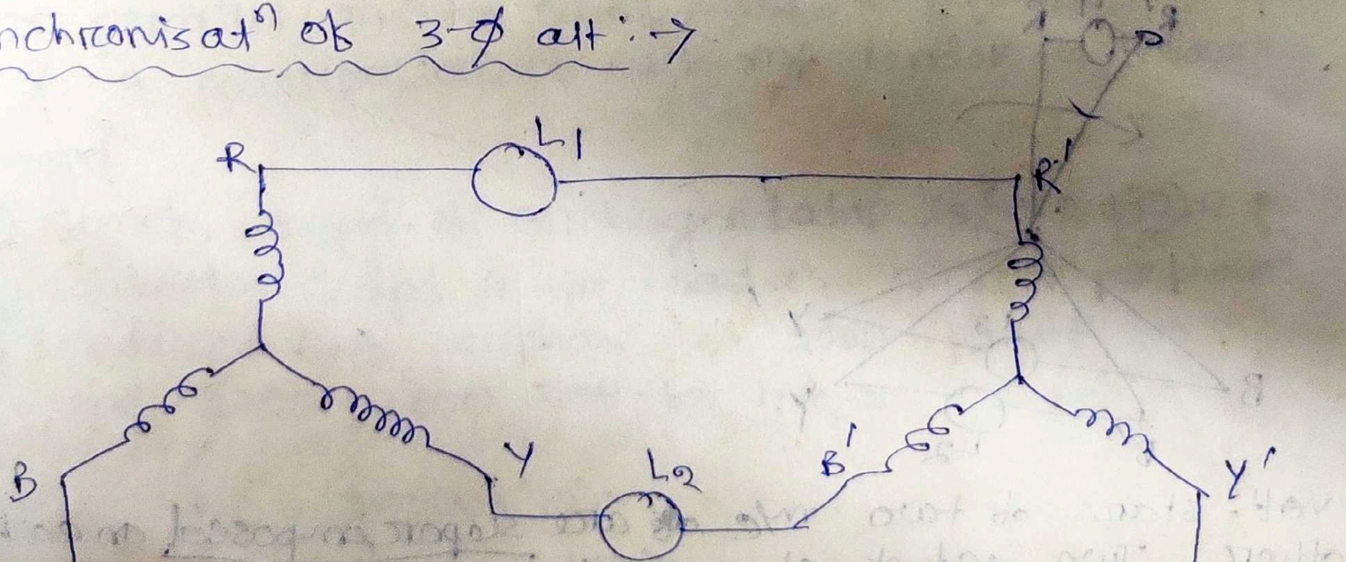
Cond^n :→

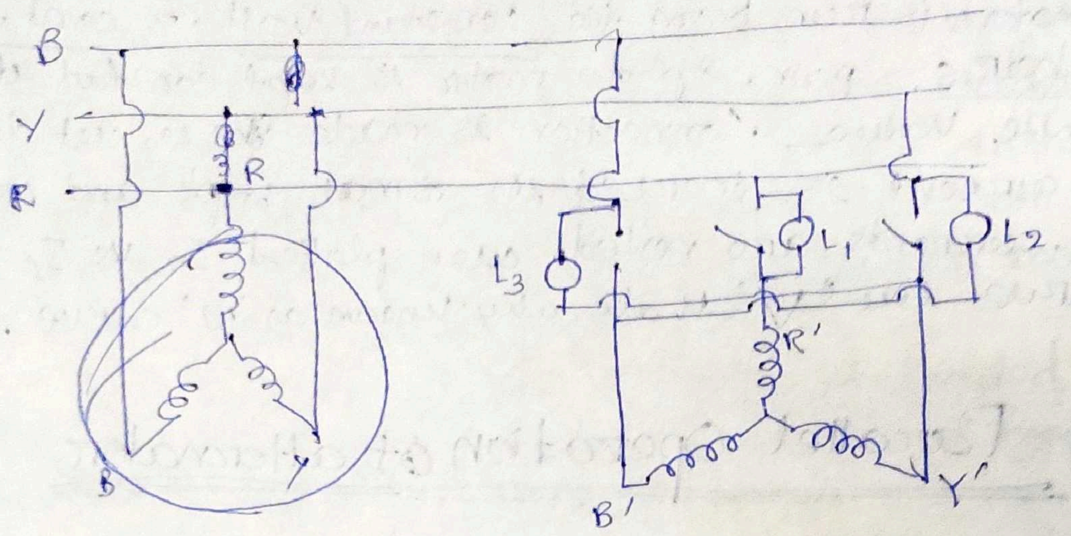
① The terminal voltage of the incoming alt. must be same as bus bar volt.

