ENGINEERING MATHEMATICS -I

FOR DIPOLMA STUDENTS

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Complex Number

28/08/2024

$x^2-4=0$
$\Rightarrow x^2=4$
$\Rightarrow x=\sqrt{4}$
$\Rightarrow x=\pm 2$

$x^2+1=0$
$\Rightarrow x^2=-1$
$\Rightarrow x=\sqrt{-1}$ (no Solution in Real)
$\Rightarrow x=i$

$Z = \underbrace{x}_{\text{Real part}} + \underbrace{iy}_{\text{Imaginary part}}$

(iota) $i=\sqrt{-1}$

$C.N = Z = x+iy$

$\{x+iy : x,y \in R\}$

Ex/- $2+3i$, $\sqrt{2}-\frac{1}{2}i$, $5-\frac{7}{6}i$, $\frac{\sqrt{3}}{2}-i$

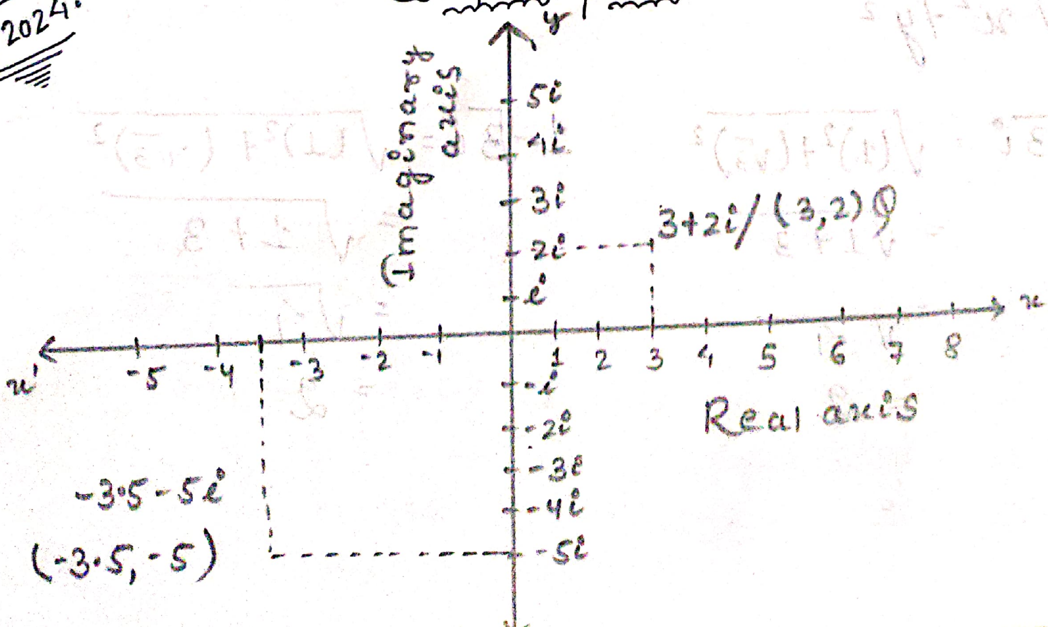
Imz $\{x+iy : x=0\}$
Purely Imaginary
Ex/- $0+6i$, $7i$, $0-\frac{\sqrt{3}}{2}i$

Rez $\{x+iy : y=0\}$
Purely real
Ex/- $6+0i$, $-\frac{1}{3}+0i$, $\frac{-7}{3}$, $\sqrt{3}$

* 0 is the number which is both purely imaginary and purely real.

30/08/2024

complex plan



Conjugate / $\bar{Z}$:-

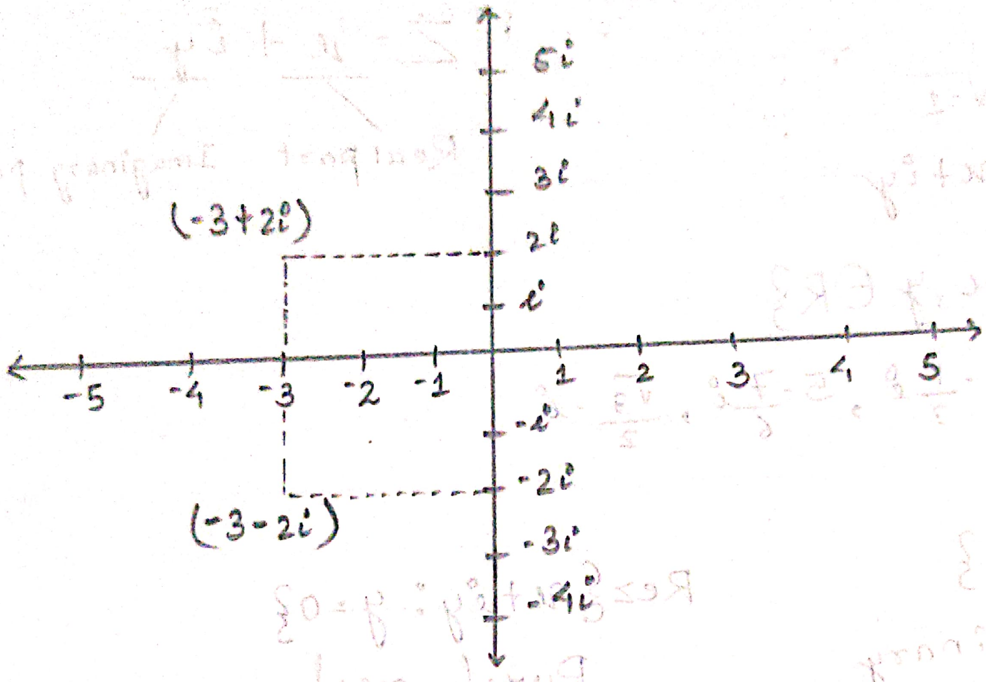
* Conjugate is the mirror image of complex number.

