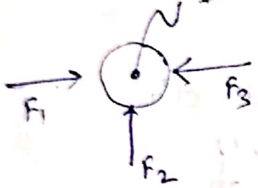
System of Forces

Engg. Mechanics
- Statics (object is in Rest)
- Dynamics (object is in motion)
  - Kinematics (Do not consider the cause of motion)
  - Kinetics (consider the cause of motion)

System of Forces
- Coplanar
  - Colinear
  - Concurrent
  - Parallel
- Non-Coplanar
  - Non-concurrent
  - Non-parallel

$\perp^r$ to plane of paper (F_4)

Coplanar:-

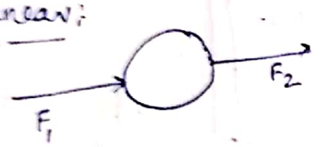


F_1 F_2 F_3 are coplanar (lies in same plane)

NOCOPLANAR:-

F_1 F_4 are non-coplanar

Colinear:



F_1 F_2 are Colinear (lies along same line)

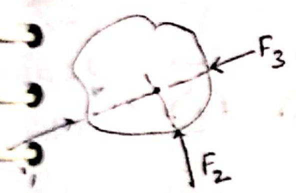
* only for coplanar.

Note:- Imp

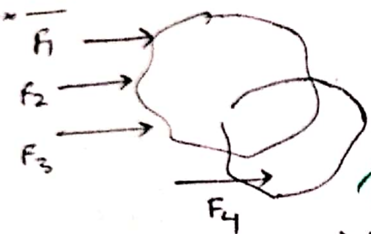
> If colinear then necessarily coplanar, vice-versa may or maynot.

Concurrent:-

> More than two forces, meets @ single point.



Parallel:-



F_1 F_2 F_3 are parallel (Coplanar)

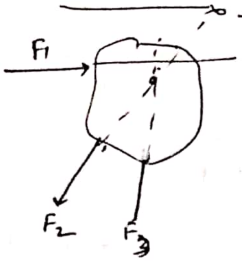
F_4 out of plane, but still parallel to F_1 F_2 F_3 (Noncoplanar)

Imp

> Note:- Parallel forces cannot be concurrent, vice-versa also true.

Scanned with CamScanner

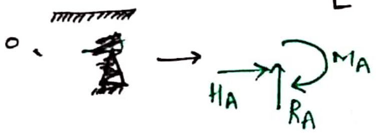
Non- concurrent Non parallel :-



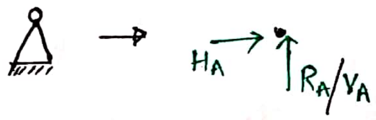
does not meet @ a point } F_1 F_2 F_3
are not parallel also }

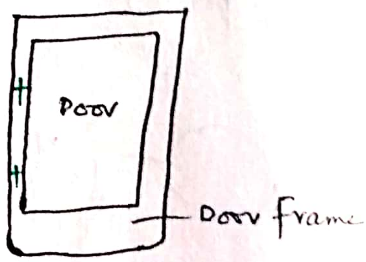
Types of Supports:

① Fixed Support :- ∘ Prevents ~~of~~ Moment/Movement of beam in all direction.
[Horizontal, vertical, Rotation]
∘ Prevents movement due to reactions

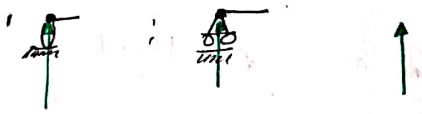


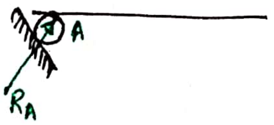
② Hinge / Pin Support :- ∘ prevents moment in two directions. due to reactions
(H, V)
∘ Not resisting rotation.
∘ Door Hinges



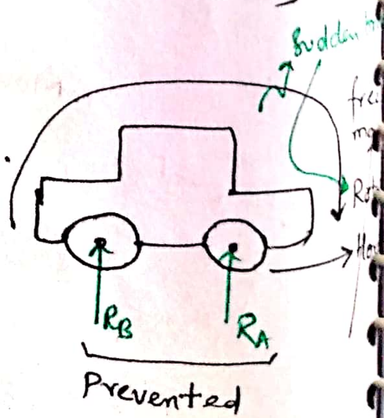


③ Roller Support :- ∘ prevents moment in one direction [⊥ʳ to plane of contact]
[H or V]

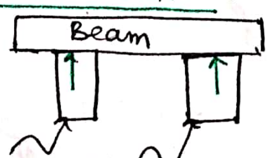




∘ Roller used in construction (Road).
∘ wheel



④ Simple Support :- ∘ allows to rest.



Simple Support

Type of body
- Rigid body Mechanics [Stone, duster, wood]
  - No change in shape
- Deformable body mechanics [Rubber band,] ✗
- Fluid body mechanics [Liquid, Gas] ✗

Motion of body
- (Rest) ✓ Statics
- (motion) ✓ Dynamics
  - Cause of motion considered (Yes/No)
    - NO → Kinematics
    - Yes → Kinetics

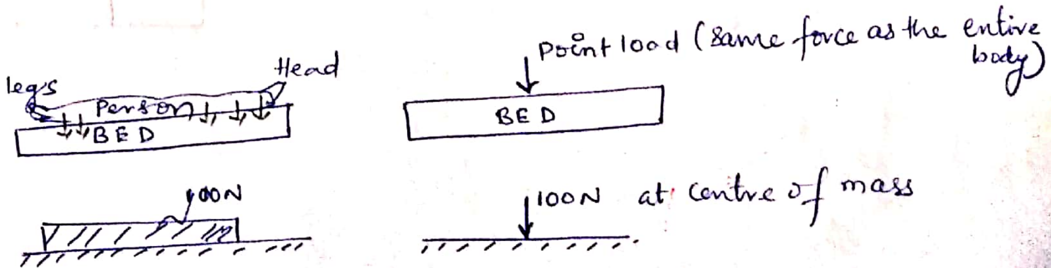
- "Branch of physics", which deals with the effect of force or multiple forces acting on a body/system.

Fundamental Principles

- Newton's laws of motion
  - first law
  - second law
  - third law
- Newton's law of Gravitation
- Principle of transmissibility of force

Assumptions / Idealisation : (Why) [∘ accepted as true w/o proof]

- Particle [when car is moving ?] Consider the entire body a particle. (Centre of mass)
- Rigid body
- Point load



Newton's first law :- Every body continues to be the state of rest or of uniform motion unless acted upon by an external unbalanced force

- Every external unbalanced force will produce motion /or change the state of motion.

Newton's second law :-

→ Rate of change of momentum is directly proportional to the applied force and takes place in the dirn of the force

- 'Football' rolling
to ↑ momentum, kick @ high force.
- cricket :- bat and ball

Newton's third law :-

→ For every action there is equal, ~~[illegible]~~ opposite and instantaneous reaction.

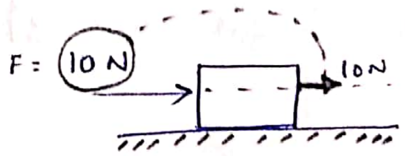
- 'Single' force never exists
- force 'always' in pair [Action & Reaction]

Newton's law of Gravitation :-

$$F \propto \frac{m_1 m_2}{r^2} = \frac{G M_1 M_2}{r^2}$$

G = gravitational constant

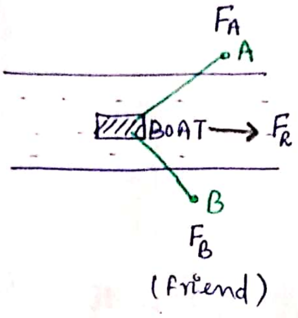
Principle of transmissibility of force :-

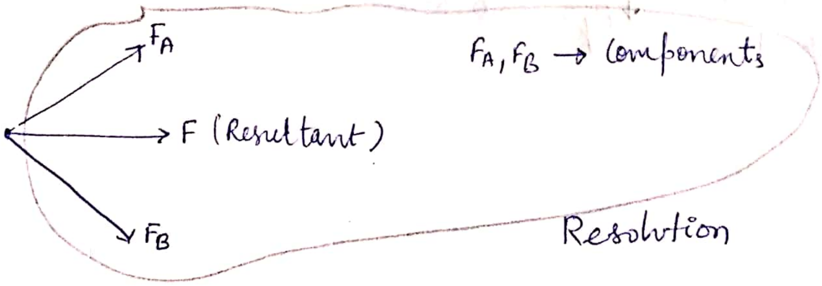


- Force can be transmitted (along the same line)

Resolution of Force :

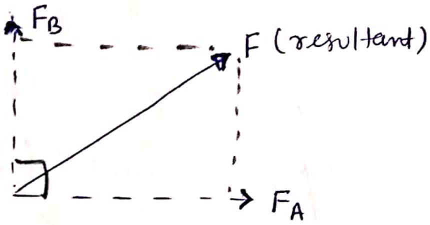
Force :- physical quantity that can change the state of motion of a given body.



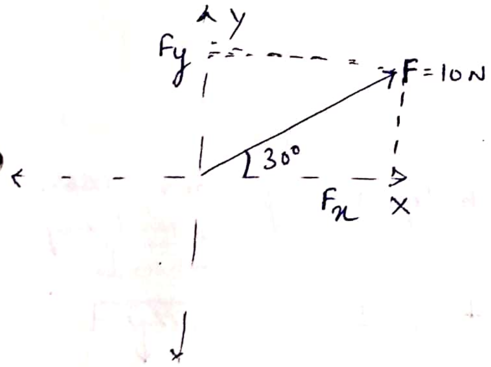


Resolution of force is the process in which we divide/resolve the resultant force into two/∞ components.

F_A, F_B $\perp^r$ to each other [Rectangular components]



Ex:-



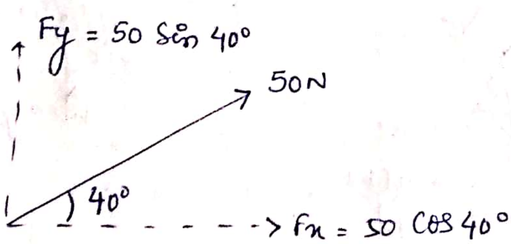
F_x, F_y ? if $F_R = 10N$

"θ must be given"

$F_x = F\cos\theta = F\cos 30° \simeq 10\cos 30°$

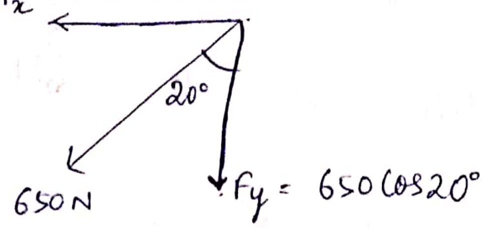
$F_y = F\sin\theta = F\sin 30° \simeq 10\sin 30°$

Ex:-



$\left.\begin{array}{l} F_x = \text{horizontal} \\ F_y = \text{vertical} \end{array}\right\}$ Components

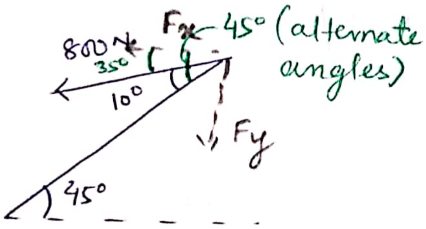
Ex:-



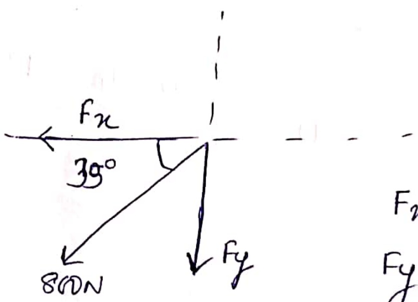
$F_x = 222.3$ N (←) OR -222.3 N

$F_y = 610.8$ N (↓) OR -610.8 N

Ex:-



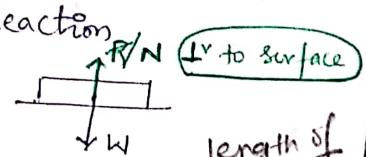
- angle should be measured with either x or y axis.

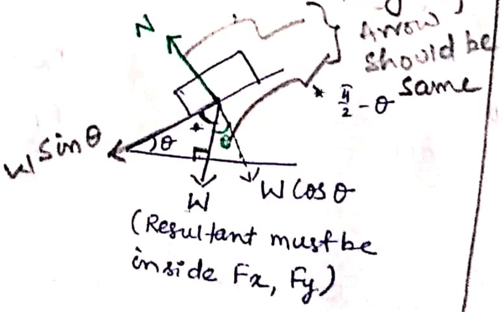


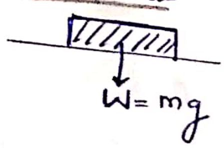
$F_x = 800 \cos 35°$ (←)

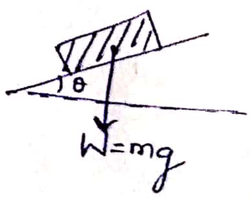
$F_y = 800 \sin 35°$ (↓)

- Three ~~special~~ ~~specif~~ SPECIAL forces
  - Weight :- gravitational force acting on a body having some mass. acts vertically downwards
  - Normal reaction :- Normal (Perpendicular) + Reaction
  - Friction :-









Types of Force system

Force system

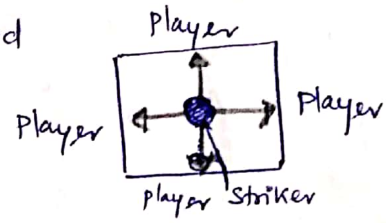
- Coplanar [2D-System] ($\vec{F} = F_x\hat{i} + F_y\hat{j}$) (OR $= F_y\hat{j} + F_z\hat{k}$) (OR $= F_x\hat{i} + F_z\hat{k}$)
  - Concurrent
  - Parallel
  - General
- Non-coplanar [3D-System] (space force) system — $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

- Coplanar :- in one plane [Carrom board; when striker hit the coins, coins move w/o flying]
- Non-coplanar:- more than one plane [when coins fly in carrom ; Kicking a football]

- Force system ; Group of force acting on a body

- Force ; vector quantity
  - direction (horizontal/vertical/angle) [sense]
  - magnitude
  - point of application of force

o) Coplanar :- Concurrent :- Carrom board



- Magnitude
- Direction
- X point of application

- each player wants to have 1st strike pull the striker towards them.
- coplanar, concurrent
- striker moves towards Larger force, towards the direction.

- concurrent :- meeting at one point

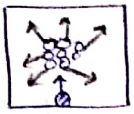
o Coplanar :- Parallel :-
- Magnitude
- Direction
- point of application of force

- in a carrom board, coins are placed (No one is playing)
- weight forces are parallel

Coplanar :- General :-

- magnitude
- Direction
- point of application of force

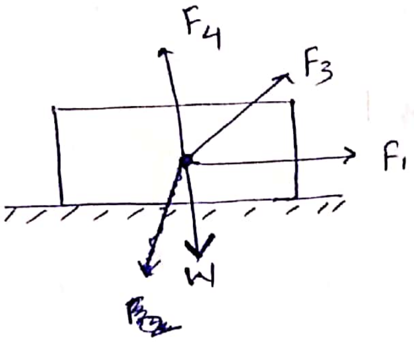
- neither Concurrent nor parallel

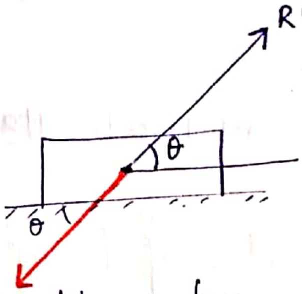


- neither concurrent
- nor parallel

Non-coplanar :- - Kicking of football (not rolling on ground)

Equilibrium :-





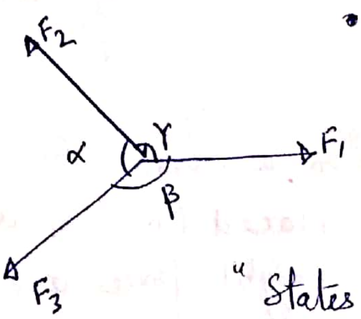
- Resultant acts as a disturbing force
- Resultant causes unbalance in the system

Equilibrant force

- cancels the effect of resultant force
- makes the system Stable

Lami's Theorem :-

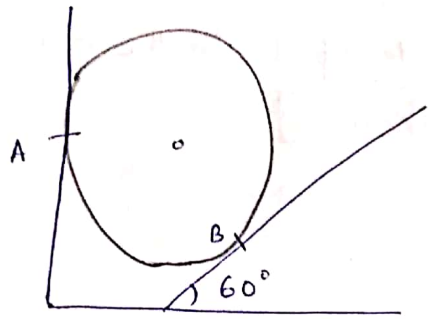
- Body is in equilibrium
- 3 forces are acting [Concurrent]



$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

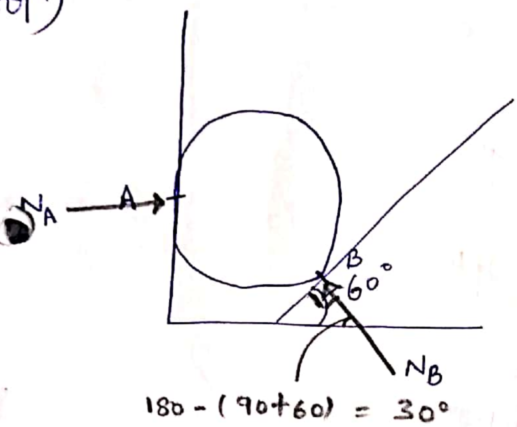
"States that for three Concurrent Coplanar forces acting on a body, each force is proportional to the Sine of angle b/w the other two forces"

(Q)



A smooth sphere weighing 1000 N is resting in a through. Determine the reactions at contact points. [Lami's Theorem]

(Sol^n)

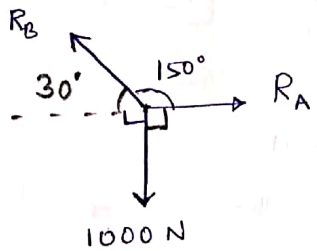


$R_A, R_B = ?$

Imp

Trick:- Determination of angle is the most important thing)

FBD



○ 3 concurrent forces

→ Applying Lami's Theorem :-

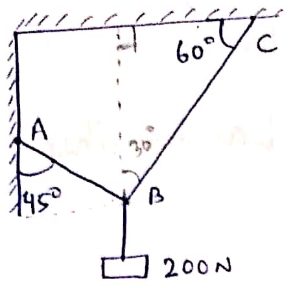
$$\frac{W}{\sin 150^\circ} = \frac{R_A}{\sin 120^\circ} = \frac{R_B}{\sin 90^\circ}$$

or, $\boxed{R_A = 1732.05 \text{ N}}$

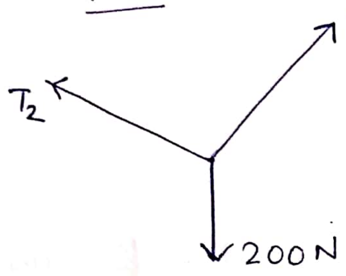
$\boxed{R_B = 2000 \text{ N}}$ } (Ans)

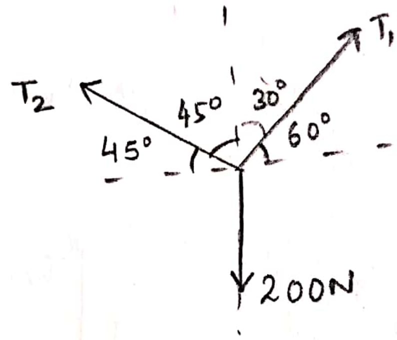
(Q) Find the tension developed in the wire.