(ii) The speed of the incoming m/c must be such that its frequency $f = \frac{PN}{120}$ equals the bus bar freq.

(iii) The phase of the alt. voltage must be identical with the phase of the bus bar voltage.

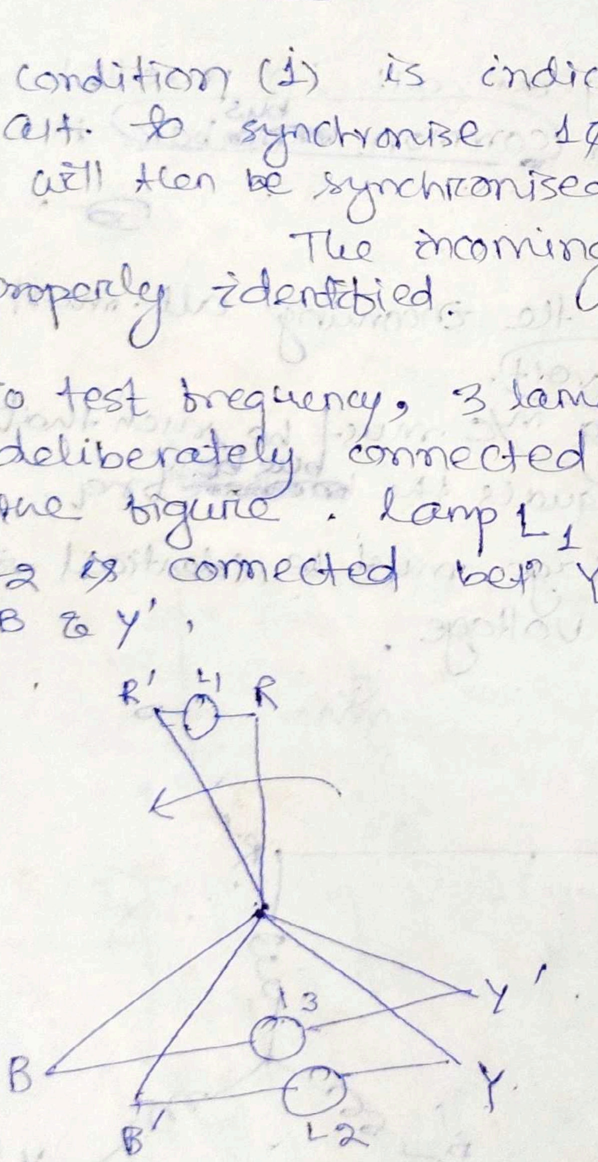
Synchronisat^n of 3-ϕ alt :→





condition (1) is indicated by voltmeter. In 3ϕ alt. to synchronise 1ϕ only, the other 2 phases will then be synchronised automatically. The incoming alt. R-Y-B should be properly identified.

→ To test frequency, 3 lamps are used, but they are deliberately connected assymetrically as shown in the figure. Lamp L_1 is connected betn R & R'. L_2 is connected betn Y & B'. L_3 is connected betn B & Y',



volt. stars of two m/c ~~as~~ are superimposed on each other. Two set of star vectors will rotate at unequal speeds if the freq. of two m/c are different. If the incoming alt. is running faster, then volt. star RY'B' will appear to rotate anticlockwise w.r.t. bush bar volt. star RYB, at a speed corresponding to the difference bet^n their frequencies.

Volt. across L_1 is RR' which is increasing from zero, that across L_2 is YB' which is decresing from its max^m and that across L_3 is BY' which is incresing from zero.

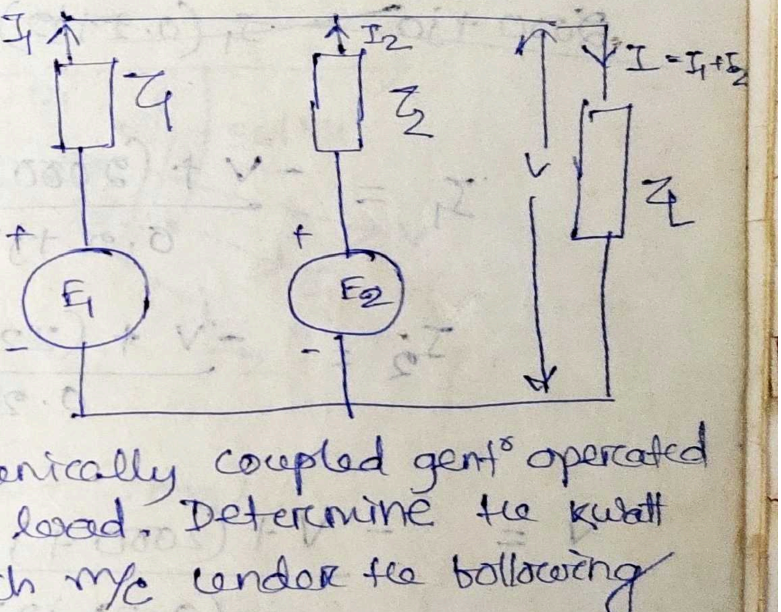
→ Hence the lamp will light up one after the other. when both the m/c freq. are same, uncrossed lamp L_1 will become dark and crossed lamp L_2 & L_3 will glow at same intensity.