$Z = x + iy$

Conjugate of Z / $\bar{Z} = \overline{x+iy} = x - iy$

Ex/- $\overline{x - iy} = x + iy$

$\overline{-3-2i} = -3 + 2i$



Modulus / $|Z|$:-

$|Z|$ = Distance from origin

$Z = x + iy$

$|Z| = \sqrt{x^2 + y^2}$

Ex/- $1 + \sqrt{3}i = \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4}$

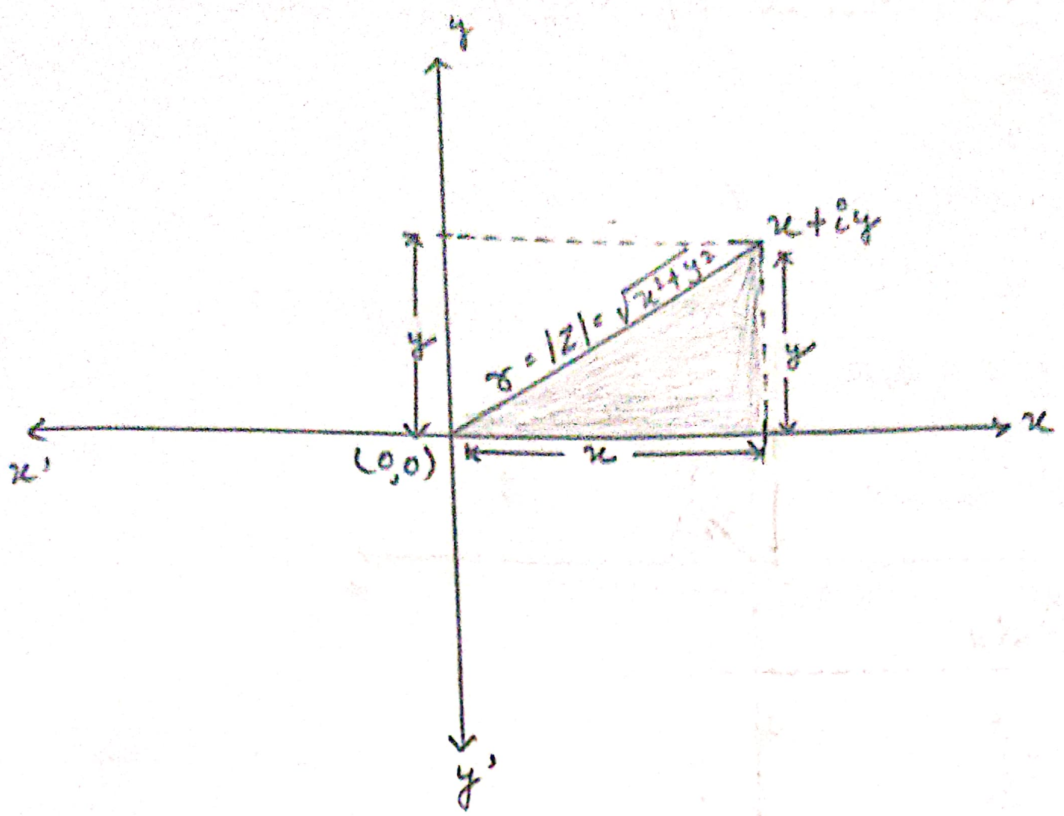
$= 2$

$1 - \sqrt{3}i = \sqrt{(1)^2 + (-\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4}$