[Lami's Theorem]



(Sol^n) FBD





Applying Lami's Theorem:-

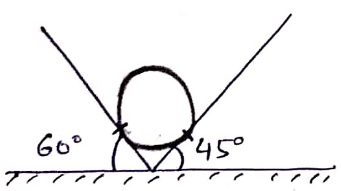
$$\frac{200}{\sin 75^\circ} = \frac{T_1}{\sin 135^\circ} = \frac{T_2}{\sin 150^\circ}$$

or, $\boxed{T_1 = 146.4\ N}$ & $\boxed{T_2 = 103.5\ N}$ (Ans)

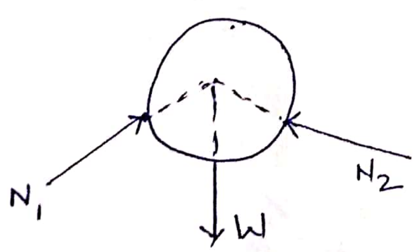
(Q) $W_{sphere} = 300\ N$

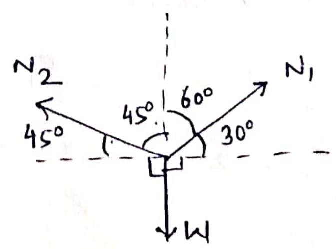
Determine the reactions developed at contact surface

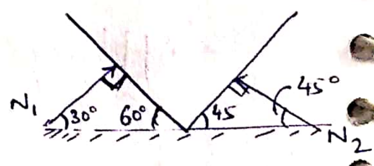
[Lami's Theorem]



(Sol^n) FBD







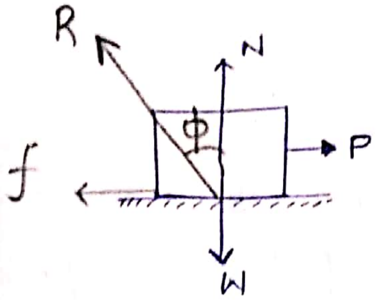
Applying Lami's Theorem:-

$$\frac{W}{\sin 105^\circ} = \frac{N_1}{\sin 135^\circ} = \frac{N_2}{\sin 120^\circ}$$

$N_1 = 2$[illegible]$9\ N$
$N_2 = 269\ N$

(Ans)

Friction

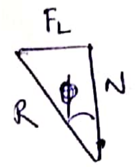


$f = \mu N$

- Contact surfaces

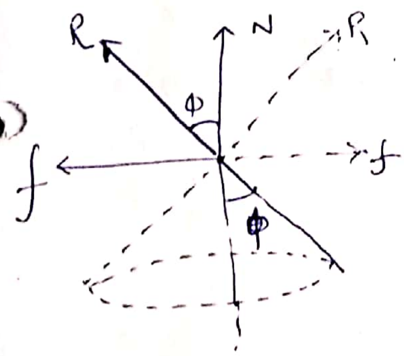
- Limiting friction force: max^m frictional force exerted at the time of impending motion. (i.e. when the motion is about to begin)

- Angle of friction (ϕ):- angle b/w normal reaction force and resultant (R) of normal reaction force and friction force when ($F_L = \mu_s N$) (Just about to move)

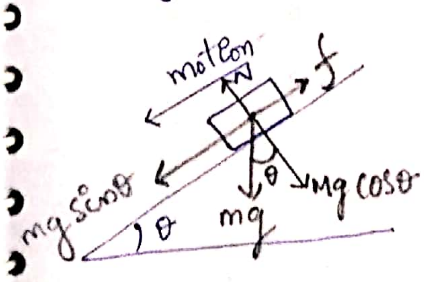


$\tan\phi = F_L/N = \mu_s N/N = \mu_s$

- Cone of friction :- If the resultant reaction is rotated about normal reaction force, it will form a cone, which is known as cone of friction.



- Angle of repose (θ):- when a body is about to slide down on an inclined plane due to its own weight, then the angle made by the plane with the horizonatal is known as Angle of Repose.



- $f = mg\sin\theta$ —— ①

$N = mg\cos\theta$ —— ②

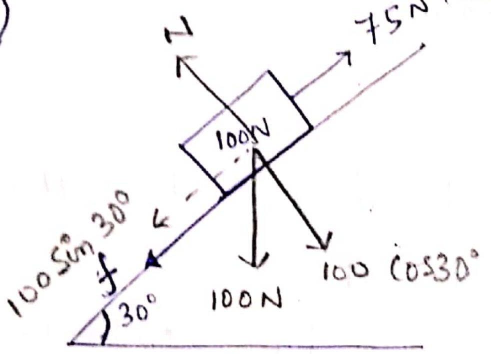
$\tan\theta = \frac{f}{N}$

or, $\mu_s = \tan\theta \left\{ = \frac{f}{N} \right. \Rightarrow \boxed{f = \mu N}$

or, $\boxed{\tan\phi = \tan\theta}$

- The maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by means of friction only.

(Q)



$\mu = ?$

(Sol^n) • Considering the block to be in limiting equilibrium;-

$\sum F_x = 0$ ($\rightarrow$ (+) & $\leftarrow$ (-))

$75 - 100\sin 30° - f(\mu N) = 0$ or, $75 - 100\sin 30° = \mu N$

or, $\mu = \frac{25}{N}$ —— (1)

$\sum F_y = 0$ ($\uparrow +$, & $\downarrow -$)

$N - 100\cos 30° = 0$ —— (2)

Substitute eq^n (2) in eq^n (1):

$\boxed{\mu = 0.288}$ (Ans)

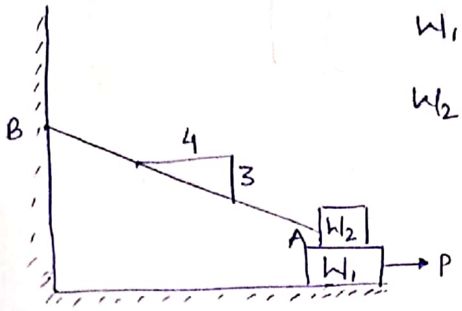
(Q)

$\mu = 0.3$ (for all contacting surfaces)

$W_1 = 1000$ N

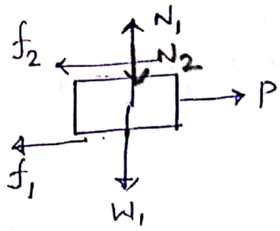
$W_2 = 250$ N

$P = ?$ (to cause slipping to impend)

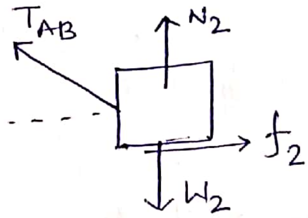


(Sol^n)

FBD for block W_1 :-



FBD for block W_2 :-



Friction :-
- oppose relative motion
- acts tangentially along the contact

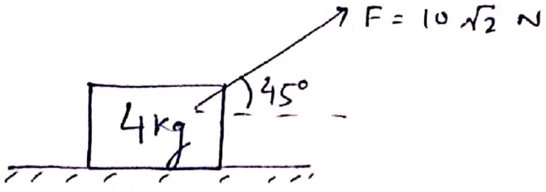
Static friction : when tendency of relative motion is there (Rest)

Kinetic friction : Relative motion is there (motion)

Kinetic friction :-
- Relative motion
- f_K = Constant
- $f_K \propto N$

  $f_K = \mu_K N$ (μ_K coefficient of kinetic friction)
- direction :- opposes relative motion

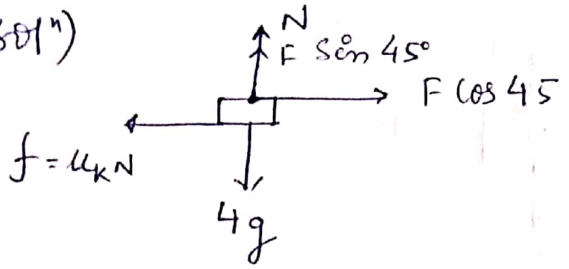
(Q)



$\boxed{N, f_K, a = ?}$

$\mu_K = 0.3$

(Sol^n)



$\Sigma f_y = 0$

$N + F \sin 45° = 4g$

or, $\boxed{N = 30 \text{ N}}$ (Ans)

$f_K = \mu_K N = 0.3 \times 30 = 9$ N (Ans)

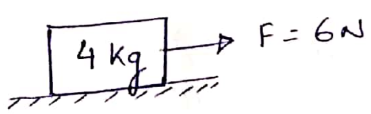
$F \cos 45 - f_K = 4 \times a$; $a = 0.25 \text{ m/s}^2$

Static friction :-
- tendency of relative motion (rest)
- variable (self adjusting)
- $0 \leq f_s \leq f_{limiting}$

f_s (No formula)

Imp
$$\begin{cases} f_{limiting} = \mu_s N \\ f_{kinetic} = \mu_k N \end{cases}$$

(Q)



$f_s = ?$

$\mu_s = 0.2$

(Sol^n) Body moves only if applied force > max^m static friction

$f_{limiting} = \mu_s N = 0.2 \times 4 \times g = 0.2 \times 4 \times 10 = 8$ N (max^m static friction)

applied force (6 N) < max^m static friction [Body is in rest]

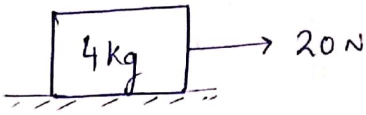
$f = 6$ N (Not 8N) (because body will move backwards)

Scanned with CamScanner

At Rest; $\underset{\text{static}}{\text{friction}} = F_{applied}$

o variable; thats why becomes 6N (not 8N)

(Q.)



find friction force

$\mu_s = 0.6$, $\mu_k = 0.4$

(sol^n)

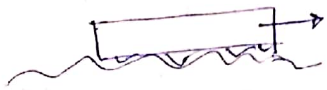
$F_{applied}$ (20N) < Max^m static force
(0.6 × 4 × 10)
(24 N)

o Rest (static friction)

o $f_s = 20$ N

———o———o———

why friction? o irregularities in surfaces

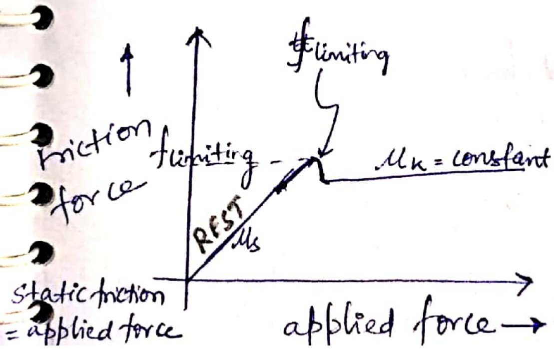


o difficult to start motion

o easy to maintain motion (irregularities breaks down)

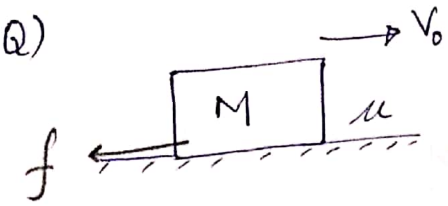
∴ $\boxed{f_{limiting} > F_{kinetic}}$

o just about to move



23.01.2018

(Q)



Find the time after which it stops & also calculate $\vec{S}$ of block.

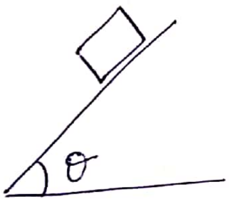
(Sol^n) $u = V_0$

$V = 0$

time 't'

$$a = \frac{F}{M} = -\frac{\mu mg}{m} = -\mu g$$

$V = u + at$ or, $0 = V_0 - (\mu g)t$ or, $t = V_0/\mu g$



$\theta < \theta_{Repose}$

- No sliding
- Rest
- Static friction = applied force

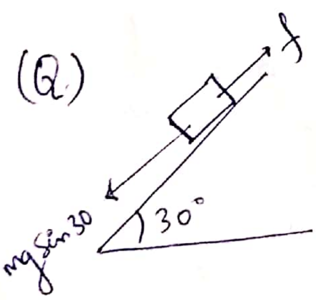
$\theta = \theta_R$

- Just about to slide
- limiting friction
- $f_L = \mu_s N$

$\theta > \theta_R$

- sliding starts
- motion
- kinetic friction
- $f_K = \mu_K \cdot N$

(Q)



$\theta_R = 45°$. Find if body is at rest/motion.

(Sol^n) $\theta < \theta_R$: Rest ; No sliding

static friction = applied force

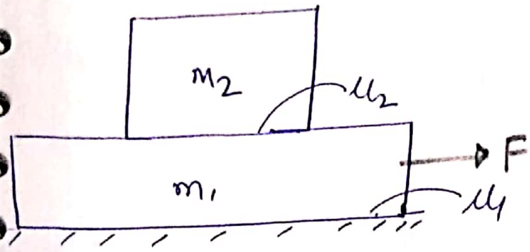
$$f_s = mg \sin 30$$

$$a = 0$$

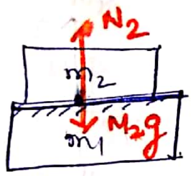
Scanned with CamScanner

Block on Block

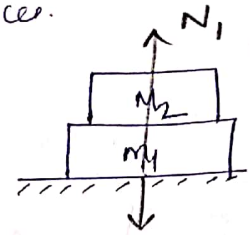
F_{max} ? blocks moves together



1. find $F_{limiting}$ on both surface.



$N_2 = m_2 g$



$N_1 = (m_1 + m_2) g$

$(m_1 + m_2) g$

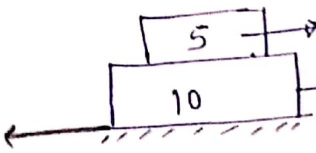
2. Search that block which only has frictional force, find a_{max} of this block

3. Use this a_{max} in second (other) block and find F_{max} for together motion

(Q) Find the max^m value of "F" such that both blocks move together.

(Sol^n)

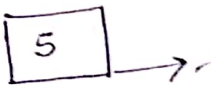
Step 1



$f_{lim} = \mu N = 0.2 \times 5 \times 10 = 10\,N$

$f_{lim} = 0$

Step 2

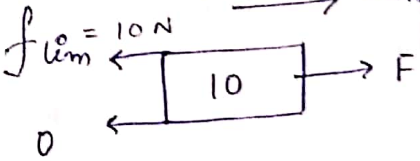


$f_{lim} = 10$

$a_{max} = \frac{10}{5} = 2\ m/s^2$

Step 3

$\longrightarrow a_{max} = 2\,m/s^2$ (to move together)

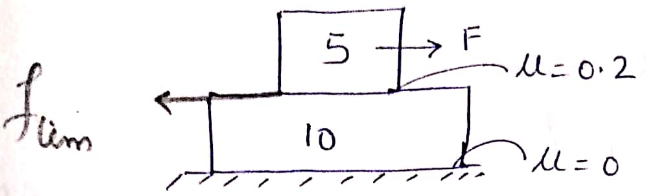


$F - f_{lim} = 10 \times a$

or, $F_{max} = 30\,N$

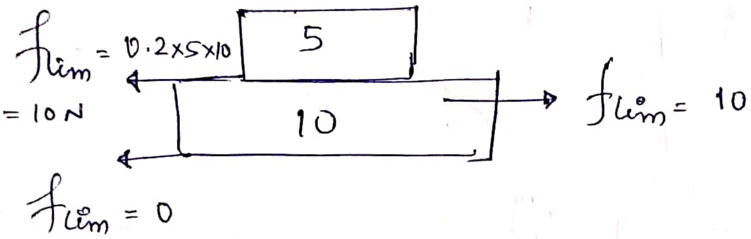
—o—o—

(Q) Find F_{max} so that Both blocks move together. (9)

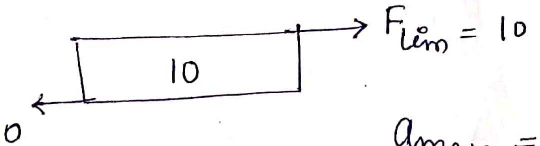


(Sol^n) Friction opposes relative motion.