→ Synchronising is done at the moment, the uncrossed lamp 'L_1' is in the middle of the dark period. when L_1 is dark L_2 & L_3 are equally bright. This method is known as two bright & one dark method.

$E_1 - I_1 Z_1 = V$ — ①

$E_2 - I_2 Z_2 = V$ — ②

$V = I Z_L = (I_1 + I_2) Z_L$ — ③



Q/ two 3-ϕ, synch. mechanically coupled gen^r operated in parallel on the same load. Determine the kwatt o/p & powerfactor of each m/c under the following cond^n

① synch. impd. of each generator is $(0.2 + j2)\Omega$ per phase

(ii) equivalent impd of the load is $3 + j4\ \Omega$ per phase.

(iii) induced emf perphase is $2000 + j0$ volt for m/c-1 and $2200 + j100$ volt. for m/c-2.

$E_1 = (2000 + j0)$ V.

$E_2 = (2200 + j100)$ V

$Z_1 = Z_2 = (0.2 + j2)\Omega$

$Z_L = (3 + j4)\Omega$

Induct^n Motor.

It converts ele. pwr. to mechanical pwr. by ind^n motor

Advantage :→

→ It has very simple & almost unbreakable construct^n.
→ Its cost is very low.
→ It requires less maintainance.
→ It has sufficiently high eff.

disadvantge →

→ Its speed decreses with encrese in load
→ Its startig torque is inferior to that of dc shunt motor

Construction →

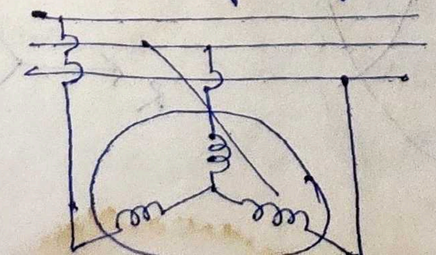
① STATOR → The stator is wound for a definite no. of poles, when supplied with 3Φ current it produces a magnetic flux which is of constant magnitude but rotates at synchronous speed $N_s = \frac{120f}{P}$.