$= 2$



From Pythagoras Theorem,

$$r^2 = x^2 + y^2$$

$$\Rightarrow r = \sqrt{x^2 + y^2}$$

$$= |z|$$

Power of iota (i) :-

$$i^{365} = i^{4 \times 91} \cdot i^1$$

$$=$$

$$i = \sqrt{-1}$$

$$i^2 = \sqrt{-1} \times \sqrt{-1} = -1$$

$$i^3 = \frac{\sqrt{-1} \times \sqrt{-1}}{1} \times \frac{\sqrt{-1}}{i} = -i$$

$$i^4 = \frac{\sqrt{-1} \times \sqrt{-1}}{-1} \times \frac{\sqrt{-1} \times \sqrt{-1}}{-1} = 1$$

$$i^{97} = i^{4 \times 24 + 1} = i^{4 \times 24} \cdot i^1$$

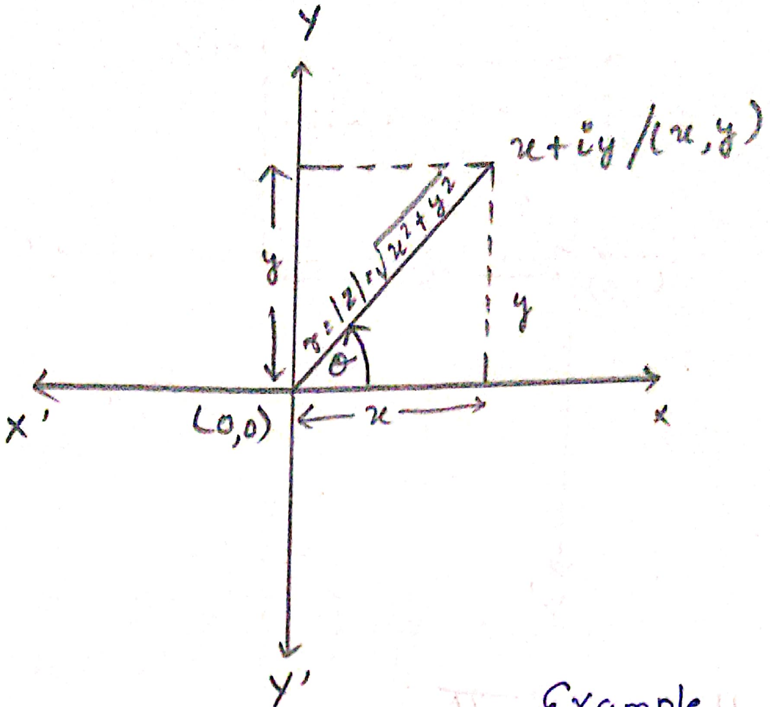
$$= 1 \cdot i$$

$$= i$$

$$\begin{array}{r|l} & 24 \\ \hline 4 & 97 \\ & -8 \\ \hline & 17 \\ & -16 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} & 91 \\ \hline 4 & 365 \\ & 36 \\ \hline & 05 \\ & 4 \\ \hline & 1 \end{array}$$

04/09/2021

Polar form of Complex Number



$\cos\theta = \frac{x}{r}$

$\Rightarrow x = r\cos\theta$

$\sin\theta = \frac{y}{r}$

$\Rightarrow y = r\sin\theta$

$Z = x + iy$

$= r\cos\theta + ir\sin\theta$

$Z = r(\cos\theta + i\sin\theta)$

Example

$1+i$

$r = |z| = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \cancel{\cos^{-1}}(\cancel{\cos} 45^\circ)$

$= 45^\circ$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \cancel{\sin^{-1}}(\sin 45^\circ)$

$= 45^\circ$

$Z = r(\cos\theta + \sin\theta)$

$= \sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$

Q- $1+\sqrt{3}i$

$r = |z| = \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4}$

$= 2$

$\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}(1/2)$

$= \cancel{\cos^{-1}}(\cancel{\cos} 60°)$

$= 60°$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}(\sqrt{3}/2)$

$= \cancel{\sin^{-1}}(\cancel{\sin} 60°)$

$= 60°$

$Z = r(\cos\theta + i\sin\theta)$

$= 2(\cos 60° + i \sin 60°)$

$= 2\left(\frac{1}{2} + i \times \frac{\sqrt{3}}{2}\right)$

$= 2\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$

$= \frac{\cancel{2}}{\cancel{2}} + \frac{\cancel{2}i\sqrt{3}}{\cancel{2}}$

$= 1 + \sqrt{3}i$

Q- $-\sqrt{3}+i$

$r = |z| = \sqrt{(-\sqrt{3})^2 + (1)^2}$

$= \sqrt{3+1}$

$= \sqrt{4}$

$= 2$

$\cos\theta = \frac{-\sqrt{3}}{2}$

$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

$\theta = (150°)$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}(1/2)$

$\theta = (150°)$

$Z = r(\cos\theta + i\sin\theta)$

$= 2(\cos 150° + i \sin 150°)$

Argument / Amplitude

(arg z / amp z)