Step 1

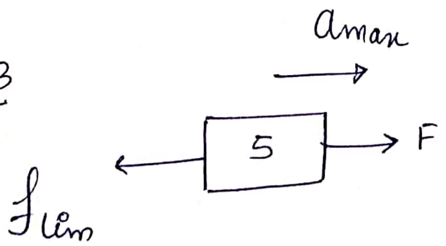


Step 2



$$a_{max} = \frac{F_{lim}}{m} = \frac{10}{10} = 1\,m/s^2$$

Step 3

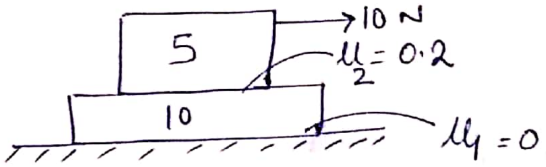


$$F - f_{lim} = m \times a$$

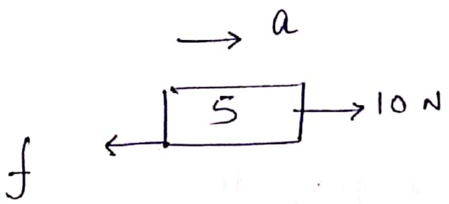
or, $F_{max} - 10 = 5 \times 1$

or, $F_{max} = 15\,N$ (for/to move together)

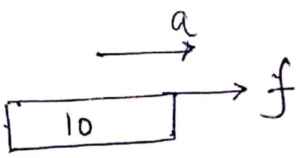
(Q) Find acceleration of both blocks. ~~[Fmax so that both blocks move together.]~~



(sol^n) both moves together → no relative motion → static friction (f_s)
(a_{same})



$$10 - f = 5a \quad \text{— (1)}$$



$$f = 10a \quad \text{— (2)}$$

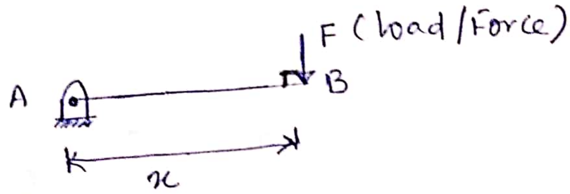
$$\therefore \; a = 10/15 = 2/3 \ m/s^2$$

$$\boxed{\begin{array}{l} F_{max} = 15 \text{ N} \\ \text{to move together} \end{array}}$$

—— o —— o ——

Varignon's theorem of moments :

→ Moment can be as the product of force and perpendicular distance.



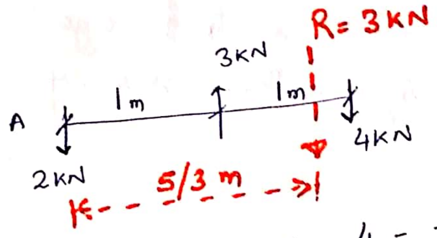
- $M_A = F * x$ (clockwise moment)
- moment "means" rotation
- unit: N-mm ; N-m ; N-cm

M = Force × perpendicular distance

" The sum of moments of all the forces about a point will be equal to the moment produced by the resultant of forces" about that point

$$\sum M_A = R \times x$$

Ex:-



- Parallel force system

$F_R = R = \Sigma F_y = -2+3-4 = -3\ KN = 3KN (\downarrow)$

$(\uparrow +, \downarrow -)$

By varignon's theorem of moments :

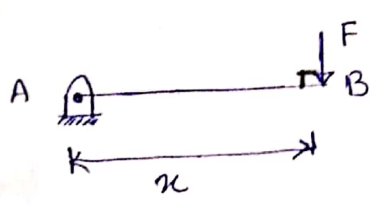
$$\sum M_A = R \times x$$

or, $-3 \times 1 + 4 \times 2 = -3 \times x$

or, $x = \frac{5}{3}$ m (Ans) w.r.t. point A

Varignon's theorem of moments :-

- Moment can be as the product of force and perpendicular distance.



- $M_A = F * x$ (clockwise moment)
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- unit :- N-mm ; N-m ; N-cm

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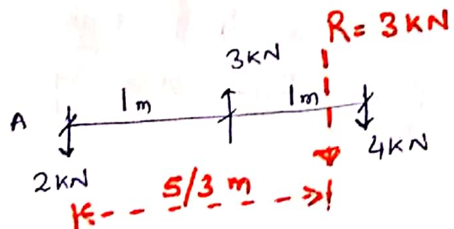
" The sum of moments of all the forces about a point will be equal to the moment produced by the resultant of forces" about that point"

$$\sum M_A = R \times x$$

Ex:-



- Parallel force system

$F_R = R = \Sigma F_y = -2+3-4 = -3\ KN = 3KN (\downarrow)$

$(\uparrow +, \downarrow -)$

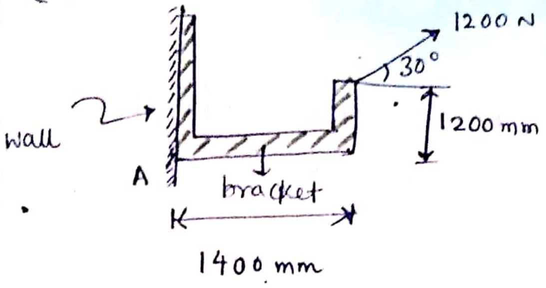
By varignon's theorem of moments :-

$$\Sigma M_A = R \times x$$

or, $\quad -3\times1 + 4\times2 = -3\times x$

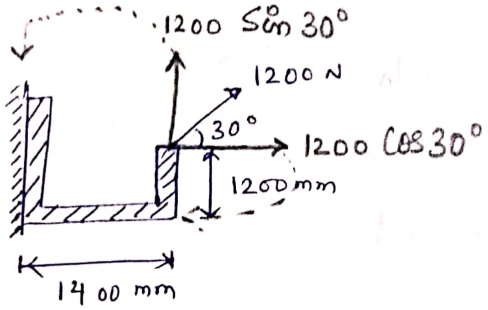
or, $\quad x = \frac{5}{3}$ m (Ans) w.r.t. point A

(Q) Coplanar - non-concurrent Force system



Find the moment of this force about point A.

(Sol^n)



Taking moments of components of 1200 N about point 'A',

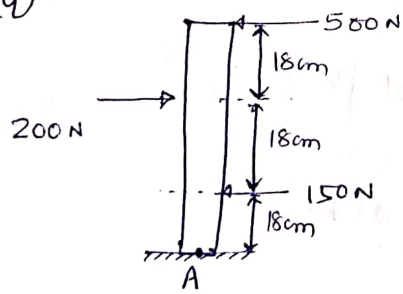
$M_A = -1200 \sin 30° * 1400 + 1200 \cos 30° * 1200$

$\boxed{M_A = \cancel{1163.076} \text{ N-m}}$ (Ans)

$M_A \simeq 407$ N-m

---o---o---

(Q)



Calculate the total moment about point 'A' for the force system :-

(Sol^n) Taking moments of all the forces about point 'A',

$M_A = (\circlearrowright + \text{ and } \circlearrowleft -)$

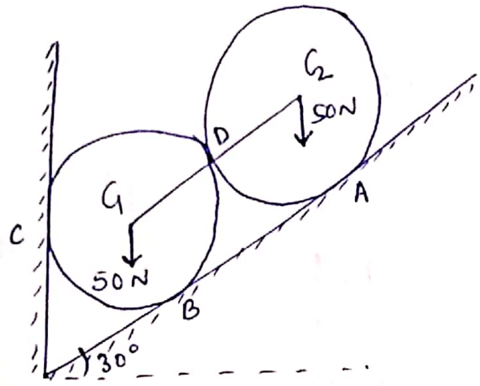
$= (-150 \times 18) + (200 \times 36) + (-500 \times 54)$

$\boxed{M_A = -22500 \text{ N-cm}}$

(Ans)

Equilibrium and FBD ;

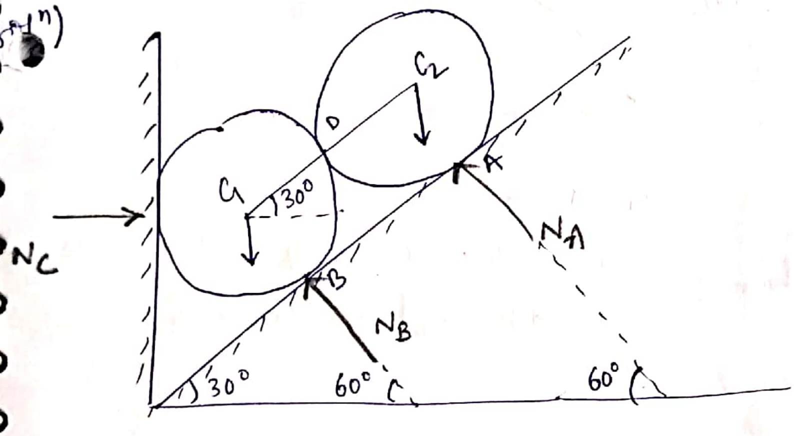
(Q)



Weigh of each rollers (circular) 50N

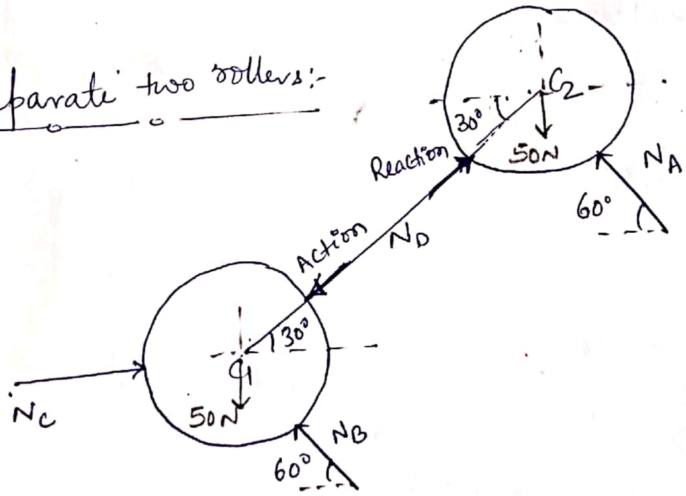
Find the reaction at the points of supports A, B and C.

(Sol^n)

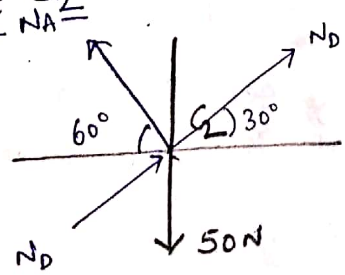


$G_1 G_2 \parallel AB$

Separate two rollers :-



FBD about C_2

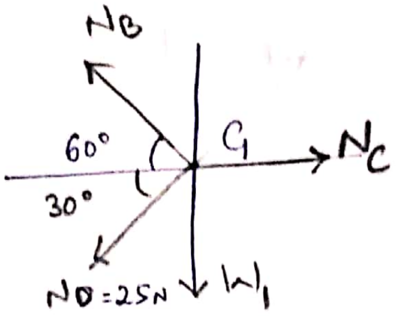


Applying Lami's theorem ;-

$$\frac{50}{\sin 90} = \frac{N_A}{\sin 120^\circ} = \frac{N_D}{\sin 150^\circ}$$

$N_A = 43.03\,N$, $N_D = 25\,N$

FBD about G :



Applying lami's theorem:-

$\sum F_x = 0$

$N_C - N_B \cos 60 - 25 \cos 30 = 0$ —— ①

$\sum F_y = 0$

$N_B \sin 60 - 50 - 25 \sin 30 = 0$ —— ②

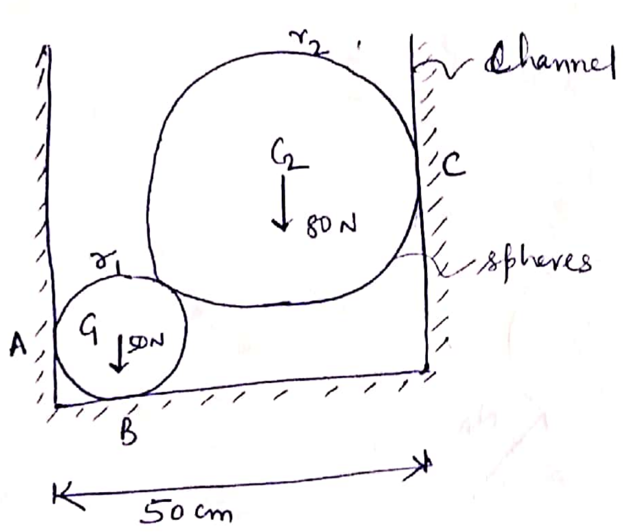
$N_B = 72.16\ N$

$N_C = 57.73\ N$

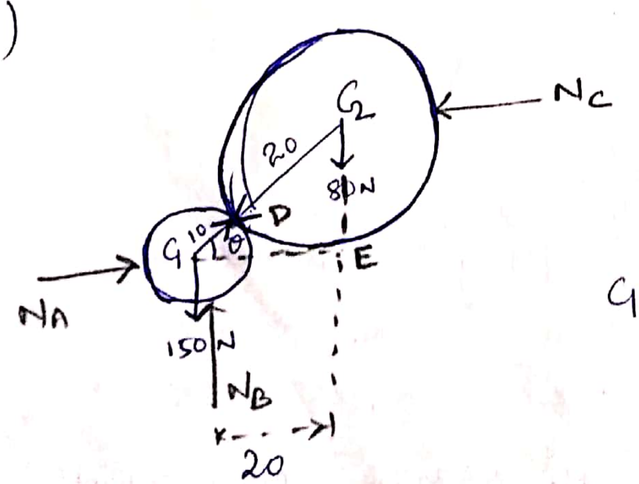
(Q)

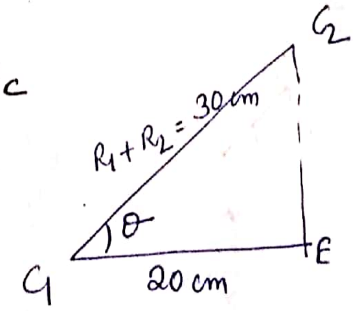
$r_1 = 10\ cm$ $r_2 = 20\ cm$

find the reactions to the walls and base of the channel.

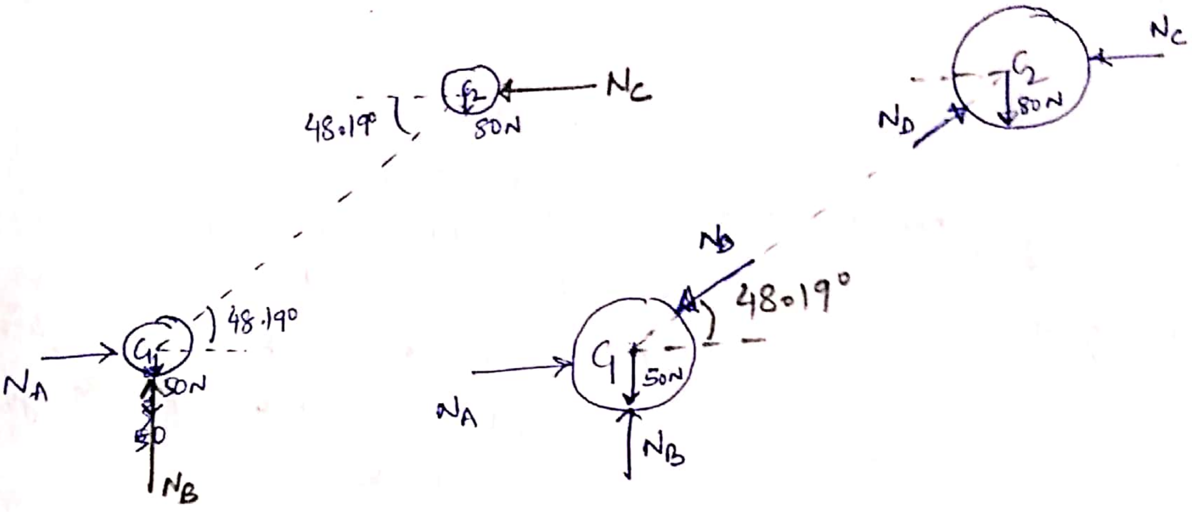


(sol^n)



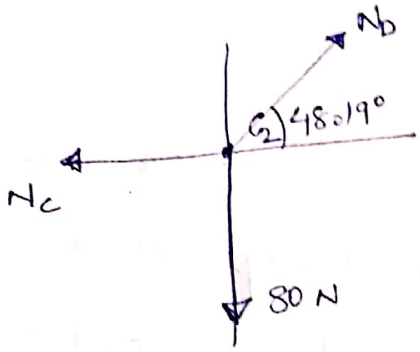


$\cos\theta = \frac{20}{30} \Rightarrow \theta = 48.19°$



Considering FBD at G_2 :-

Applying Lami's Theorem:-

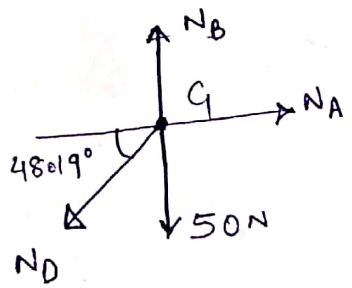


$$\frac{80}{\sin 131.81^\circ} = \frac{N_D}{\sin 90^\circ} = \frac{N_C}{\sin 138.19^\circ}$$

or, $\boxed{N_D = 107.33\ N}$

$\boxed{N_C = 71.55\ N}$ (Ans)

Considering FBD at G_1 :-



$$\Sigma F_x = 0$$

$$N_A - N_D \cos 48.19^\circ = 0 \quad \text{--- (1)}$$

$$\Sigma F_y = 0$$

$$N_B - 50 - N_D \sin 48.19^\circ = 0 \quad \text{--- (2)}$$

$\boxed{N_A = 71.55\ N}$

$\boxed{N_B = 130\ N}$ (Ans)

Cross-check by $\Sigma F_x = 0$ & $\Sigma F_y = 0$

Types of forces

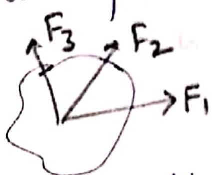
1. External forces
   - a) Normal Reaction } Newton's IIIrd law
   - b) Contact forces }
   - c) friction forces

2. Internal forces
   - a) Tension
   - b) Compression

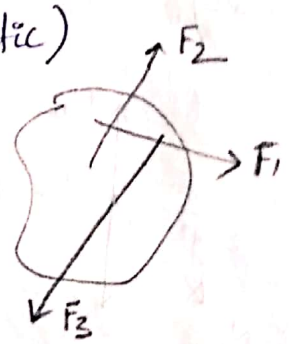
- All flexible members are always in Tension. Ex:- wire, cable, rope, string, belt

FBD: A FBD is a simplified representation of a particle or rigid body that is isolated from its surrounding & on which all applied forces and reactions are shown.