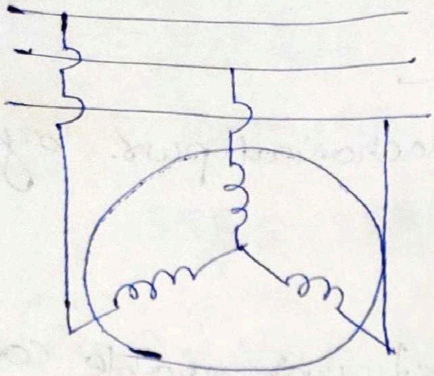
② ROTOR → ① Squireel cage type rotor :→

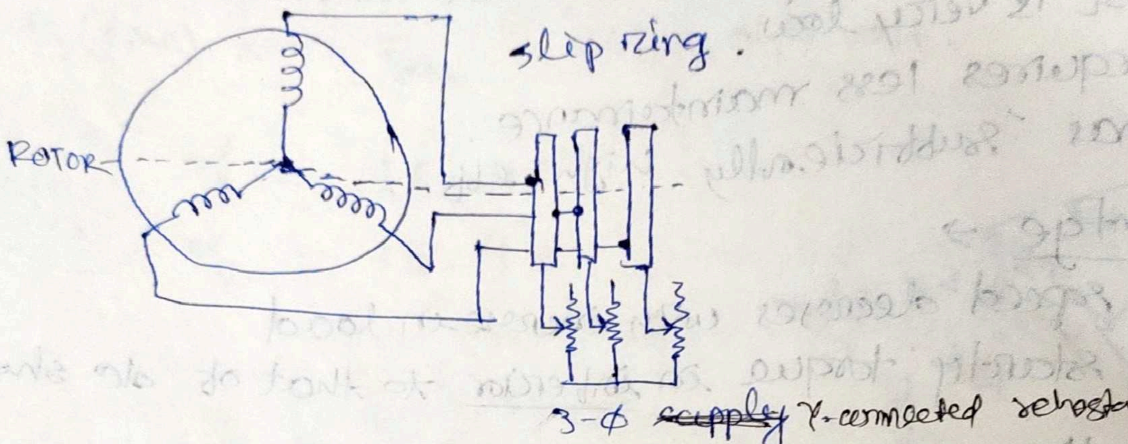
Induct^n motor using squirell cage type rotor is known as squirelcage induction motor (SCIM). The rotor consist of a cylindrical laminated core with parallel slots for carrying the rotor conductor which aren't wires but consists of heavy bars of copper, aluminium or alloys. The rotor bars are electrically welded or bolted to heavy and stout sht. cuteing end rings.

② Slip Ring Rotor :→

The induct^n motor using this type of rotor is known as slip ring ind^n motor (SRIM).



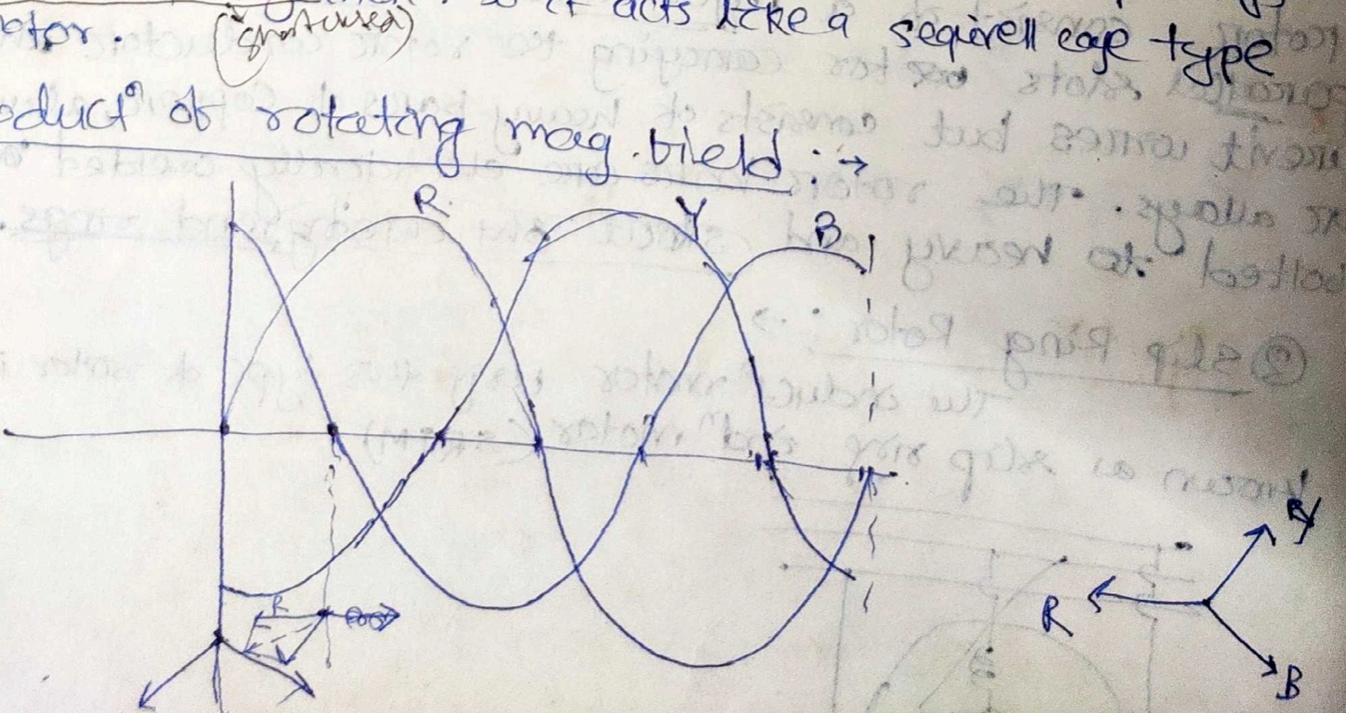




The 3-Ø of the rotor are starred externally, the other 3 wdg terminals are brought out & connected to 3 insulated slip rings mounted on shaft with brushes resting on them. These 3 brushes are further externally connected to a 3Ø Y-connected rheostat. This makes possible the introduction of additional resistance in the rotor ckt during starting period for increasing the starting torque of the motor.