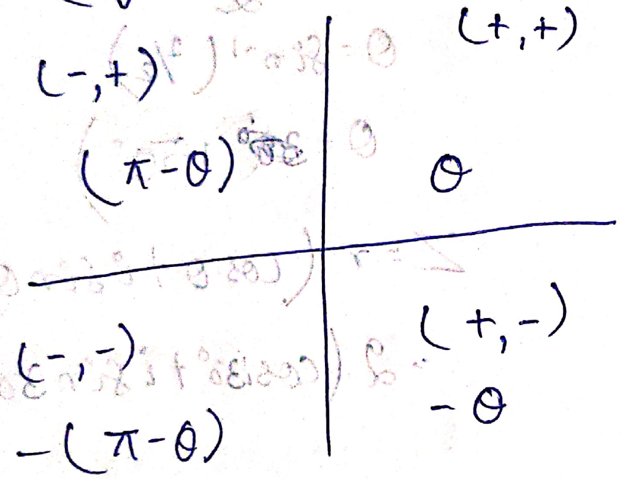
$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$\arg z = \theta = \tan^{-1}\left|\frac{1}{\sqrt{3}}\right|$

$\Rightarrow \theta = 30^\circ$

Principle argument

(Arg z)



$-\sqrt{3}+i$

$\text{Arg} = \pi - 30^\circ$

$= 150^\circ$

06/09/2024

Algebra of Complex Number:

1. Addition

$(a+ib)+(c+id)$

$\Rightarrow (a+c)+(ib+id)$

$\Rightarrow (a+c)+i(b+d)$

Ex/- $(7+3i)+(1-i)$

$\Rightarrow (7+1)+i(3-1)$

$\Rightarrow 8+2i$ (Ans.)

2. Substraction

$(a+ib)-(c+id)$

$\Rightarrow (a-c)+(ib-id)$

$\Rightarrow (a-c)+i(b-d)$

Ex/- $(1-i)-(3+i)$

$\Rightarrow (1-3)-(i-i)$

$\Rightarrow -2-2i$ Ans.

3. Multiplication

$(a+ib)(c+id)$

$\Rightarrow (a\times c)+(a\times id)+(ib\times c)+(ib\times id)$

$\Rightarrow (ac)+i(ad)+i(bc)+i^2(bd)$

$\Rightarrow ac+i(ad)+i(bc)-bd$

$\Rightarrow (ac-bd)+i(ad+bc)$

Ex/- $(3-i)(2+3i)$

$\Rightarrow (3\times 2)+(3\times 3i)+(-i\times 2)+(-i\times 3i)$

$\Rightarrow 6+i(9)+(-2i)+(-3i^2)$

$\Rightarrow 6+9i-2i-(3i^2)$

$\Rightarrow 6+9i-2i+3$

$\Rightarrow (6+3)+i(9-2)$

$\Rightarrow 9+7i$ (Ans.)

Ex/- $(3+2i)(1+2i)$

$\Rightarrow 3+6i+2i+4i^2$

$\Rightarrow 3+6i+2i-4$

$\Rightarrow (3-4)+i(6+2)$

$\Rightarrow -1+8i$ (Ans.)

4. Division

$$\frac{a+bi}{c+di}$$

$$\Rightarrow \frac{(a+ib)(c-id)}{(c+id)(c-id)}$$

$$\Rightarrow \frac{(a+ib)(c-id)}{c^2-(id)^2}$$

$$\Rightarrow \frac{(ac)-(aid)+(ibc)-(i^2bd)}{c^2-(d^2)(i^2)}$$

$$\Rightarrow \frac{ac-iad+ibc+bd}{c^2+d^2}$$

$$\Rightarrow \frac{(ac+bd)+i(bc-ad)}{c^2+d^2}$$

$$\Rightarrow \frac{ac+bd}{c^2+d^2} + i\frac{(bc-ad)}{c^2+d^2}$$

Ex/- $\frac{3+4i}{1+2i}$

$$\Rightarrow \frac{(3+4i)(1-2i)}{(1+2i)(1-2i)}$$

$$\Rightarrow \frac{(3)-(6i)+(4i)-(8i^2)}{(1)^2-(2i)^2}$$

$$\Rightarrow \frac{3-6i+4i+8}{1+4}$$

$$\Rightarrow \frac{11-2i}{5} = \frac{11}{5} - i\frac{2}{5}$$ (Ans.)

Ex/- $\frac{3-2i}{2-4i}$

$$\Rightarrow \frac{(3-2i)(2+4i)}{(2-4i)(2+4i)}$$

$$\Rightarrow \frac{(6)+(12i)-(4i)-(8i^2)}{4+16}$$

$$\Rightarrow \frac{6+12i-4i+8}{20}$$

$$\Rightarrow \frac{14+8i}{20}$$

$$\Rightarrow \frac{\cancel{14}\,7}{\cancel{20}_{10}} + i\frac{\cancel{8}\,\cancel{4}\,2}{\cancel{20}\,\cancel{10}\,5}$$

$$\Rightarrow \frac{7}{10} + i\frac{2}{5}$$ (Ans.)

10/09/2024.

Square Root of Complex Number.