- Conditions of equilibrium for concurrent coplaner force systems are $\sum f_x = 0$, $\sum f_y = 0$ (Static)
  - No translation



- conditions of equilibrium for non-concurrent coplaner force systems are $\sum f_x = 0$, $\sum f_y = 0$, & $\sum M = 0$ (Static)
  - no translation
  - no Rotation



Introduction :

- Curosity to cenderstand the envirooment and the laws that govern it.
- why objects fall onto the ground?
- what happened when you push an object?
- If an object is in motion; than how fast? OR how long it would take to reach its destination?

For objects at rest :- $F_{net} = 0$

For objects in motion :- $F_{net} = ma$

Laws of motion

Impulse

momentum

Friction

→

Velocity

acceleration

Dir^n of motion

Frictional force

" Engineering mechanics can be defined as the branch of (Physics) Science which deals with body in rest/motion under the action of forces"

Aristotle — Archimedes — Christ era — Newton — Einstein

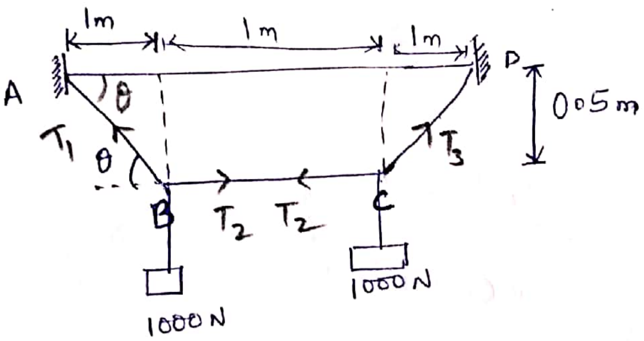
384-322 BC — 287-212 BC — — 1642-1727 AD — 1905 AD

Speed →

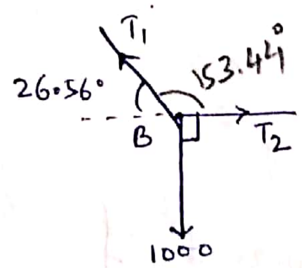
Size ↑	$< 3\times10^{8}$ m/s	$> 3\times10^{8}$ m/s
$> 10^{-9}$ m	Classical mechanics ✓	Relativistic mechanics
$\lesssim 10^{-9}$ m	Quantum mechanics	Quantum field theory

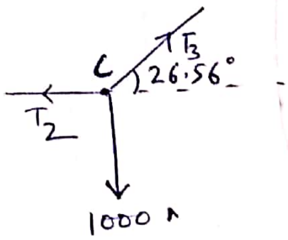
—o—o—

(Q) Calculate tensions in the portions AB, BC, & CD of the string



(Sol^n) $\tan\theta = 0.5/1 \Rightarrow \theta = 26.56°$





Applying Lami's theorem:

$$\frac{T_1}{\sin 90°} = \frac{T_2}{\sin 116.56°} = \frac{1000}{\sin 153°.44}$$

$$\left.\begin{array}{l}\boxed{T_1 = 2236.46\text{ N}} \\ \boxed{T_2 = 2000.44\text{ N}}\end{array}\right\} \text{(Ans)}$$

$$\frac{T_2}{\sin 116.56°} = \frac{T_3}{\sin 90°} = \frac{1000}{\sin 153°.44°}$$

or, $\boxed{T_3 = 2236.46\text{ N}}$ (Ans)

—o—o—

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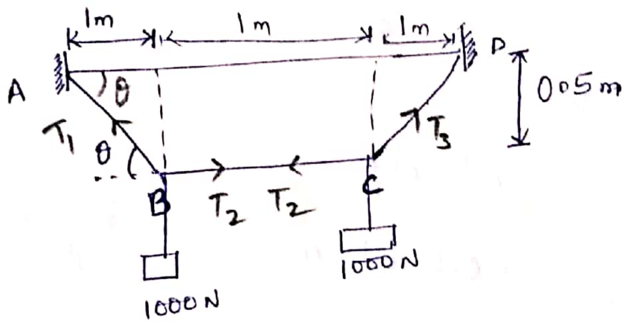
Speed →

Size ↑

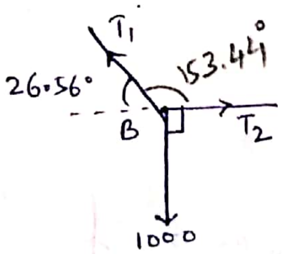
	$< 3\times10^8$ m/s	$> 3\times10^8$ m/s
$> 10^{-9}$ m	Classical mechanics ✓	Relativistic mechanics
$\leq 10^{-9}$ m	Quantum mechanics	Quantum field theory

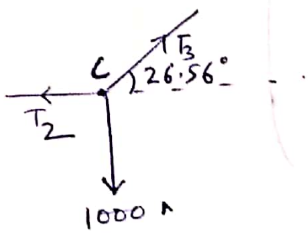
—o—o—

(Q) 

Calculate tensions in the portions AB, BC, & CD of the string

(Sol^n) $\tan\theta = 0.5/1 \Rightarrow \theta = 26.56°$





Applying Lami's theorem:

$$\frac{T_1}{\sin 90°} = \frac{T_2}{\sin 116.56°} = \frac{1000}{\sin 153°.44'}$$

$$\left.\begin{array}{l}\boxed{T_1 = 2236.46\ N} \\ \boxed{T_2 = 2000.44\ N}\end{array}\right\} \text{(Ans)}$$

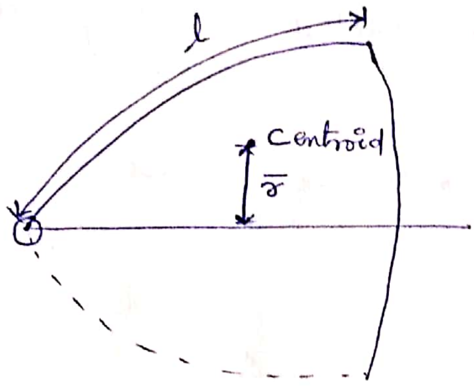
$$\frac{T_2}{\sin 116.560} = \frac{T_3}{\sin 90°} = \frac{1000}{\sin 153°.44°}$$

or, $\boxed{T_3 = 2236.46\ N}$ (Ans)

—o—o—

Theorem of Pappus and Guldinus:

Surface area (Theorem I)



$$A = \theta \bar{r} L$$

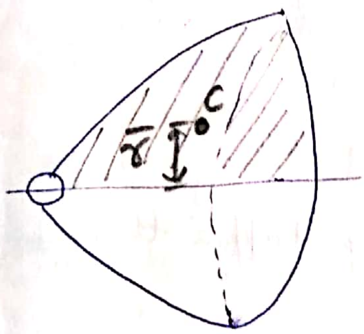
A = Surface area of revolution

θ = angle of revolution, measured in radians $\theta \leq 2\pi$

$\bar{r}$ = $\perp^{r}$ distance from axis of revolution to the centre

L = length of the generating curve

Volume (Theorem II)



$$V = \theta \bar{r} A$$

V = volume of the revolution

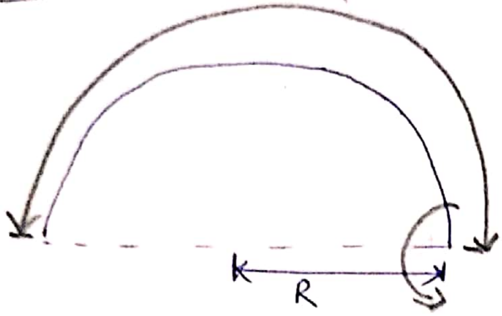
A = generating area

Composite area

$$A = \theta \sum(\bar{r} L)$$

$$V = \theta \sum(\bar{r} A)$$

Ex:- $L = \pi R$ Ring



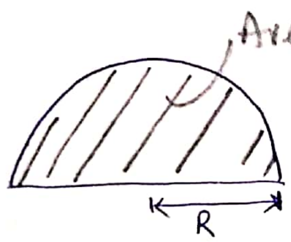
θ
$\uparrow$
Area of revolution $= 2\pi \times$ (length of arc) $\times \bar{y}_{COM}$

- sphere will be generated when Ring is rotated

or, $4\pi R^2 = 2\pi \times \pi R \times \bar{y}_{COM}$ → centre of mass

or, $\bar{y}_{COM} = \dfrac{2R}{\pi}$

Ex:- Disc



volume of revolution $= 2\pi \times (\text{Area}) \times \bar{y}_{COM}$

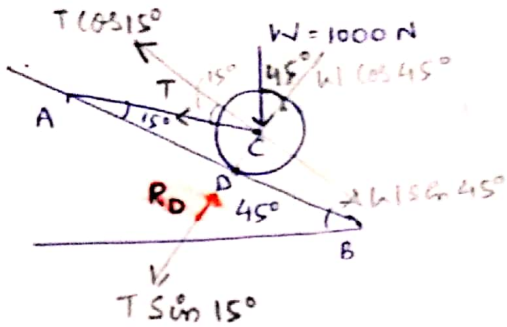
or, $\dfrac{4}{3}\pi R^3 = 2\pi \times \dfrac{\pi R^2}{2} \times \bar{y}_{COM}$

or, $\bar{y}_{COM} = \dfrac{4R}{3\pi}$

- COM w/o calculus can be obtained by using Pappus theorem

—o—o—

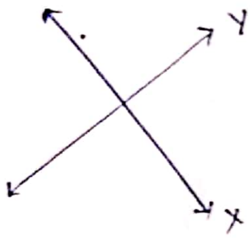
(Q)



Find the tension in the string and the reaction at the point of contact D.

[Forces on a plane]

(Sol^n)



$\sum F_x = 0$

or, $1000 \sin 45° - T \cos 15° = 0$

or, $\boxed{T = 732\,N}$ (Ans)

$\sum F_y = 0$

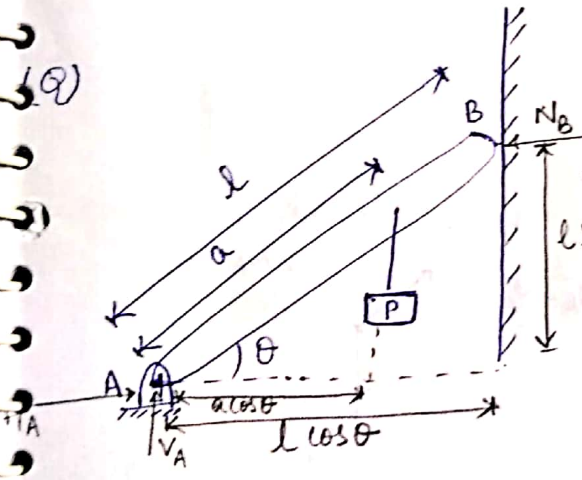
or, $R_D - 1000 \cos 45° - T \sin 15° = 0$

or, $\boxed{R_D = 896.5\,N}$ (Ans)

(Q)

A prismatic bar AB of negligible weight is hinged at 'A' ~~with~~ the end B resting against a vertical wall. Find the reaction R_B (N_B)

[Forces on a plane]

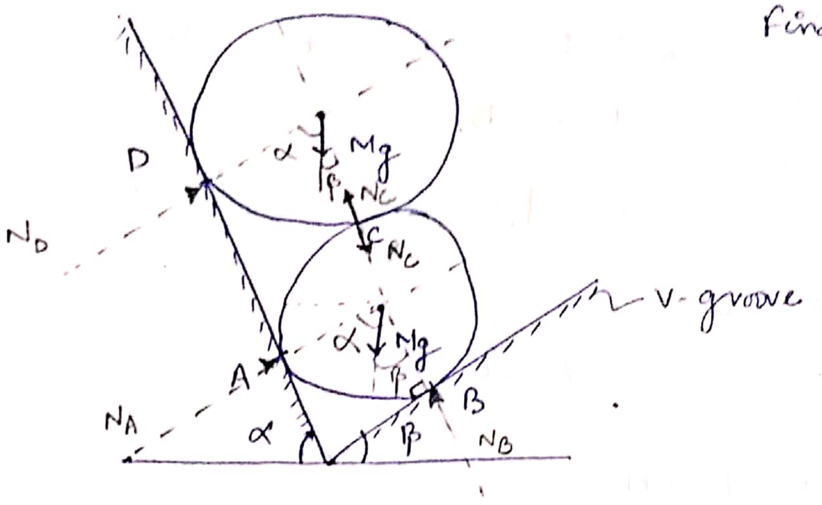


(Sol^n) $\sum M_A = 0$

$-R_B \times l \sin\theta + P \times a \cos\theta = 0$

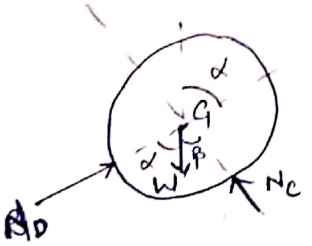
or, $R_B = \dfrac{P \times a \times \cos\theta}{l \sin\theta} = \dfrac{Pa}{l} \times \cot\theta$ (Ans)

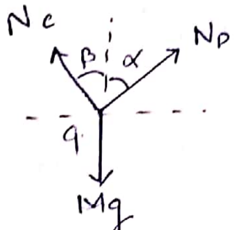
(Q)



Find the reaction N_B.

(Sol^n) Separate the two spheres:-

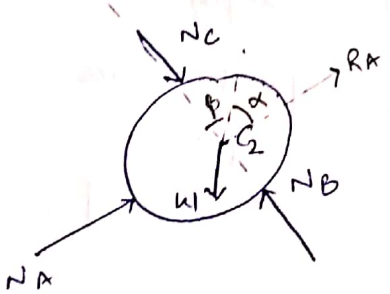




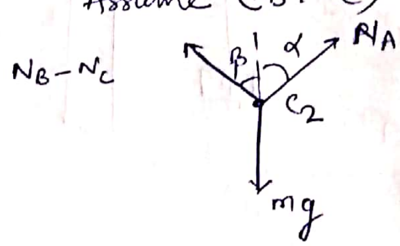
Applying Lami's theorem:-

$$\frac{mg}{\sin(\alpha+\beta)} = \frac{N_D}{\sin(180-\beta)} = \frac{N_C}{\sin(180-\alpha)}$$

or, $$\frac{mg}{\sin(\alpha+\beta)} = \frac{N_D}{\sin\beta} = \frac{N_C}{\sin\alpha}$$



Assume ($R_B > R_C$)



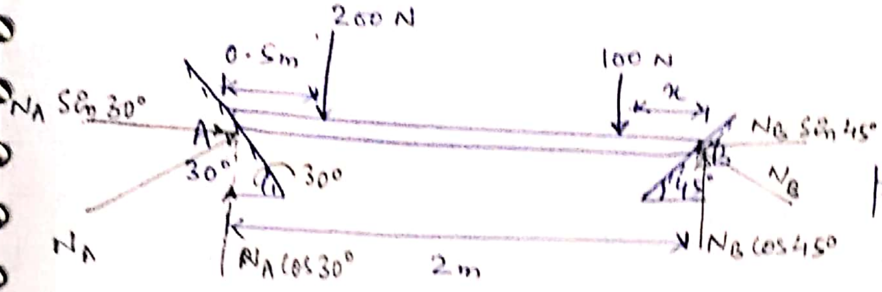
Applying Lami's theorem:-

$$\frac{mg}{\sin(\alpha+\beta)} = \frac{N_B - N_C}{\sin(180-\alpha)} = \frac{N_A}{\sin(180-\beta)}$$

or, $$\frac{mg}{\sin(\alpha+\beta)} = \frac{N_B - N_C}{\sin\alpha} = \frac{N_A}{\sin\beta}$$

$$\boxed{R_B = \frac{2mg\sin\alpha}{\sin(\alpha+\beta)}}$$
(Ans)

—o—o—



○ Determine the distance x at which the load 100 N should be placed from point B to keep the bar horizontal.