Under normal running cond^n, all the slip rings connected together (shorted). So it acts like a squirrel cage type motor.

Product^n of rotating mag. field :→



(ii) This resulting flux rotates

by $N_s = \frac{120f}{P}$.

Synch. speed $\omega = \boxed{\frac{4\pi f}{P}}$

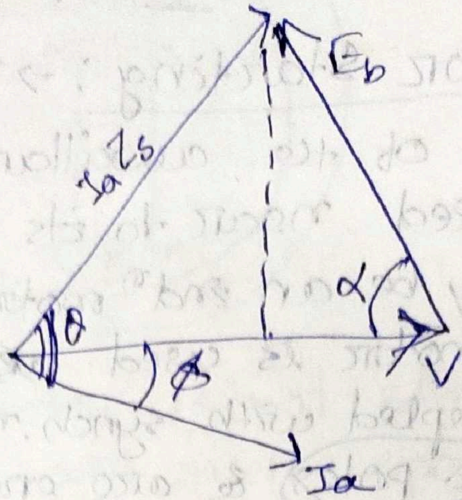
Principle :→

→ When 3ϕ supply is given to the stator is produce a rotating mag. flux which rotates at sync. speed $N_s = \frac{120f}{P}$.

→ This rotating flux cuts the rotor conductors, so an emf is induced in it according to Faraday's laws of electromag. induct^n $e = N\frac{d\phi}{dt}$

→ As the rotor is shortcircuited, this emf produces a current in the rotor conductor. This currents interacts with the flux to produce a torque (Fleming left hand rule). As a result rotor rotates.

lagging p.f



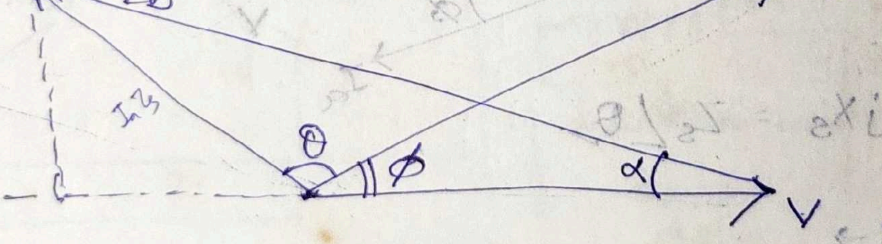
$$E_b = \sqrt{(V - I_aZ_s\cos(\theta-\phi))^2 + (I_aZ_s\sin(\theta-\phi))^2}$$

load angle or angle of retardation

$$\tan\alpha = \frac{I_aZ_s\sin(\theta-\phi)}{V - I_aZ_s\cos(\theta-\phi)}$$

Leading p.f.

$(E_b > V)$



$$E_b = \sqrt{\{V + I_aZ_s\cos(180-(\theta+\phi))\}^2 + \{I_aZ_s\sin(180-(\theta+\phi))\}^2}$$

$$\tan\alpha = \frac{I_aZ_s\sin(180-(\theta+\phi))}{V + I_aZ_s\cos(180-(\theta+\phi))}$$

led
lagg $E < V$
E V
la $E > V$

gen ~~$E < V$~~
E_b V
$E_b > V$
led → $V > E_b$
Cong → $< E$

Here ele. pwr is converted into mechanical pwr not by conduction, but by induction like a X^r. So sometime knowns rotating X^r.

Slip: In practice rotor never succeeds in catching up with stator field. If it did so, there would be no relative speed betn the two, hence no rotor emf, no rotor curr, & so no torque to maintain rotation.

The diff. betn synch. speed (N_s) & Actual speed (N_r) of the rotor is known as slip. express as percentage of synch. speed. (Actually the term 'slip' is descriptive of the way in which the rotor slips back from synchronism.

$$S = \frac{N_s - N_r}{N_s}$$

$$\%\ \text{slip} = \frac{N_s - N_r}{N_s} \times 100.$$

$$S = 1 - \frac{N_r}{N_s}$$

$$\Rightarrow \frac{N_r}{N_s} = 1 - S$$

$$\Rightarrow \boxed{N_r = (1-S) N_s}$$

slip speed $= N_s - N_r$

Note

④ It may be noted that as the load is applied, the natural effect of the load is to slow down the motor. Hence the slip (diff of N_s & N_r) increases and with it increases the current & torque, till the driving torque of the motor balances the retarding torque of the load.