$\sqrt{x+iy} = a+ib$

$\Rightarrow x+iy = (a+ib)^2$

$\Rightarrow x+iy = a^2+i^2b^2+2iab$

$\Rightarrow x+iy = a^2-b^2+2iab$

So, $a^2-b^2 = x$
$2ab = y$ ——— (1)

$(a^2+b^2)^2 = (a^2-b^2)^2+(2ab)^2$

$\Rightarrow (a^2+b^2)^2 = x^2+y^2$

$\Rightarrow a^2+b^2 = \pm\sqrt{x^2+y^2}$ ——— (11)

Now, Solving eqn (1) and (11)

$a^2-\cancel{b^2} = x$

$a^2+\cancel{b^2} = \pm\sqrt{x^2+y^2}$

$2a^2 = x\pm\sqrt{x^2+y^2}$

$\Rightarrow a = \sqrt{\dfrac{x\pm\sqrt{x^2+y^2}}{2}}$

Ex/> $5-12i$

$\Rightarrow a-ib = \sqrt{5-12i}$

$\Rightarrow (a-ib)^2 = 5-12i$

$\Rightarrow a^2-b^2+2iab = 5-12i$

$a^2-b^2 = 5$

$2ab = -12$

$(a^2+b^2)^2 = (a^2-b^2)^2+(2ab)^2$

$\Rightarrow (a^2+b^2)^2 = 5^2+12^2$

$= 25+144$

$= 169$

$\Rightarrow a^2+b^2 = \pm\sqrt{169}$

$= \pm 13$

Solving eqn (1) and (11)

$a^2-\cancel{b^2} = 5$

$a^2+\cancel{b^2} = 13$

$2a^2 = 18$

$a^2 = 9$

$a = \pm 3$

$a^2-b^2 = 5$

$\Rightarrow 3^2-b^2 = 5$

$\Rightarrow 9-b^2 = 5$

$\Rightarrow b^2 = 4$

$\Rightarrow b = \pm 2$

$\therefore (-3+2i), (3+2i), (3-2i), (-3-2i)$

Date:- 11.09.2024

Q → $-15-8\sqrt{-1}$

$\Rightarrow a+ib = \sqrt{-15-8i}$

$\Rightarrow -15-8i = (a+ib)^2$

$\Rightarrow (a+ib)^2 = -15-8i$

$\Rightarrow a^2+i^2b^2+2aib = -15-8i$ $[\because i^2=-1]$

$\therefore a^2-b^2+2aib = -15-8i$

So, $a^2-b^2=-15$ —— (1)

$2ab = -8$

Now, $(a^2+b^2)^2 = (a^2-b^2)^2+(2ab)^2$

$= (-15)^2+(-8)^2$

$= 225+64$

$= 289$

$\Rightarrow a^2+b^2 = \pm\sqrt{289}$ —— (11)

$= \pm 17$

Solving eqn (1) and (11)

$a^2-\cancel{b^2} = -15$

$a^2+\cancel{b^2} = 17$

$2a^2 = 2$

$a^2 = 1$

$a = \pm\sqrt{1}$

$= \pm 1$

$a^2-b^2=-15$

$(1)^2-b^2=-15$

$-b^2=-15-1$

$-b^2=-16$

$b^2=16$

$b=\pm\sqrt{16}$

$=\pm 4$

$\therefore a+ib = (1+4i), (1-4i), (-1-4i), (-1+4i)$

13/09/2024

Cube Root Of Unity

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$*\boxed{1+\omega+\omega^2=0}$

$*\boxed{\omega^3=1}$

Let $\sqrt[3]{1}=x$

$\Rightarrow (1)^{1/3}=x$

$\Rightarrow x^3-1=0$

$\Rightarrow x^3-1^3=0$

$\Rightarrow (x-1)(x^2+x+1^2)=0$

$(x-1)=0$

$\Rightarrow x=0$

A.T. Formula, $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-1\pm\sqrt{(1)^2-4\cdot1\cdot1}}{2\cdot1}$

$=\frac{-1\pm\sqrt{1-4}}{2}$

$=\frac{-1\pm\sqrt{-3}}{2}$

$=\frac{-1\pm\sqrt{3}i}{2}$

$\omega=\frac{-1+\sqrt{3}i}{2}$

$\omega^2=\frac{-1-\sqrt{3}i}{2}$

$\therefore x=1,\omega,\omega^2$

$1+\left(\frac{-1+\sqrt{3}i}{2}\right)+\left(\frac{-1-\sqrt{3}i}{2}\right)$

$\Rightarrow \frac{2+(-1+\sqrt{3}i)+(-1-\sqrt{3}i)}{2}$

$\Rightarrow \frac{2-1+\cancel{\sqrt{3}i}-1-\cancel{\sqrt{3}i}}{2}$

$\Rightarrow \frac{2-2}{2}$

$\Rightarrow \frac{0}{2}$

$=0$

20/09/2024.