(Sol^n)

Equation of equilibrium:-

$\sum F_x = 0$, $N_A \sin 30° - N_B \sin 45° = 0$ —— (1)

or, $N_A = \sqrt{2}\, N_B$

$\sum F_y = 0$, $N_A \cos 30° + N_B \cos 45° - 200 - 100 = 0$

or, $N_B = 155.3$ N

$N_A = 219.6$ N

$\sum M_A = 0$

$- N_B \cos 45° \times 2 + 100\,(2 - x) + 200 \times 0.5$

or, $\boxed{x = 0.81 \text{ m}}$

(Ans)

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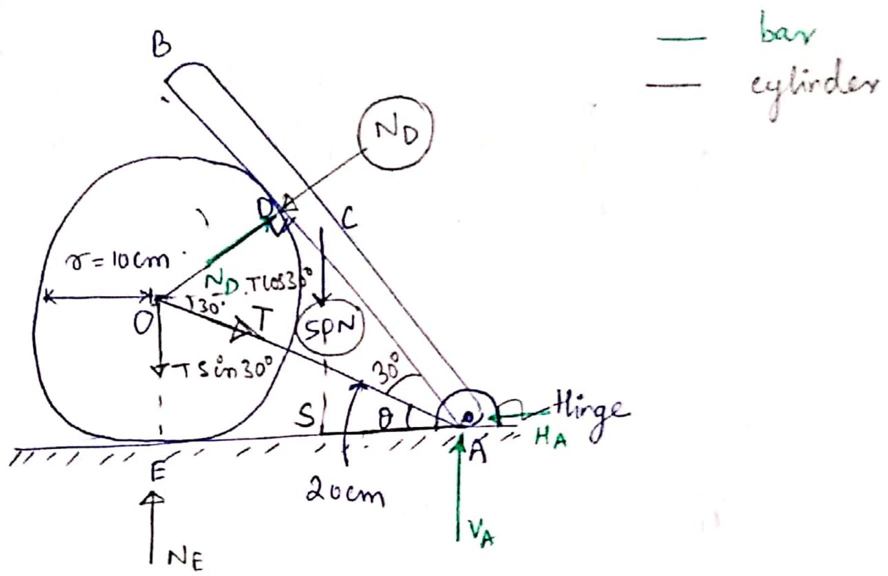
(Q) $l_{AB} = 30\,cm$. find the tension in the string (OA)



(Sol^n) Analyse AB

$\sum M_A = 0$

$$-50 \times AS + N_D \times AD = 0$$

$$\text{or, } N_D = 50\,\frac{AS}{AD} \quad \text{—— (1)}$$

In ΔOEA $\quad \sin\theta = \frac{OE}{OA} \Rightarrow \theta = 30°$

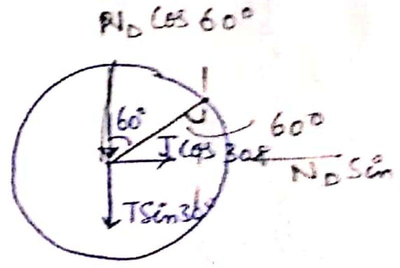
In ΔODA $\quad AD = 20\cos 30°$

In ΔCSA :-

$$CA = \frac{30}{2} = 15\,cm \quad \text{(weight acts at Centre)}$$

$$AS = 15\cos 30° \text{ cm}$$

In eq^n (1) ; $\quad N_D = 21.65\,N$



$\sum F_x = 0$

$$-N_D \sin 60° + T\cos 30° = 0$$

$$\text{or, } \boxed{T = 21.65\,N} \text{ (Ans)}$$

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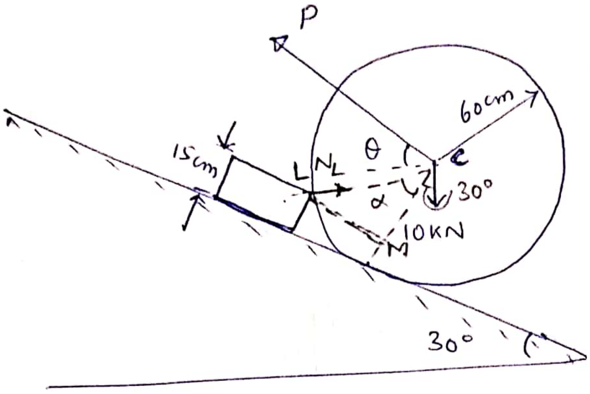
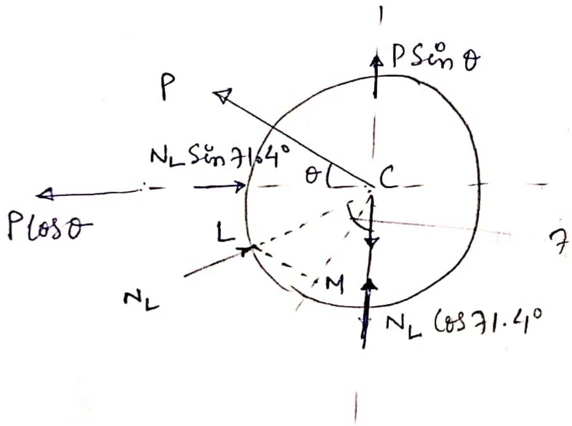
(Q)



Smallest force 'P' required to start the wheel over the block.

(Sol^n) In Δ CLM : $\cos\alpha = \frac{CM}{CL} = \frac{60-15}{60} \Rightarrow \alpha = 41.4°$



Eq^ns of equilibrium:-

$\sum F_x = 0$

$-P\cos\theta + N_L \sin 71.4° = 0$

or, $N_L = \frac{P\cos\theta}{\sin 71.4°}$

$\sum F_y = 0$

$P\sin\theta + N_L\cos 71.4° - 10 = 0$

$P\sin\theta + \frac{P\cos\theta}{\sin 71.4}\cdot\cos 71.4 = 10$

or,

$P[\sin\theta\cdot\sin 71.4 + \cos\theta\cdot\cos 71.4] = 10\sin 71.4$

or, $P = \frac{10\sin 71.4}{\cos(\theta - 71.4)°}$

For P_{min}, when $\cos(\theta - 71.4°)$ is max^m

hence $\theta - 71.4° = 0$

$\theta = 71.4°$

hence, $P_{min} = 10\sin 71.4 = 9.47\ kN$ (Ans)

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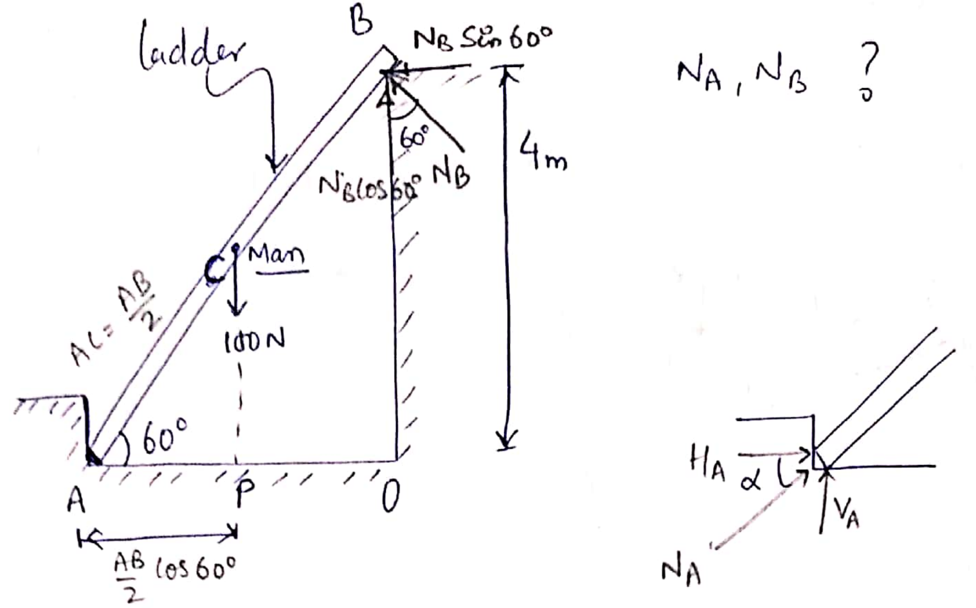
(Q)



N_A, N_B ?

(Sol^n)

$\sum F_x = 0$

$$H_A - N_B \sin 60° = 0$$

$$\text{or, } H_A = N_B \sin 60° \quad \text{--- (1)}$$

$\sum F_y = 0$

$$V_A + N_B \cos 60° - 100 = 0 \quad \text{--- (2)}$$

$\sum M_A = 0$

$$-N_B \times AB + 100 \times \frac{AB}{2} \times \cos 60° = 0 \quad \text{--- (3)}$$

$$\boxed{N_B = 25 \text{ N}} \text{ (Ans)}$$

In eq^n (1),(2) $V_A = 87.5$ N, $H_A = 21.65$ N

$$N_A^2 = V_A^2 + H_A^2$$

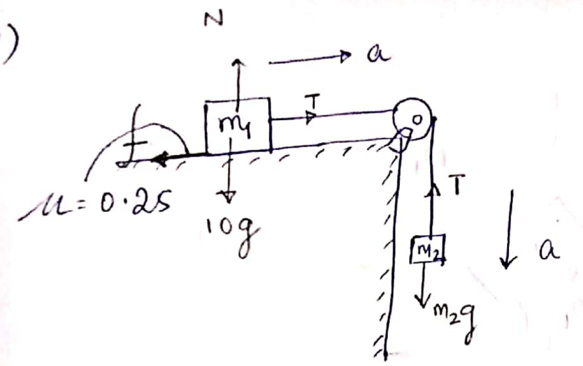
$$\text{or, } \boxed{N_A = 91.13 \text{ N}} \text{ (Ans)}$$

$$\tan\alpha = V_A / H_A$$

$$\boxed{\alpha = 76.1°} \text{ (Ans)}$$

—o—o—

(Q)



$m_1 = 10kg$

$m_2 = 5kg$

$T = ?$ $a = ?$

$g = 9.81 m/s^2$

[Dynamics - Newtons 2nd Law]

(Sol^n) Block M_1 :- $\sum F_y = 0$ $N - 10g = 0$

$N = 10g$ N

$f = \mu N = 0.25 \times 10g = 2.5g$ N

Newton's 2nd law of motion

$\sum F_x = m_1 a$

$T - 2.5g = m_1 \times a$ —— ①

[(Dominating) force is taken as positive]

Block M_2 :-

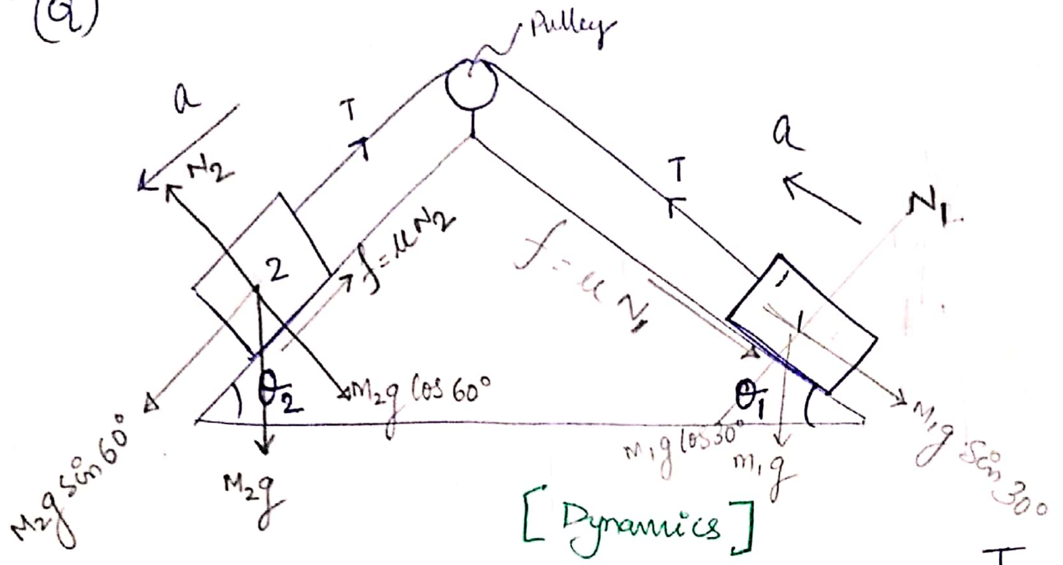
$\sum F_y = m_2 a$

$m_2 g - T = m_2 a$ —— ②

Solving ① & ② ; $T = 40.875$ N
$a = 1.635 m/s^2$ (Ans)

—— o —— o ——

(Q)



[Dynamics]

$m_1 = 5\,kg$

$m_2 = 10\,kg$

$\theta_1 = 30^\circ$

$\theta_2 = 60^\circ$

$\mu = 0.33$

T, a_{m_1}) a_{m_2} ?

(Sol^n) For Block 2

$\sum F_y = 0$, $N_2 - 10g \cos 60^\circ = 0$

$N_2 = 10g \cos 60^\circ$ —— (1)

$\sum F_x = m_2 a.$

$10g \sin 60^\circ - \mu(10g \cos 60^\circ) - T = m_2 a$

or, $10a + T = 68.77$ —— (2)

For Block 1

$\sum F_y = 0$; $N_1 - 5g \cos 30^\circ = 0$

$N_1 = 5g \cos 30^\circ$ —— (3)

$\sum F_x = m_1 a$; $T - 5g \sin 30^\circ - \mu(5g \cos 30^\circ) = m_1 a$;

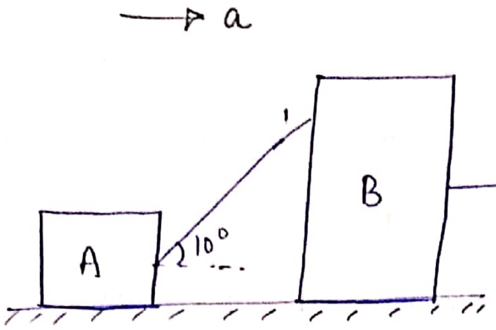
or, $5a - T = -38.54$ —— (4)

Solving (2) & (4)

$\boxed{a = 2.01\ m/s^2}$ (Ans)

$\boxed{T = 48.67\ N}$ (Ans)

(Q)



$P = 100\ N$

$m_A = 5\ kg$

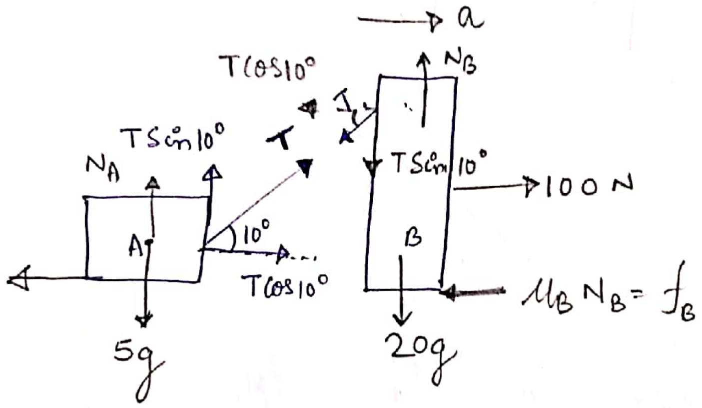
$m_B = 20\ kg$

$\mu_A = 0.5$

$\mu_B = 0.25$

Tension, acceleration ?