Frequency of rotor current:→

let f = supply frequency

$$N_s = \frac{120 f}{P} \quad \text{—— ①}$$

let f' → rotor current freq.
= rotor emf frequency.

$$N_s - N_r = \frac{120 f'}{P} \quad \text{—— ②}$$

$$\Rightarrow \frac{N_s - N_r}{N_s} = \frac{f'}{f}$$

$$\Rightarrow \frac{f'}{f} = S$$

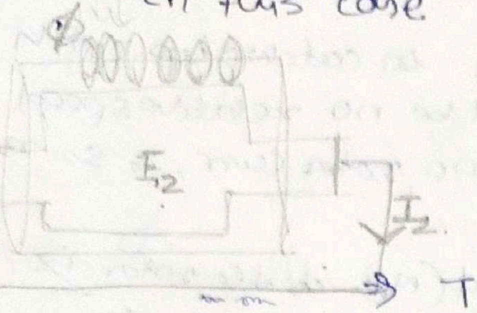
$$\Rightarrow \boxed{f' = S f}$$

The torque of rotor [illegible] interaction of strength of field and the [illegible] depends on the relation betn them. Thus the torque developed by the rotor of IM is directly proportional to (i) rotor current I_2 (ii) stator [illegible] per pole, & (iii) powerfactor of rotor ckt,

Torque equation: →

In case of dc motor $T \propto \phi I$

in this case $\boxed{T \propto \phi (I_2 \cos\phi_2)}$



where I_2 → rotor * curr. at standstill

ϕ_2 → angle betn rotor emf & rotor current * i.e, $T \propto \phi I_2 \cos\phi_2$

since, rotor emf/phase at standstill

where k → constant. $E \propto \phi$

$$\Rightarrow T = k\phi I_2 \cos\phi_2 .$$

so, $T \propto E_2 I_2 \cos\phi_2$

$\Rightarrow T = k_1 E_2 I_2 \cos\phi_2$

as $E_2 \propto \phi$

where E_2 → rotor emf per phase at standstill.

so $T \propto E_2 I_2 \cos\phi_2$

$$\boxed{\Rightarrow \boxed{T = k_1 E_2 I_2 \cos\phi_2}} \leftarrow *$$

Starting torque (T_{st}) →

Let E_2 → rotor emf/phase at standstill.

X_2 → rotor reactance/phase at "

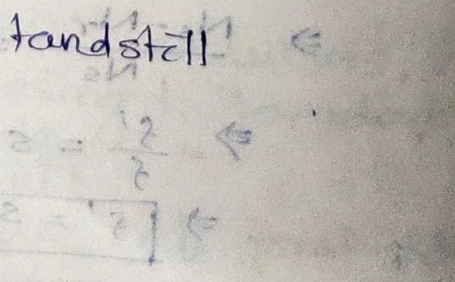
R_2 → rotor resistance/phase

Z_2 → rotor impd/phase at standstill.

$$\Rightarrow Z_2 = \sqrt{R_2^2 + X_2^2}$$

$$\cos\phi_2 = \frac{R_2}{Z_2}$$

$$= \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$



I_2 → rotor current/phase at standstill

$$= \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

$$T = k_1 E_2 I_2 \cos\phi_2$$

$$= k_1 \cdot E_2 \cdot \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

(*) $$\boxed{T_{st} = k_1 R_2 E_2^2 / R_2^2 + X_2^2}$$

experimentaly. $K_1 = \frac{3}{\left(2\pi \cdot \frac{N_s}{60}\right)}$ $N_s = \frac{120f}{P}$ rpm.

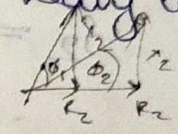
$N_s = \frac{120f}{P}$

$\omega = \frac{4\pi f}{P}$

(*) starting torque of squirrel cage motor.

st. curr I_2 very large but st. Torque very poor

The resistace of a squrrell cage motor is fixed and small as compared to reactance which is very large. Hence the starting current I_2 though very large in magnitude lags by a very large angle behind E_2, which with a result that starting torque perc ampere is very poor. Hence these motors aren't very useful, when the motor has to start against heavy loads



starting torque of a slip ring motor.

Startg torque of such a motor is incresed by improving p.f by adding external resistance from star connected rehostant.