Q- $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5) = 9$

$\Rightarrow (1-\omega)(1-\omega^2)(1-\omega^3.\omega)(1-\omega^3.\omega^2)$

$\Rightarrow (1-\omega)(1-\omega^2)(1-\omega)(1-\omega^2)$

$\Rightarrow (1-\omega)^2(1-\omega^2)^2$ $[\because \omega^3 = 1]$

$\Rightarrow \left((1-\omega)(1-\omega^2)\right)^2$

$\Rightarrow (1-\omega^2-\omega+\omega^3)^2$

$\Rightarrow (1-\omega^2-\omega+1)^2$

$\because 1+\omega+\omega^2 = 0$

$\Rightarrow 1 = -\omega-\omega^2$

$\Rightarrow [1+1+1]^2$

$\Rightarrow (3)^2$

$= 9$ proved.

Q- $(1+5\omega^2+\omega^4)(1+5\omega+\omega^2)(5+\omega+\omega^2) = 64$

$\Rightarrow (1+5\omega^2+\omega^3.\omega)(-\omega+5\omega)(5\cancel{+\omega}-1\cancel{-\omega})$

$\Rightarrow (1+5\omega^2+\omega)(4\omega)(4)$

$\Rightarrow (-\omega^2+5\omega^2)(4\omega)(4)$

$\Rightarrow (4\omega^2)(4\omega)(4)$

$\Rightarrow 64(\omega^2 \times \omega)$

$\Rightarrow 64\omega^3$ $(a^m \times a^n = a^{m+n})$

$\Rightarrow 64 \times 1$ $[\because \omega^3 = 1]$

$= 64$ proved

27/09/2024.

De-Movire's Theorem

Theorem :- Where 'n' is integer, positive, negative or '0'.

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

Case-1 when 'n' is +ve

$$(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$$

Ex/- $(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$

Ans- $(\cos\theta + i\sin\theta)(\cos\theta + i\sin\theta)$

$= (\cos\theta\cdot\cos\theta + i\sin\theta\cdot\cos\theta + i\sin\theta\cdot\cos\theta + i^2\sin\theta\cdot\sin\theta)$

$= (\cos\theta\cdot\cos\theta - \sin\theta\cdot\sin\theta) + i(\sin\theta\cdot\cos\theta + \sin\theta\cdot\cos\theta)$

$= (\cos\theta+\theta) + i(\sin\theta+\theta)$

$= \cos 2\theta + i\sin 2\theta.$

Case-2 when 'n' is -ve $(n=-m)$

$$(\cos\theta + i\sin\theta)^{-m} = \cos(-m\theta) + i\sin(-m\theta)$$

$$= \cos m\theta - i\sin m\theta.$$

Proof :- $(\cos\theta + i\sin\theta)^{-m} = \dfrac{1}{(\cos\theta + i\sin\theta)^m}$

$= \dfrac{1}{(\cos m\theta + i\sin m\theta)}$

$= \dfrac{\cos m\theta - i\sin m\theta}{(\cos m\theta + i\sin m\theta)(\cos m\theta - i\sin m\theta)}$

$= \dfrac{\cos m\theta - i\sin m\theta}{\cos^2 m\theta - i^2\sin^2 m\theta}$

$= \dfrac{\cos m\theta - i\sin m\theta}{\cos^2 m\theta + \sin^2 m\theta}$

$= \cos m\theta - i\sin m\theta$ proved.

Case-3 When 'n' is 0.

$$(\cos\theta + i\sin\theta)^0 = \cos(0\cdot\theta) + i\sin(0\cdot\theta)$$
$$= \cos 0 - i\sin 0$$
$$= 1$$

n = fraction, $\frac{P}{q}$, P and q are prime number.

$$(\cos\theta + i\sin\theta)^{P/q} = \cos\left(\frac{[2k\pi+\theta]P}{q}\right) + i\sin\left(\frac{[2k\pi+\theta]P}{q}\right)$$

$$(\cos\theta + i\sin\theta)^{1/2}$$
$$= \cos\left(\frac{2k\pi+\theta}{2}\right) + i\sin\left(\frac{2k\pi+\theta}{2}\right)$$

when, $k=1$ $= \cos\left(\frac{2\pi+\theta}{2}\right) + \sin\left(\frac{2\pi+\theta}{2}\right)$

Q- Sq. root of $-5+12\sqrt{-1}$

Ans- $-5+12\sqrt{-1}$

$(a+ib)^2 = -5+12i$

$\Rightarrow a^2+(ib)^2+2iab = -5+12i$

$\Rightarrow a^2-b^2+2iab = -5+12i$

Equating Real and imaginary part

$a^2-b^2 = -5$ —— (i)

$2iab = 12i$

$2ab = 12$

$ab = 6$ —— (ii)

$(a^2+b^2)^2 = (a^2-b^2)^2+4(ab)^2$

$= (-5)^2+4(6)^2$

$= 25+4\times36$

$= 25+144$

$a^2+b^2 = \pm\sqrt{169}$

$= \pm13$. —— (iii)

Solving eqn (i) and (iii)