(soln)



Block A :-

$\sum F_y = 0$; $N_A + T\sin 10° = 5g$

$\sum F_x = m_A \cdot a$; $T\cos 10° - \mu_A N_A = m_A \cdot a$

or, $T\cos 10° - 0.5[5g - T\sin 10] = 5a$

or, $1.071T - 5a = 24.525$ —— (1)

Block B :-

$\sum F_y = 0$; $N_B - T\sin 10° - 20g = 0$

$\sum F_x = m_B a$; $100 - \mu_B \cdot N_B - T\cos 10° = 20a$

or, $100 - 0.25[T\sin 10° + 20g] - T\cos 10° = 20a$

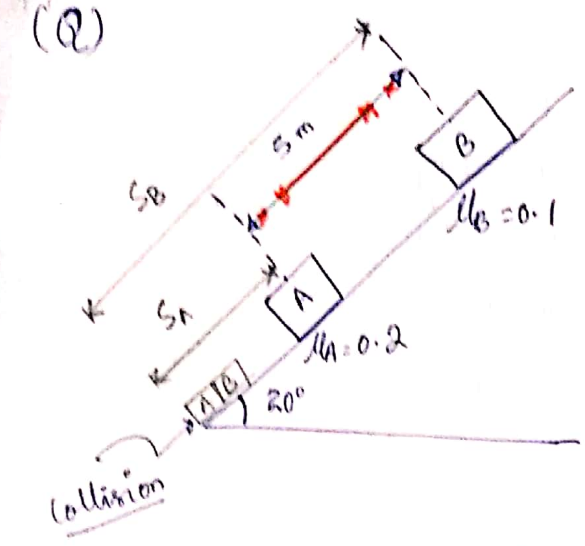
or, $1.028T + 20a = 50.95$ —— (2)

Solving eqn (1) & (2)

$T = 28.06\ N$
$a = 1.105\ m/s^2$

(Ans)

(Q)



If the blocks begin to slide down the plane simultaneously, calculate the time and distance travelled by each block before collision

(Sol^n)

$t_A = t_B = t \quad | \quad S_B = S_A + 5$

Block A :-

$\sum F_y = 0 ; \quad N_A - M_A g \cos 20° = 0$

$N_A = M_A g . \cos 20°$

$\sum F_x = m_A \cdot a_A \quad ; \quad M_A g \sin 20° - 0.2 \times M_A g \times \cos 20° = M_A \times a_A$

or, $a_A = g . \sin 20° - 0.2 g \cos 20° \simeq 1.51 \, m/s^2$

Block B :- $\sum F_y = 0 \quad ; \quad N_B = m_B g \cos 20$

$m_B g \sin 20° - 0.1 \times m_B g \times \cos 20° = m_B \times a_B$

or, $a_B = 2.43 \, m/s^2$

$\boxed{S = \cancel{ut}^{\,0} + \frac{1}{2} a t^2}$

$S_B - S_A = 5 \quad ; \quad \frac{1}{2} a_B t^2 - \frac{1}{2} a_A t^2 = 5$

or, $\boxed{t = 3.3 \text{ Sec}}$ (Ans)

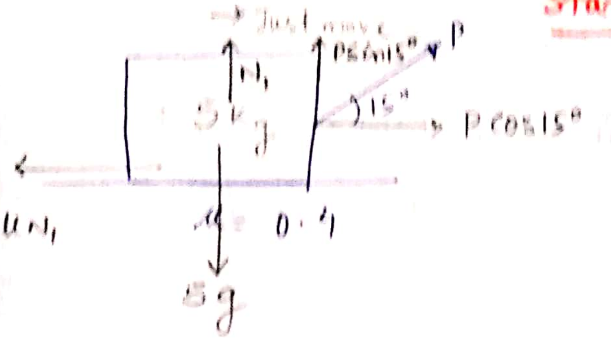
$S_A = \frac{1}{2} a_A t^2 = 8.22 \, m$

$S_B = \frac{1}{2} a_B t^2 = 13.22 \, m$ (Ans)

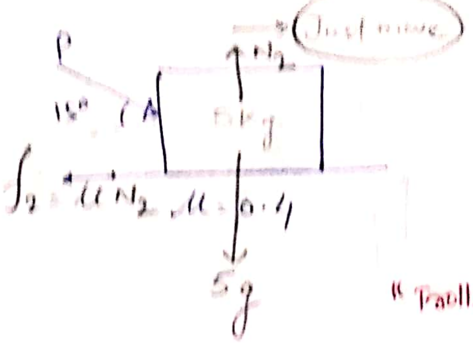
(Q)

Friction
Static

Case I (Pull)



Case II (Push)



"Trolley Bag"

(Sol^n)

$\sum F_x = 0$

$P\cos 15° - \mu N_1 = 0$ —— (1)

$\sum F_y = 0$

$N_1 + P\sin 15° - 5g = 0$ —— (2)

In eq^n (1)

$P = 18.34\,N$

$\sum F_x = 0$

$P\cos 15° - \mu N_2 = 0$ —— (3)

$\sum F_y = 0$

$N_2 - 5g - P\sin 15° = 0$ —— (4)

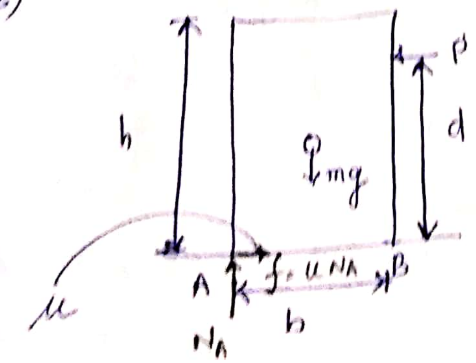
In eq^n (3); $P = 22.8\,N$

"easier to Pull an object than Push"

—o—o—

(Qns)

Highest position for a horizontal force 'P' that would permit to just move the block w/o tilting.



$\sum F_x = 0$, $\mu N_A - P = 0$ —— (1)

$\sum F_y = 0$, $N_A - mg = 0$ —— (2)

$\sum M_A = 0$, $-P\times d + mg \times \frac{b}{2} = 0$ —— (3)

(Sol^n) "Explain by taking the force from bottom of the block"

$d = \frac{Mg \cdot b}{2P}$

From eq^n(1) $P = \mu N_A = \mu mg$

$\boxed{d = b/2\mu}$

—o—o—

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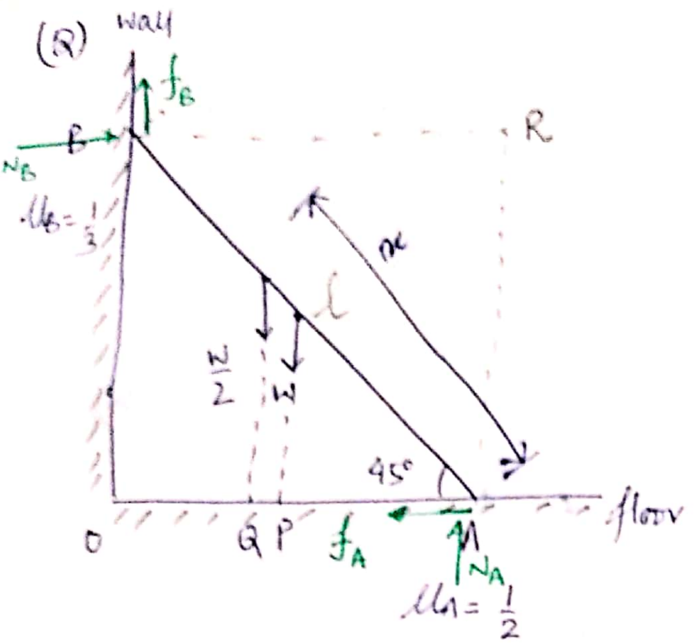
(Q)



$l = 20$ cm

Man $= \frac{W}{2}$

(i) how much length x of the ladder a man shall climb before ladder slips

(sol^n)

$$f_A = \mu_A N_A = \frac{N_A}{2}$$

$$f_B = \mu_B N_B = \frac{N_B}{3}$$

$\sum F_x = 0$; $+N_B - f_A = 0$; $N_B - \frac{N_A}{2} = 0$; $N_B = \frac{N_A}{2}$

$\sum F_y = 0$; $+f_B + N_A - W - \frac{W}{2} = 0$; $N_B = \frac{9W}{14}$

$N_A = \frac{9W}{7}$

$\sum M_A = 0$ [select those point, where more number of forces are acting]

$$-\frac{W}{2} \times AQ - W \times AP + N_B \times AR + f_B \times AO = 0$$

or, $AQ = x \cos 45°$; $AP = \frac{l}{2} \cos 45°$; $AR = OB = 20 \sin 45°$; $AO = 20\cos$

or, $\boxed{x = 14.28 \text{ cm}}$ (Ans)

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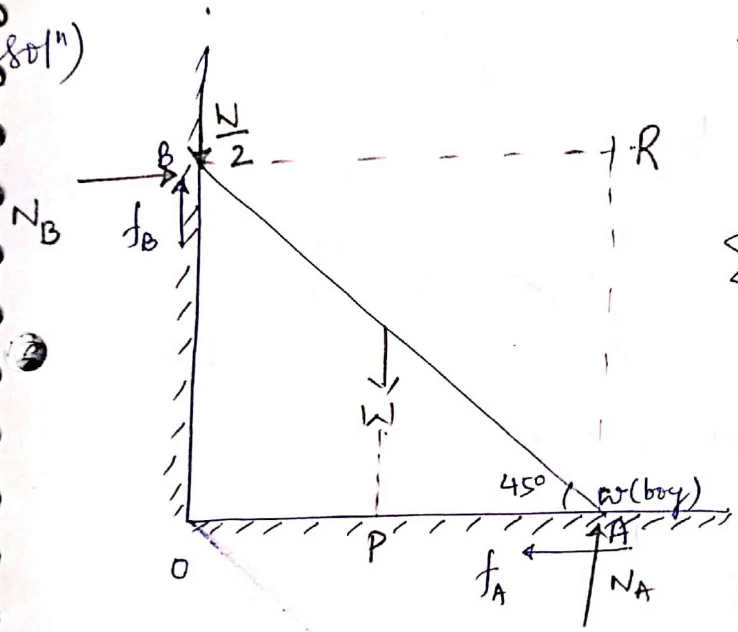
(Q) (iii) If a body now stands on the end 'A' of the ladder, what must be his least weight "w" so that the man may go on the top of the ladder?

(Sol^n)



$\sum F_x = 0,$

$N_B - \frac{N_A}{2} = 0 \ ; \ N_A = 2N_B$

$\sum F_y = 0,$

$-\frac{W}{2} + \frac{N_B}{3} - W - w + N_A = 0$

or, $N_B = \frac{9W}{14} + \frac{3w}{7} \ ; \ N_A = \frac{9W}{7} + \frac{6w}{7}$

$\sum M_A = 0$

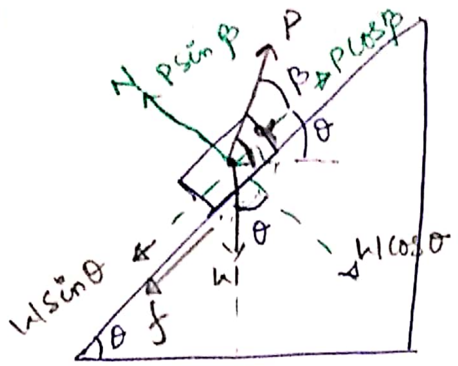
$-\frac{W}{2} \times OA + \frac{N_B}{3} \times OA + N_B \times AR - W \times AP = 0$

$OA = 20 \cos 45° ; \ OB = 20 \sin 45° = AR ; \ AP = \frac{l}{2} \cos 45°$

or, $\boxed{w = \frac{W}{4}}$ (Ans)

———o———o———

(Q)



Smallest force "P" that is enough to move the block up the plane.

(Soln)

$\mu = \tan\phi$ ← angle of friction

$\sum F_x = 0$; $P\cos\beta - W\sin\theta - \mu N = 0$ —— ①

$\sum F_y = 0$; $N + P\sin\beta - W\cos\theta = 0$

or, $N = W\cos\theta - P\sin\beta$ —— ②

From eqn ① $P\cos\beta - W\sin\theta - \mu(W\cos\theta - P\sin\beta) = 0$

or, $P\cos\beta + \mu P\sin\beta = W\sin\theta + \mu W\cos\theta$ [put $\mu = \tan\phi$, $\mu = \frac{\sin\phi}{\cos\phi}$]

or, $P\cos\beta + \frac{\sin\phi}{\cos\phi}\cdot P\sin\beta = W\sin\theta + \frac{\sin\phi}{\cos\phi}\cdot W\cos\theta$.

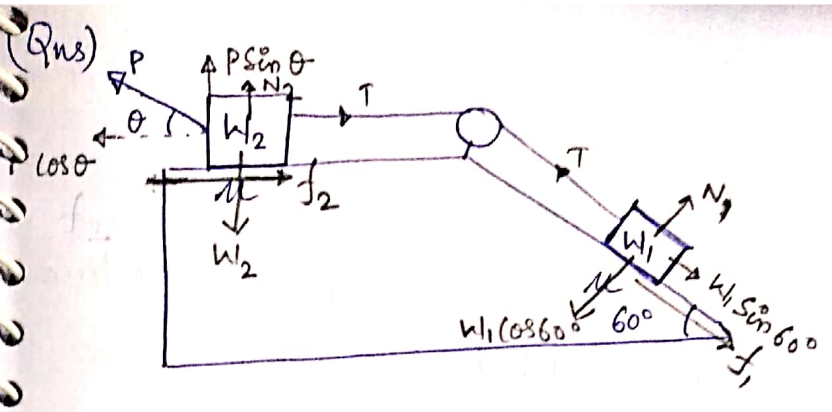
or, $P\left[\frac{\cos\beta\cdot\cos\phi + \sin\beta\cdot\sin\phi}{\cos\phi}\right] = W\left[\frac{\sin\theta\cdot\cos\phi + \sin\phi\cdot\cos\theta}{\cos\phi}\right]$

or, $P\cos(\beta - \phi) = W\sin(\theta + \phi)$

or, $\boxed{P = \frac{W\sin(\theta+\phi)}{\cos(\beta-\phi)}}$

For P_{min}; $\cos(\beta - \phi) = \cos 0$, $\beta = \phi$

$\boxed{P = W\sin(\theta+\phi)}$ (Ans)

(Qns) 

$W_1 = 150N$

$W_2 = 100N$

$\mu = 0.2$

Find the magnitude and direction of the least force 'P' at which the motion of blocks will ~~be~~ impend.

(Soln)

Block 1

$\sum F_y = 0$; $N_1 - 150\cos60° = 0$

$N_1 = 75N$

$\sum F_x = 0$; $T - 150\sin60° - 0.2\times 150\cos60° = 0$

or, $T = 145N$

Block 2

$\sum F_y = 0$; $P\sin\theta + N_2 - 100 = 0$

or, $N_2 = 100 - P\sin\theta$ — ①

$\sum F_x = 0$; $P\cos\theta - T - \overset{\mu}{0.2}\times(\overset{N_2}{100 - P\sin\theta}) = 0$

or, $\mu N_2 = T - P\cos\theta$ — ②

$\frac{eq^n ①}{eq^n ②} \Rightarrow \frac{1}{\mu} = \frac{100 - P\sin\theta}{T - P\cos\theta}$; $\mu = \tan\phi = \frac{\sin\phi}{\cos\phi}$

or, $\frac{\cos\phi}{\sin\phi} = \frac{100 - P\sin\theta}{145 - P\cos\theta} \Rightarrow 145\cos\phi - P\cos\theta\cdot\cos\phi = 100\sin\phi - P\sin\theta\cdot\sin\phi$

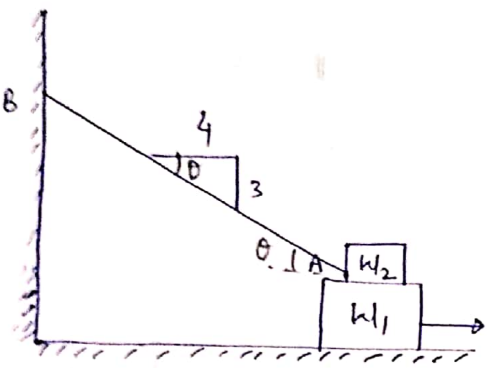
or, $P[\cos\phi\cdot\cos\theta - \sin\theta\cdot\sin\phi] = 145\cos\phi - 100\sin\phi$.

or, $P\cos(\theta+\phi) = 145(5/\sqrt{26}) - 100(1/\sqrt{26})$

or, $\boxed{P = 122N}$ # (Ans)

$\left[\mu = \tan\phi,\ \tan\phi = \frac{1}{5}\right]$ $\sqrt{26}$ 1 5

(Q.)



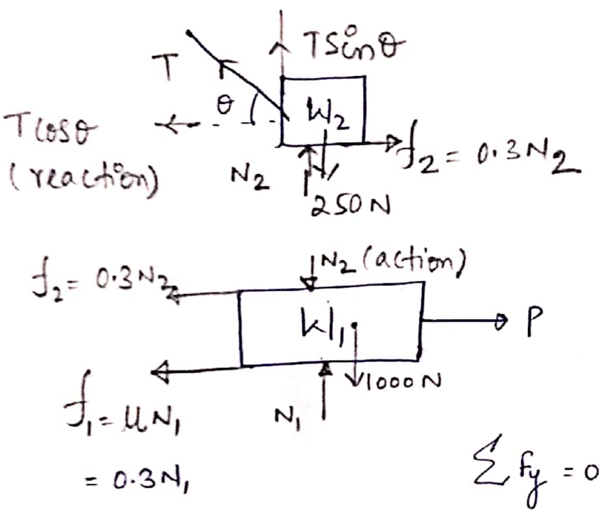
$W_1 = 1000$ N

$W_2 = 250$N

$\mu = 0.3$

Find the magnitude of horizontal force 'P' applied to the lower block to cause slipping to impend.

(Sol^n) FBD



$\tan\theta = 3/4$

Block 1 ; $\sum F_x = 0$

$\oplus$ $P - 0.3N_1 - 0.3N_2 = 0$

or, $P = 0.3(N_1 + N_2)$ —— (1)

$\sum F_y = 0$

$-1000 - N_2 + N_1 = 0$

or, $N_1 = 1000 + N_2$ —— (2)

Block 2 : $\sum F_x = 0$; $0.3N_2 - T\cos\theta = 0$

$T\cos\theta = 0.3N_2$ —— (3)

$\sum F_y = 0$: $T\sin\theta + N_2 - 250 = 0$

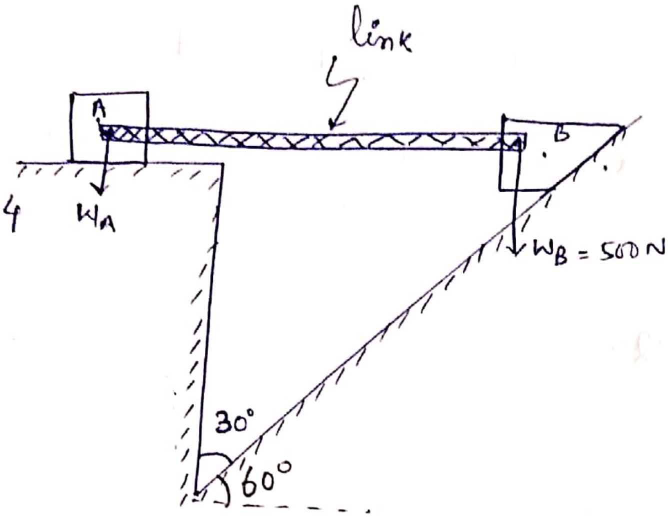
or, $T\sin\theta = 250 - N_2$ —— (4)

eq^n (4) by (3) $\tan\theta = \dfrac{250 - N_2}{0.3N_2} \Rightarrow \dfrac{3}{4} = \dfrac{250 - N_2}{0.3N_2} \Rightarrow N_2 = 204.0$

In eq^n (2) $N_1 = 1204.08$ N

In eq^n (1) $\boxed{P = 422.45\text{N}}$ (Ans)

(3)



Smallest W_A of the block for which equilibrium can exist.