Condition for max^m startg torque (T_{st})

$$T_{st} \propto \frac{E_2^2 R_2}{(R_2^2 + X_2^2)}$$

T_{st} is max^m when $\frac{dT_{st}}{dR_2} = 0$

$$\Rightarrow \frac{E_2^2\left[(R_2^2 + X_2^2) \cdot 1 - R_2 \cdot 2R_2\right]}{\left[R_2^2 + X_2^2\right]^2} = 0$$

$$\Rightarrow R_2^2 + X_2^2 = 2R_2^2$$

$$\Rightarrow R_2^2 = X_2^2$$

$$\Rightarrow \boxed{R_2 = X_2}$$

$$\boxed{T_{st}\,(\max) = \frac{K_1 E_2^2}{2R_2} = \frac{K_1 E_2^2}{2X_2}}$$

$R_2 = X_2$ region
max^m st. torque

Effect of change in supply volt. on starting torqu

as $E_2 \propto V$ (supply volt)

so $T_{st} \propto \frac{V^2 R_2}{R_2^2 + X_2^2}$

→ $\boxed{T_{st} \propto V^2}$

Rotor emf & reactance under running condⁿ:

Let E_2 → rotor induced emf/phase at standstill

X_2 → rotor reactance/phase at standstill.

f_2 = rotor current freq. at standstill = f (supply freq)

When the rotor is stationary i.e $S = 1$, the freq. of rotor emf is the same as the supply frequency. The value of emf induced in the rotor at standstill is maxᵐ becoz the relative speed betⁿ the rotor & the revolving stator flux is maxᵐ.

When rotor starts running, the relative speed betⁿ it & the rotating stator flux is decreased. Hence the rotor induced emf which is directly proportional to this relative speed is also decreased. Hence for a slip 's' the rotor emf will be 's' times the induced emf at standstill i.e

E_r = rotor emf/phase under running cond.

$\boxed{E_r = SE_2}$

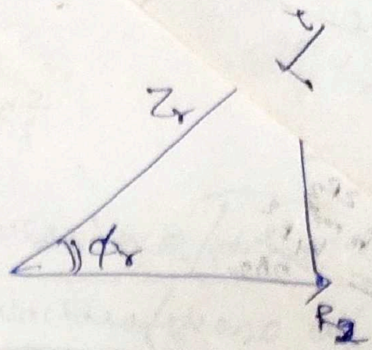
X_r = rotor reactance/phase under running Condⁿ = $\boxed{SX_2}$.

Torque under running condⁿ: →

$T \propto E_r I_r \cos\phi_r$

$T \propto \phi I$

$T \propto \phi I_r \cos\phi_r$



$$I_r = \frac{E_r}{Z_r} = \boxed{\frac{SE_2}{\sqrt{R_2^2+(SX_2)^2}}}$$

$$\cos\phi_r = \frac{R_2}{Z_r} = \frac{R_2}{\sqrt{R_2^2+(SX_2)^2}}$$

So $T_r \propto \phi \dfrac{SE_2}{\sqrt{R_2^2+(SX_2)^2}} \cdot \dfrac{R_2}{\sqrt{R_2^2+(SX_2)^2}}$

$$\Rightarrow T_r \propto \frac{\phi S E_2 R_2}{R_2^2+(SX_2)^2}$$

As $E_2 \propto \phi$

$$\Rightarrow T_r \propto \frac{S E_2^2 R_2}{R_2^2+(SX_2)^2} \checkmark$$

⊛ $$\Rightarrow \boxed{T_r = \frac{k_1 S E_2^2 R_2}{R_2^2+(SX_2)^2}}$$ wen $k_1 = \dfrac{3}{\left(2\pi \frac{N_s}{60}\right)}$

Condition for max^m torque under running $cond^n$: →

$$T_r = k_1 \frac{S E_2^2 R_2}{R_2^2+(SX_2)^2}$$

T_r is max^m wen $\dfrac{dT_r}{ds} = 0$

$$\Rightarrow k_1 E_2^2 R_2 \frac{\left[R_2^2+(SX_2)^2 \cdot \cancelto{1}{\frac{ds}{ds}} - S[2SX_2^2]\right]}{[\quad]^2} = 0$$

$$\Rightarrow R_2^2+(SX_2)^2 = 2S^2X_2^2$$

$$\Rightarrow R_2^2 = (SX_2)^2$$

$$\Rightarrow \boxed{R_2 = SX_2}$$

Effect o/s max^m $= K_1 \dfrac{SE_2^2 R_2}{2R_2^2}$

$$= \boxed{K_1 \frac{SE_2^2}{2R_2}}$$

$$= K_1 \frac{SE_2^2}{2 S X_2}$$

starting Running

$T_{st\,max} = \dfrac{K_1E_2^2}{2R_2} \neq \dfrac{(S)K_1E_2^2}{2R_2}$ $T_{r\,max}$