$a^2-\cancel{b^2} = -5$

$a^2+\cancel{b^2} = \pm13$

$2a^2 = 8$

$a^2 = 4$

$a = \pm\sqrt{4}$

$= \pm2$

Putting value of a in eqn

$a^2-b^2 = -5$

$\Rightarrow 2^2-b^2 = -5$

$\Rightarrow -b^2 = -5-4$

$\Rightarrow -b^2 = -9$

$\Rightarrow b^2 = 9$

$\Rightarrow b = \pm\sqrt{9}$

$= \pm3$

$\therefore$ Sq. root of $-5+12\sqrt{-1}$ will be :

$(-2+3i), (2-3i), (-2-3i), (2+3i)$

Q- $1+i$

Ans- $(a+ib)^2 = 1+i$

$\Rightarrow a^2+(ib)^2+2iab = 1+i$

$\Rightarrow a^2-b^2+2iab = 1+i$

Equating Real and imaginary part

$a^2-b^2 = 1$ —— (i)

$2ab = 1$

$2ab = 1$

$ab = 1/2$ —— (ii)

$(a^2+b^2)^2 = (a^2-b^2)^2+4(ab)^2$

$= (1)^2+4\times\left(\frac{1}{2}\right)^2$

$= 1+\cancel{4}\times\frac{1}{\cancel{4}}$

$= 2$

$(a^2+b^2) = \pm\sqrt{2}$ —— (iii)

Solving eqn (i) and (iii)

$a^2-\cancel{b^2} = 1$

$a^2+\cancel{b^2} = \sqrt{2}$

$2a^2 = 1+\sqrt{2}$

$a^2 = \frac{1+\sqrt{2}}{2}$

$a = \pm\sqrt{\frac{1+\sqrt{2}}{2}}$ —— (iv)

Putting the value of a in eqn (i)

$a^2-b^2 = 1$

$\Rightarrow \left(\sqrt{\frac{1+\sqrt{2}}{2}}\right)^2 - b^2 = 1$

$\Rightarrow \left(\frac{1+\sqrt{2}}{2}\right) - b^2 = 1$

$\Rightarrow -b^2 = \frac{1}{1} - \left(\frac{1+\sqrt{2}}{2}\right)$

$\Rightarrow -b^2 = \frac{2-(1+\sqrt{2})}{2}$

$\Rightarrow -b^2 = \frac{2-1-\sqrt{2}}{2}$

$\Rightarrow -b^2 = \frac{1-\sqrt{2}}{2}$

$\Rightarrow b = \pm\sqrt{\frac{1-\sqrt{2}}{2}}$

18/10/2024

De-Moivre's Theorem

Q. Show that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-\theta\right)$

Sol^n:- $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n$

$$\left[\because \begin{array}{l}\sin\theta=\cos(\pi/2-\theta)\\ \cos\theta=\sin(\pi/2-\theta)\end{array}\right]$$

$$=\left[\frac{1+\cos(\pi/2-\theta)+i\sin(\pi/2-\theta)}{1+\cos(\pi/2-\theta)-i\sin(\pi/2-\theta)}\right]^n$$

Put $\left(\frac{\pi}{2}-\theta\right)=k$

$$=\left[\frac{1+\cos k+i\sin k}{1+\cos k-i\sin k}\right]^n$$

Put $z=(\cos k+i\sin k)$

$$z^{-1}=\frac{1}{z}=(\cos k+i\sin k)^{-1}$$
$$=(\cos k-i\sin k)$$

$$=\left[\frac{1+z}{1+1/z}\right]^n$$

$$=\left[\frac{1+z}{\frac{z+1}{z}}\right]^n$$

$$=\left[\frac{z\cancel{(1+z)}}{\cancel{1+z}}\right]^n$$

$$=z^n=\left[\cos(\pi/2-\theta)+i\sin(\pi/2-\theta)\right]^n=\left[\cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)\right]$$

Q- $(1+\sqrt{3}i)^n + (1-\sqrt{3}i)^n = 2^{n+1} \cos\frac{n\pi}{3}$

Sol^n:- $(1+\sqrt{3}i)$

$r = \sqrt{(1)^2 + (\sqrt{3})^2}$

$= \sqrt{1+3}$

$= \sqrt{4}$

$= 2$

$1 = r\cos\theta$, $\sqrt{3} = r\sin\theta$

$1 = 2\cos\theta$ $\quad \sqrt{3} = 2\sin\theta$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

$\Rightarrow \theta = \pi/3$

$1+\sqrt{3}i = 2\left(\cos\pi/3 + i\sin\pi/3\right)$

$r = \sqrt{1^2 - (\sqrt{3})^2}$

$= \sqrt{1-3}$

$= \sqrt{-2}$

$= 2$

$1-\sqrt{3}i = 2\left(\cos\pi/3 - i\sin\pi/3\right)$

$(1+\sqrt{3}i)^n = \left[2\left(\cos\pi/3 + i\sin\pi/3\right)\right]^n$

$= 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$

$(1-\sqrt{3}i)^n = \left[2\left(\cos \pi/_3 - i \sin \pi/_3\right)\right]^n$

$= 2^n \left(\cos \frac{n\pi}{3} - i \sin \frac{n\pi}{3}\right)$

So, $(1+\sqrt{3}i)^n + (1-\sqrt{3}i)^n$

$= 2^n \cos \frac{n\pi}{3} + \cancel{2^n i \sin \frac{n\pi}{3}} + 2^n \cos \frac{n\pi}{3} - \cancel{2^n i \sin \frac{n\pi}{3}}$

$= 2^n . 2\cos \frac{n\pi}{3}$

$= 2^{n+1} \cos \frac{n\pi}{3}$ proved.