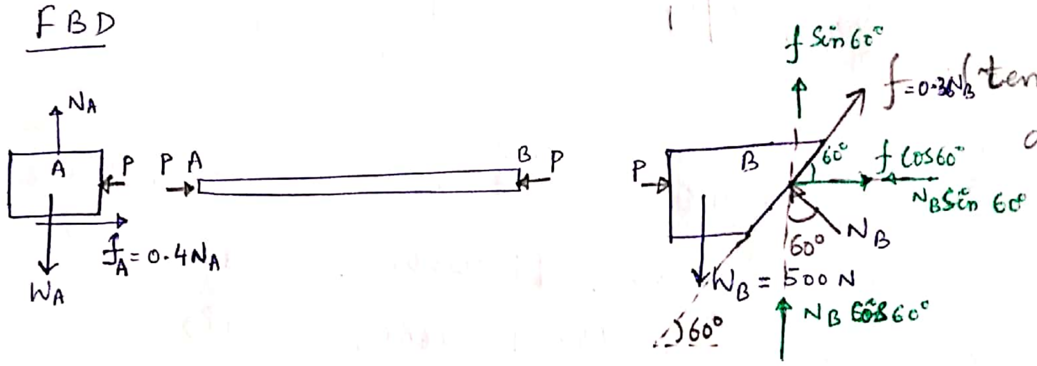
$\phi_B = 20°$ — angle of friction

(Sol^n)

link AB should stay horizontal

$\mu_B = \tan\phi_B = \tan 20° = 0.36$

FBD



Block A : $\sum F_y = 0$; $N_A - W_A = 0$; $W_A = N_A$ — (1)

$\sum F_x = 0$; $-P + 0.4N_A = 0$; $N_A = P/0.4$ — (2)

Block 4 :- $\sum F_x = 0$; $0.36\,N_B\cos 60° - N_B \sin 60° + P = 0$

or, $N_B(0.36\cos 60° - \sin 60°) + P = 0$

or, $P = 0.686\,N_B$ — (3)

$\sum F_y = 0$; $0.36\,N_B\sin 60° + N_B\cos 60° - W = 0$

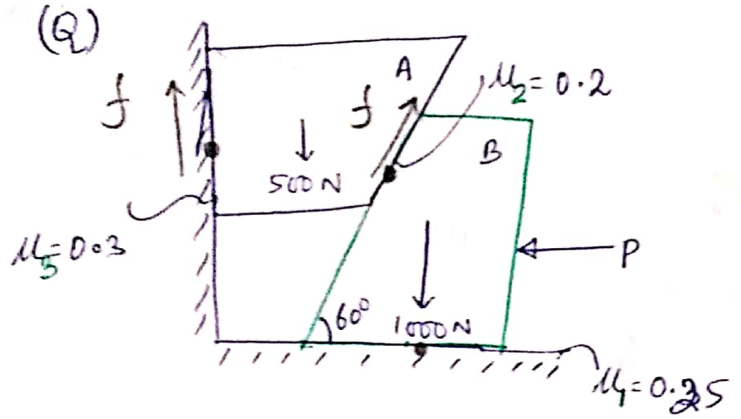
or, $N_B(0.36\sin 60° + \cos 60°) = 500$

or, $N_B = 615.94\,N$

In eq^n (3) $P = 422.5\,N$

In eq^n (2) $\boxed{N_A = 1056\,N} = W_A$ (Ans)

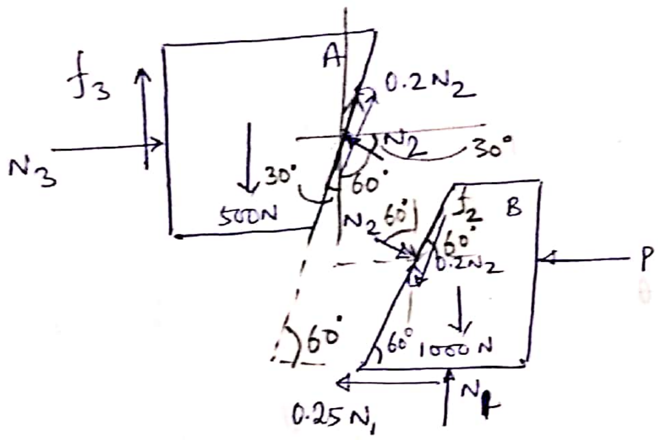
(Q)



Horizontal force "p" to hold the system in equilibrium

(sol^n)

FBD



Block B:- $\sum F_x = 0$; $-0.2N_2\cos 60° - P + N_2\sin 60° - 0.25N_1 = 0$

or, $P = -0.25N_1 - [0.2\cos 60 - \sin 60]N_2$

or, $P = -0.25N_1 + 0.766N_2$ — (1)

$\sum F_y = 0$:- $N_1 - 1000 - \{N_2(\cos 60° + 0.2\sin 60°) = 0$

or, $N_1 - 0.673N_2 = 1000$ — (2)

Block A:-

$\sum F_x = 0$:- $-N_2\cos 30° + N_3 + 0.2N_2\sin 30° = 0$

or, $N_3 - 0.766N_2 = 0$ — (3)

$\sum F_y = 0$; $0.3N_3 + [0.2\cos 30 + \sin 30]N_2 - 500 = 0$

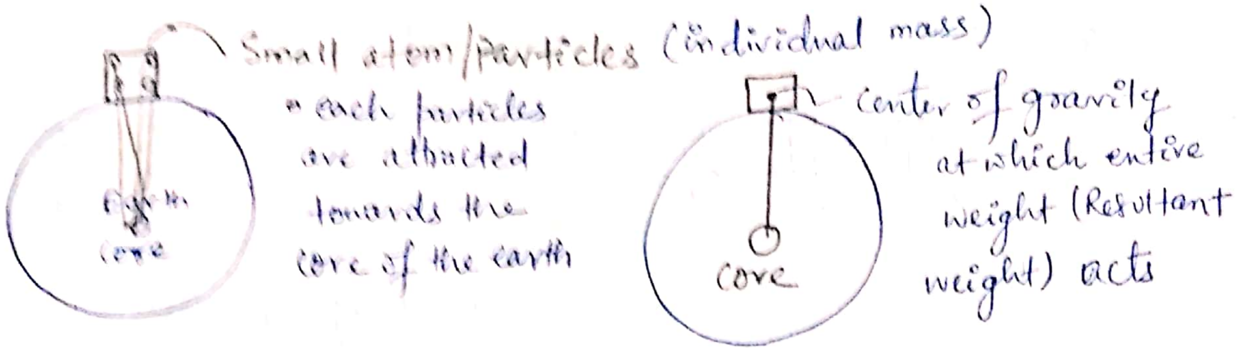
or, $0.3N_3 + 0.673N_2 = 500$ — (4)

Solve (3) & (4) : $N_3 = 424.23$ N ; $N_2 = 553.8$ N

In eq^n (2) $N_1 = 1372$ N ; In eq^n (1) $\boxed{P = 81.034 \text{ N}}$ (Ans)

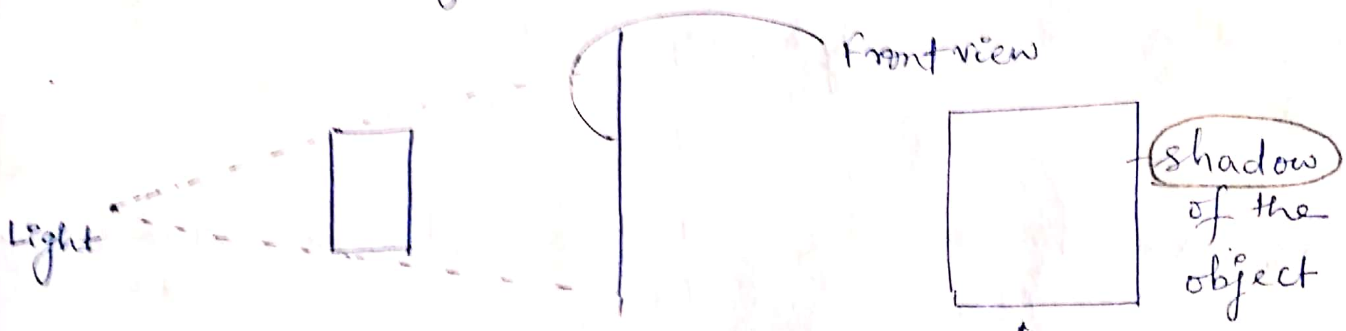
—o—o—

Centroid, Center of mass, Center of gravity



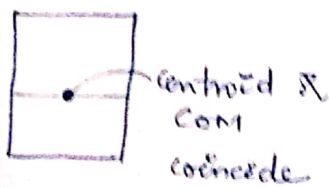
Centroid: geometric center of a body

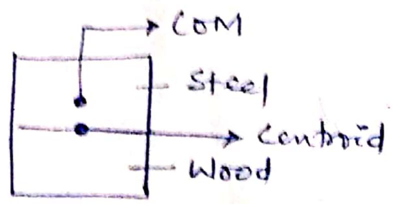
Center of mass: point about which entire mass of the body is assumed to be concentrated.



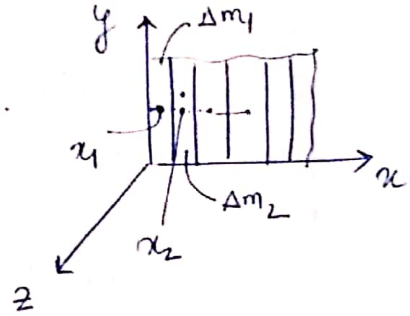
- centroid still exists
- but center of mass doesn't

Ex:- Try to balance a note copy using "finger" [balanced when equal and opposite force is provided]





Center of gravity : Method of moments



moment of resultant about Y- axis = Sum of the moments of individual pieces about y- axis

$$\bar{X} \cdot mg = \Delta m_1 g \cdot x_1 + x_2 \cdot \Delta m_2 g + \cdots + x_n \Delta m_n g$$

or, $$\bar{X} = \frac{\Delta m_1 + x_1 + x_2 \Delta m_2 + \cdots x_n \Delta m_n}{M}$$

or, $$\bar{X} = \frac{\sum x_i \Delta m_i}{M} \text{ or } = \frac{\sum x_i \Delta m_i}{\sum \Delta m_i}$$

uly ~~or~~, $$\bar{Y} = \frac{\sum y_i \cdot \Delta m_i}{\sum \Delta m_i}$$

or, $\bar{X} = \dfrac{\int x \cdot dm}{\int dm}$; $\bar{Y} = \dfrac{\int y \cdot dm}{\int dm}$; when pieces will have negligible width

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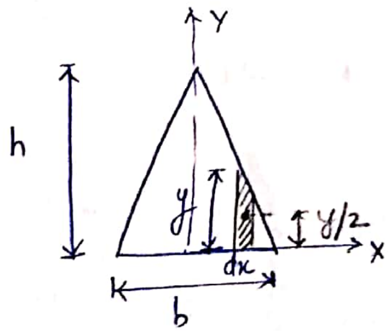
o Note;- For plane figures (homogenous, isotropic), Centroid & COG same.



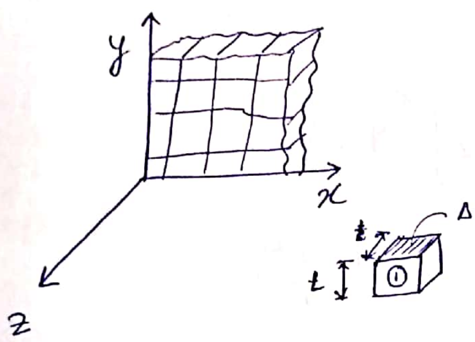
$$\bar{Y} = \int y \cdot dA$$

Scanned with CamScanner

Centroid :- Two dimensional body

Imp

Centroid $\simeq$ CG for isotropic, homogeneous (ρ = constant)



$$\bar{X} = \frac{\sum x_i \Delta M_i}{\sum \Delta M_i}$$

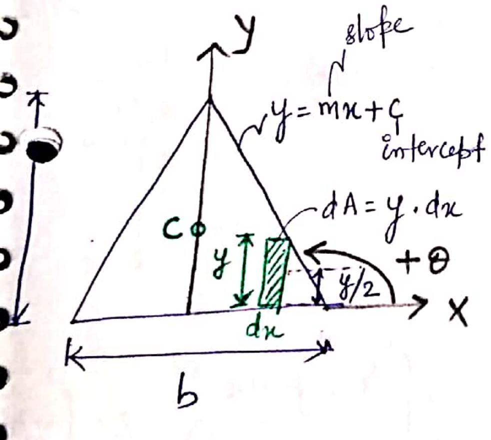
mass = $\rho * V \Rightarrow \Delta M_1 = \rho(\Delta A_1 \times t)$

or, $$\bar{X} = \frac{x_1(\cancel{\rho}\Delta A_1 \cancel{t}) + x_2(\rho \Delta A_2 t) + \cdots + x_n(\rho \Delta A_n t)}{\cancel{\rho}\Delta A_1 \cancel{t} + \rho \Delta A_2 t + \rho \Delta A_3 t + \cdots \rho \Delta A_n t}$$

or, $$\boxed{\bar{X} = \frac{\sum x_i \Delta A_i}{\sum \Delta A_i}} \simeq \frac{\int x\, dA}{\int dA}$$

First moment of area (achieved by taking moment)

IIIly $$\bar{Y} = \frac{\sum y_i \Delta A_i}{\sum \Delta A_i}$$



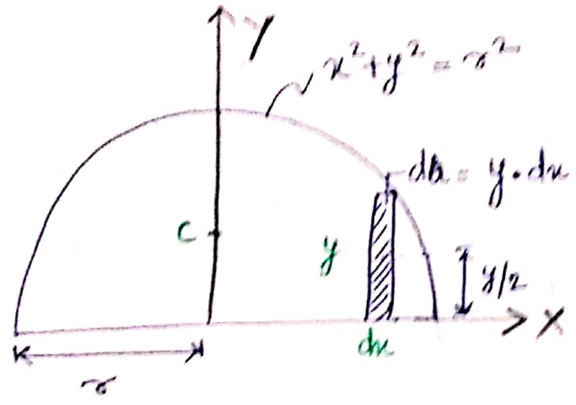
$$\bar{Y} = \frac{\int y \cdot da}{\int da} = \frac{2\int_0^{b/2} \frac{y}{2} \cdot y \cdot dx}{\frac{1}{2} b \times h} = \frac{2\int_0^{b/2} y^2 \cdot dx}{bh}$$

$\int da$: Total area of Δ

$m = \frac{-h}{b/2}$; $c = h$; $y = -\frac{2h}{b}x + h$; $\boxed{y^2 = \frac{4h^2x^2}{b^2} + h^2 - \frac{4h^2x}{b}}$

$$\therefore \bar{Y} = \frac{2}{bh}\int_0^{b/2}\left[\frac{4h^2x^2}{b^2} + h^2 - \frac{4h^2x}{b}\right] dx$$

or, $$\boxed{\bar{Y} = \frac{h}{3}}$$



$$\bar{Y} = \frac{\int y \cdot da}{\int da}$$ Centroid of the vertical strip

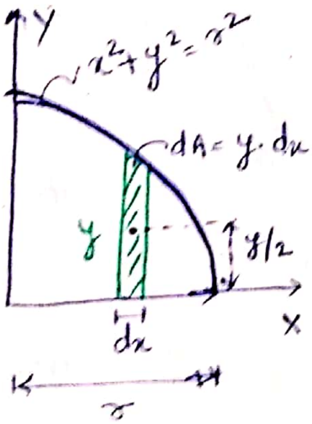
$\int da = \frac{\pi r^2}{2}$

$$\bar{Y} = \frac{2\int_0^r \frac{y}{2} \cdot y \cdot dx}{\frac{\pi r^2}{2}}$$

or, $$\bar{Y} = \frac{2}{\pi r^2}\int_0^r y^2\, dx$$

$$= \frac{2}{\pi r^2}\int_0^r (r^2 - x^2)\, dx$$

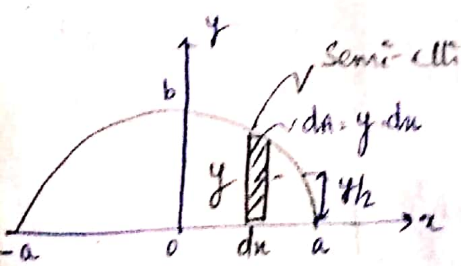
or, $$\boxed{\bar{Y} = \frac{4r}{3\pi}}$$



$$\bar{Y} = \frac{\int y\, da}{\int da} = \frac{\int_0^r \frac{y}{2} \cdot y\, dx}{\frac{\pi r^2}{4}} = \frac{2}{\pi r^2}\int_0^r y^2 \cdot dx$$

or, $$\bar{Y} = \frac{2}{\pi r^2}\int_0^r (r^2 - x^2)\, dx = \frac{4r}{3\pi}$$

n. for $\bar{X} = \frac{4r}{3\pi}$



$$\bar{Y} = \frac{\int y\, da}{\int da} = \frac{2\int_0^a \frac{y}{2} \cdot y \cdot dx}{\frac{\pi ab}{2}} = \frac{2}{\pi ab}\int_0^a y^2 dx$$