$T_{st\,max} = \dfrac{K_1E_2^2}{2X_2} = \dfrac{K_1E_2^2}{2X_2}$ $T_{r\,max}$

$$\boxed{T_r(\text{max}^m) = \frac{K_1E_2^2}{2X_2}}$$

1. (i) max^m torque is independt of rotor resistance
2. (ii) But the speed or slip at which max^m torque occures is determined by the rotor resistance.
3. (iii) By varing rotor resistance (possible only with slip ring motor) max^m torque can be made to occure at any desired slip or motor speed.
4. (iv) max^m torque varies inversly with standstill reactance, hence it should be kept as small as possible.
5. (v) max^m torque varies directly as the sq. of applied voltage.
6. (vi) For obtaining max^m torque at starting, rotor resistance must be equal to rotor reactance (By putty $S = 1$)

F/L torque & max^m torque :→

let S_f → slip corresponding to F/L torque

$$T_f \propto \frac{S_f R_2}{R_2^2 + (S_f X_2)^2}$$

$$T_{max} \propto \frac{1}{2X_2}$$

$$\Rightarrow \frac{T_f}{T_{max}} = \frac{2S_f R_2 X_2}{R_2^2 + (S_f X_2)^2}$$

$$\Rightarrow \frac{T_f}{T_{max}} = \frac{2S_f\frac{R_2}{X_2}}{\left(\frac{R_2}{X_2}\right)^2 + S_f^2}$$

let $\boxed{a = \frac{R_2}{X_2}}$ = $\frac{\text{Rotor resistance/phase at standstill}}{\text{Rotor reactance/phase at standstill}}$.

$$\Rightarrow \boxed{\frac{T_f}{T_{max}} = \frac{2aS_f}{a^2+S_f^2}} \checkmark$$

Also $\frac{R_2}{X_2} = S_m$ = slip correspondig to max^m torque

$\boxed{a = S_m} = \frac{R_2}{X_2}$

$$(*)\ \boxed{\frac{T_f}{T_{max}} = \frac{2S_mS_f}{S_m^2+S_f^2}} = \frac{2}{\frac{S_m}{S_f}+\frac{S_f}{S_m}} \quad ① \quad = a$$

Starting torque & max^m torque :→

$$T_{st} \propto \frac{R_2}{R_2^2+X_2^2}$$

**

$$T_{max} \propto \frac{1}{2X_2}$$

$$\frac{T_{st}}{T_{max}} = \frac{2R_2X_2}{R_2^2+X_2^2}$$

$$= \frac{2R_2/X_2}{\left(\frac{R_2}{X_2}\right)^2+1}$$

take $\frac{R_2}{X_2} = a = S_m$ ✓

$$\Rightarrow \boxed{\frac{T_{st}}{T_{max}} = \frac{2a}{a^2+1}} = \frac{2S_m}{S_m^2+1} \;(*) \quad —— ②$$

$$\frac{T_f}{T_{max}} \times \frac{T_{max}}{T_{st}} = \frac{2aS_f}{a^2+S_f^2} \cdot \frac{a^2+1}{2a}$$

$$(*)\ \boxed{\frac{T_f}{T_{st}} = \frac{(a^2+1)S_f}{a^2+S_f^2}} \quad —— ③$$

$$\frac{T_f}{T_{max}} = \frac{2aS_f}{a^2+S_f^2} = \frac{2}{\left(\frac{a^2+S_f^2}{aS_f}\right)}$$

$$= \frac{2}{\left(\frac{a^2}{aS_f} + \frac{S_f^2}{aS_f}\right)}$$

$$= \frac{2}{\left(\frac{a}{S_f} + \frac{S_f}{a}\right)}$$

** $\Rightarrow \boxed{T_f = T_{max}\left[\frac{2}{\frac{S_m}{S_f} + \frac{S_f}{S_m}}\right]} = T_b \times \left(\frac{2}{\frac{S_m}{S_f} +}\right.$

were T_b = Breakdown torque

= Tmax / Pull out torque.

** $\boxed{S_b = S_m = a = \frac{R_2}{X_2}}$

Effect of change in supply volt. on torque & speed

$$\text{Torque} \propto \frac{SE_2^2R_2}{R_2^2+(SX_2)^2}$$

As $E_2 \propto V$

$$\boxed{T \propto SV^2}$$

considering freq. $\boxed{T \propto \frac{SV^2}{f}}$

Ex