Q- $\dfrac{(\cos\theta + i\sin\theta)^3}{(\cos\theta + i\sin\theta)^{-2}} = \dfrac{\cos 3\theta + i\sin 3\theta}{\cos 2\theta - i\sin 2\theta}$

$= (\cos\theta + i\sin\theta)^3 (\cos\theta + i\sin\theta)^2$

$= (\cos 3\theta + i\sin 3\theta)\ (\cos 2\theta + i\sin 2\theta)$

$= (\cos 3\theta \cdot \cos 2\theta + i\sin 2\theta \cdot \cos 3\theta + i\sin 3\theta \cdot \cos 2\theta + i^2 \sin^2 3\theta \cdot 2\theta)$

Q- $\dfrac{(\cos\theta - i\sin\theta)^2}{(\cos\theta + i\sin\theta)^{-3}}$

$= \dfrac{\cos 2\theta - i\sin 2\theta}{\cos 3\theta - i\sin 3\theta}$

$= \dfrac{(\cos 2\theta - i\sin 2\theta)(\cos 3\theta + i\sin 3\theta)}{(\cos 3\theta - i\sin 3\theta)(\cos 3\theta + i\sin 3\theta)}$

$= \dfrac{\cos 2\theta \cdot \cos 3\theta + i\sin 3\theta \cdot \cos 2\theta - i\sin 2\theta \cdot \cos 3\theta - i^2 \sin^2 6\theta}{\cos 3\theta \cdot \cos 3\theta + i\sin 3\theta \cdot \cos 3\theta - i\sin 3\theta \cdot \cos 3\theta - i^2 \sin^2 9\theta}$

or

$= \dfrac{\cos 2\theta \cdot \cos 3\theta + i\sin 3\theta \cdot \cos 2\theta - i\sin 2\theta \cdot \cos 3\theta + \sin^2 6\theta}{\cos^2 3\theta + \sin^2 3\theta}$

$= \dfrac{\cos 2\theta \cdot \cos 3\theta + i(\sin 3\theta \cdot \cos 2\theta - \sin 2\theta \cdot \cos 3\theta) + \sin 2\theta \cdot \sin 3}{\cancel{\cos}\ 1}$

$= \cos(2\theta - 3\theta) + i(\sin(3\theta - 2\theta))$

$= \cos\theta + i\sin\theta.$

Q - If $x + \frac{1}{x} = 2\cos\theta$ then show that (i) $x^n + \frac{1}{x^n} = 2\cos\theta$

(ii) $x^n - \frac{1}{x^n} = 2i\sin\theta$

Soln:- $x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow \frac{x^2+1}{x} = 2\cos\theta$

$\Rightarrow x^2+1 = 2x\cos\theta$

$\Rightarrow x^2+1-2x\cos\theta = 0$

$\Rightarrow x^2+\cos^2\theta+\sin^2\theta-2x\cos\theta = 0$ $(\because \cos^2\theta+\sin^2\theta = 1)$

$\Rightarrow x^2+\cos^2\theta-2x\cos\theta+\sin^2\theta = 0$

$\Rightarrow (x-\cos\theta)^2 = -\sin^2\theta$

$\Rightarrow (x-\cos\theta) = \sqrt{-\sin^2\theta}$ $(\because \sqrt{-1} = i)$

$\Rightarrow x-\cos\theta = i\sin\theta$

$\Rightarrow \boxed{x = i\sin\theta+\cos\theta}$

(i) $x^n = (\cos\theta + i\sin\theta)^n$

$= \cos n\theta + i\sin n\theta$

$\frac{1}{x^n} = x^{-n} = (\cos\theta + i\sin\theta)^{-n}$

$= (\cos n\theta - i\sin n\theta)$

$\therefore$ Now, $x^n + \frac{1}{x^n}$

$= \cos n\theta + i\cancel{\sin n\theta} + \cos n\theta - i\cancel{\sin n\theta}$

$= 2\cos n\theta$ proved.

(ii) Now,

$$x^n - \frac{1}{x^n}$$

$$= (\cos n\theta + i\sin n\theta) - (\cos n\theta - i\sin n\theta)$$

$$= \cancel{\cos n\theta} + i\sin n\theta - \cancel{\cos n\theta} + i\sin n\theta$$

$$= 2i\sin(n\theta)$$ proved