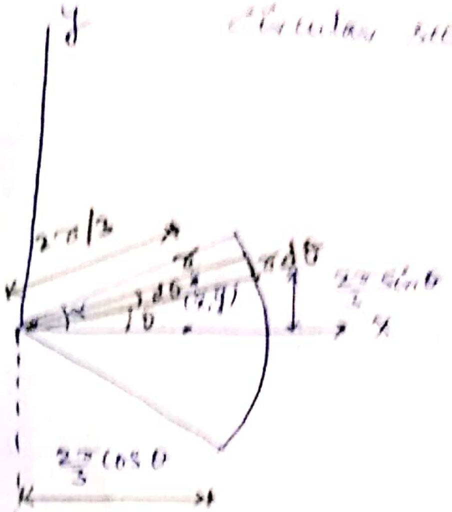
Total Area = $\pi \times a \times b$

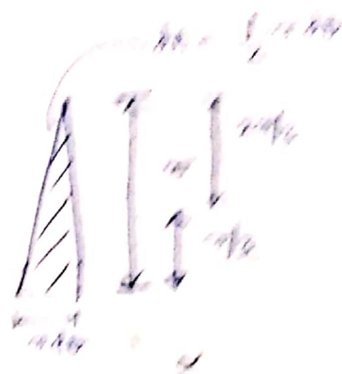
$x^2/a^2 + y^2/b^2 = 1$; $y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$

or, $$\bar{Y} = \frac{2}{\pi ab}\int_0^a b^2\left(1 - \frac{x^2}{a^2}\right)dx$$

or, $$\boxed{\bar{Y} = \frac{4b}{3\pi}}$$

Circular sector

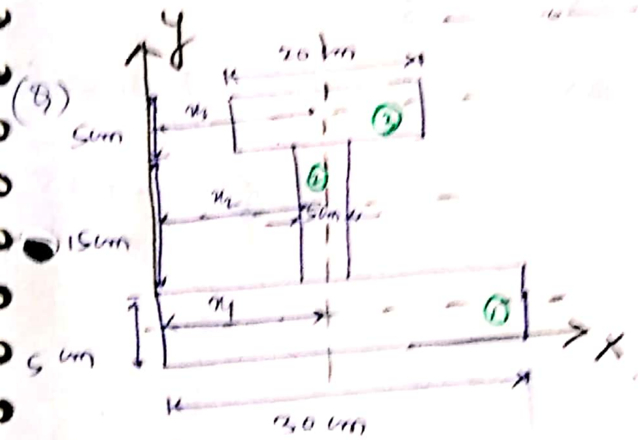




$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{2\int_0^{\alpha} \frac{2r}{3}\cos\theta \cdot \frac{r^2\,d\theta}{2}}{2\int_0^{\alpha} \frac{r^2\,d\theta}{2}}$$

$$\text{So, } \bar{x} = \frac{\frac{r^2}{2}\int_0^{\alpha}\cos\theta\,d\theta}{\frac{r^2}{2}\int_0^{\alpha} d\theta} = \frac{2r\sin\alpha}{3\alpha}$$

(Q)



A_1 150	x_1 15	y_1 2.5
A_2 75	x_2 15	y_2 5+7.5
A_3 100	x_3 15	y_3 $20+\frac{5}{2}$

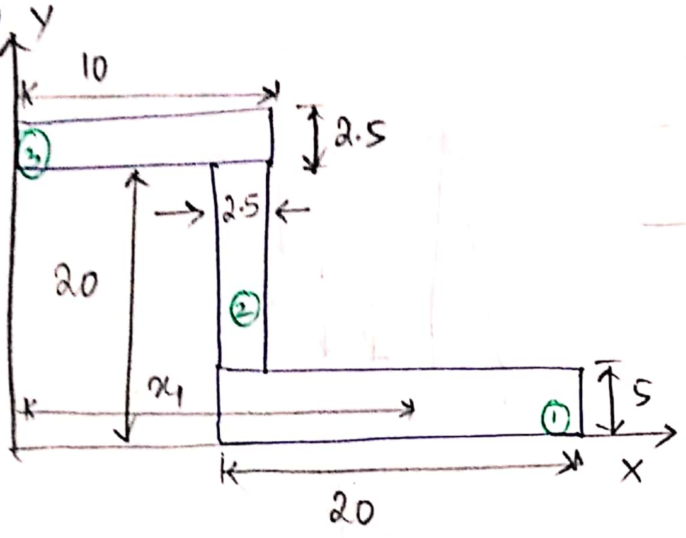
$\Sigma A = 325\,cm^2$

(Soln)

$$x_c = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1+A_2+A_3} = 15\,cm$$

$$y_c = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1+A_2+A_3} = 10.96\,cm$$

(Qns)



A_1	100	x_1	17.5	y_1	$5/2$
A_2	37.5	x_2	$7.5 + \frac{2.5}{2}$	y_2	$7.5 + 5$
A_3	25	x_3	5	y_3	$\frac{2.5}{2} + 20$

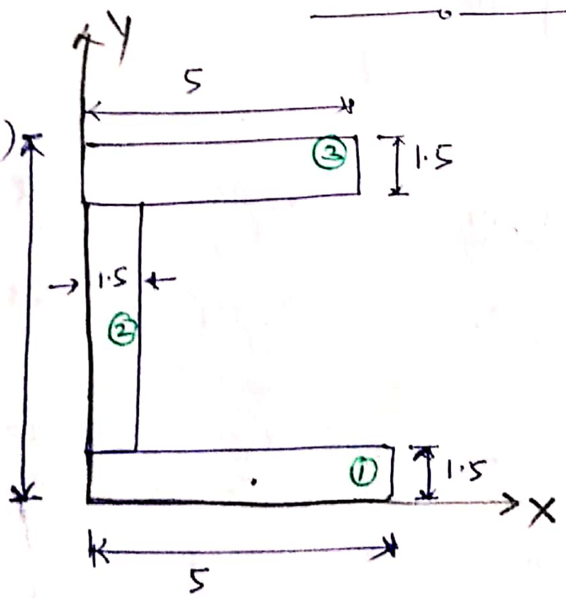
(Sol^n)

$$x_c = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = 13.55\ cm$$

$$y_c = 7.69\ cm$$

———o———o———

(Qns)



A_1	7.5	x_1	$\frac{5}{2}$	y_1	$\frac{1.5}{2}$
A_2	10.5	x_2	$\frac{1.5}{2}$	y_2	$\frac{7}{2} + 1.5$
A_3	7.5	x_3	$\frac{5}{2}$	y_3	$8.5 + \frac{1.5}{2}$

(Sol^n)

$x_c = 1.78\ cm$

$y_c = 5\ cm$

———o———o———

Centroid and Centre of gravity :

Centroid :→ geometric centre of the object

(Centre of mass) → point about which the total area of a plane figure is assumed to be concentrated.

→ Plane figure (having area but no volume) [2D]

→ rectangle, triangle, square, circle.

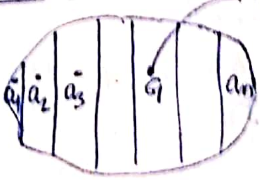
Centre of gravity :→ point where the total weight of the body acts.

Difference :
- CG applies to the bodies with mass and weight. (3D)
- Centroid applies to plane areas. (2D)

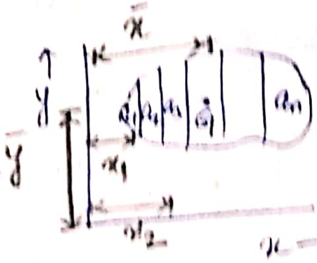
→ CG of a body is the point through which the resultant gravitational force (weight) of the body acts, for any orientation of the body.

* Centroid is the point in a plane area such that the moment of area about any axis, through that point is zero.

Centroid :



Total area (A)

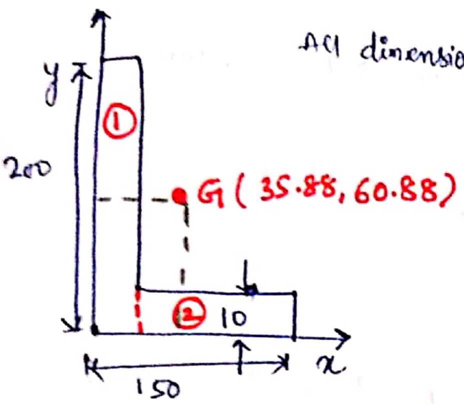


Moment of area about y-axis is $a_1x_1 + a_2x_2 + \dots + a_nx_n = \sum ax = A\bar{x}$

moment of total area about y axis

$$\boxed{\bar{x} = \frac{\sum ax}{A}} \quad \text{ııly} \quad \boxed{\bar{y} = \frac{\sum ay}{A}}$$

(Q) Find the position of Centroid i.e., $(\bar{x}, \bar{y})$



(Sol^n) Divide into two sub-section

For rectangle ① $\begin{cases} A_1 = 10 \times 200 = 2000\ mm^2 \\ x_1 = \frac{10}{2} = 5\ mm; \ y_1 = \frac{200}{2} = 100\ mm \end{cases}$

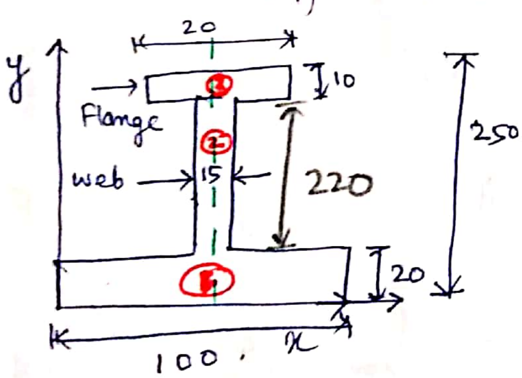
For rectangle ② $\begin{cases} A_2 = (150-10) \times 10 = 1400\ mm^2 \\ x_2 = 10 + \frac{140}{2} = 80\ mm \\ y_2 = \frac{10}{2} = 5\ mm \end{cases}$

location of centroid w.r.t y-axis, $\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1+A_2} = 35.88\ mm$

location of centroid w.r.t. x-axis, $\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1+A_2} = 60.88\ mm$

(Ans)

(Q) Find the centroid for unequal I-section :-



x_1, x_2, x_3 (w.r.t. Origin)

(Sol^n) $A_1 = 20 \times 100 = 2000\ mm^2$

$x_1 = \frac{100}{2} = 50\ mm; \ y_1 = \frac{20}{2} = 10\ mm$

$A_2 = 15 \times 220 = 3300\ mm^2$

$x_2 = \frac{100}{2} = 50\ mm; \ y_2 = 20 + \frac{220}{2} = 130\ mm$

$A_3 = 20 \times 10 = 200\ mm^2$

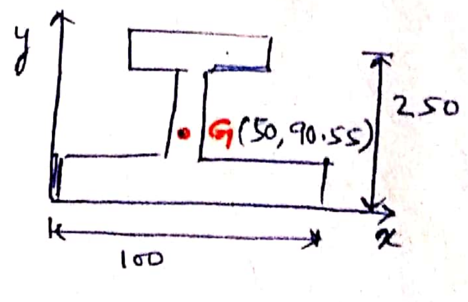
$x_3 = \frac{100}{2} = 50\ mm; \ y_3 = 240 + \frac{10}{2} = 245\ mm$

$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$ (w.r.t. y-axis)

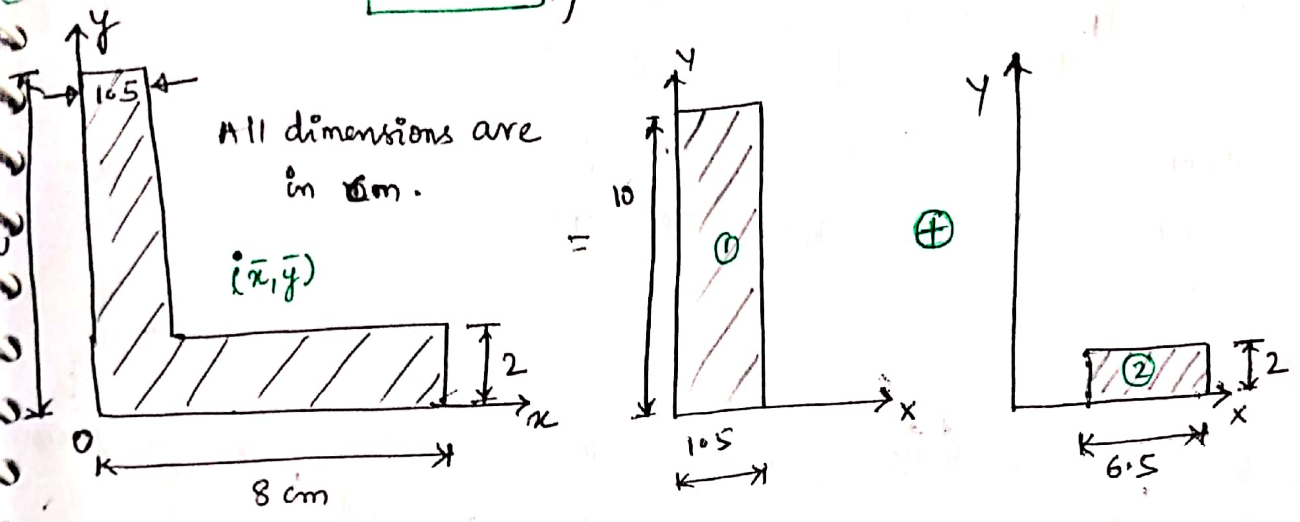
$\bar{x} = 50\ mm$

$\bar{y} = 90.55\ mm$

(Ans)



(Q) Locate the $\boxed{\text{centroid}}$ of the L-Section shown in fig:-



Ans) $A_1 = 10 \times 1.5 = 15\ cm^2$; $A_2 = 6.5 \times 2 = 13\ cm^2$

$x_1 = \frac{1.5}{2} = 0.75\ cm$; $x_2 = 1.5 + \frac{6.5}{2} = 4.75\ cm$ [From reference-y-axis]

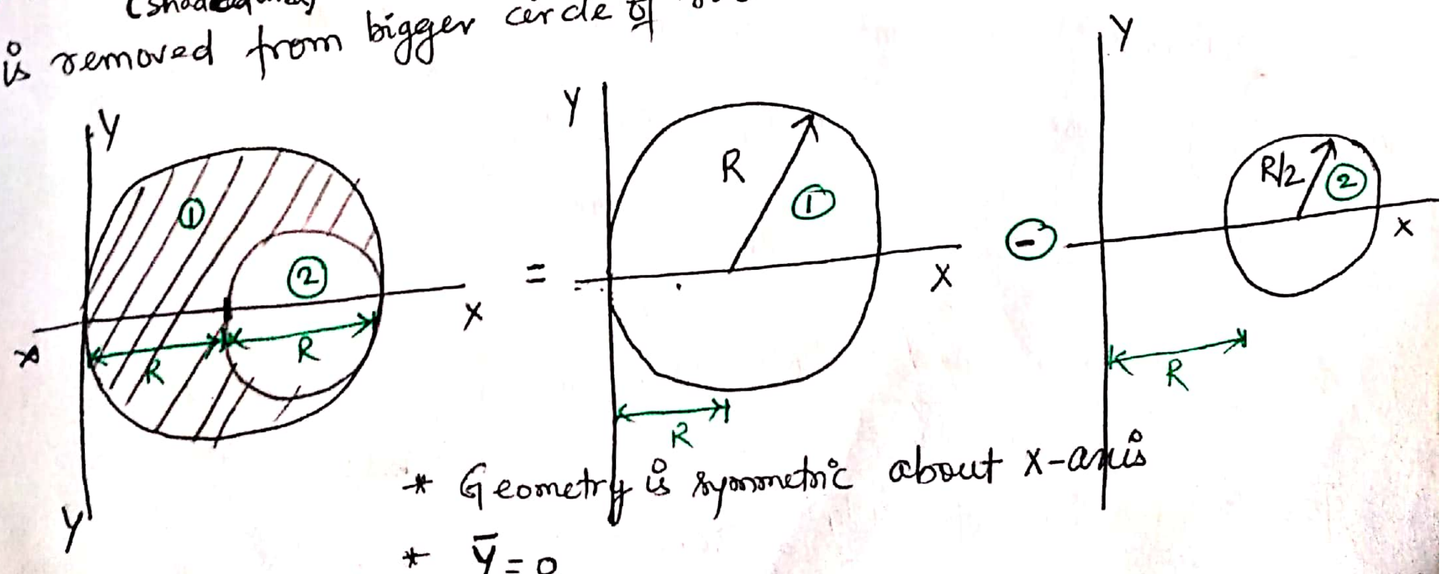
$y_1 = \frac{10}{2} = 5\ cm$; $y_2 = \frac{2}{2} = 1\ cm$

$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum A_i} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{15 \times 0.75 + 13 \times 4.75}{15 + 13} = 2.607\ cm \text{ (Ans)}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{15 \times 5 + 13 \times 1}{15 + 13} = 3.14\ cm \text{ (Ans)}$$

—o—o—

(Q) Find centroid of the fig. (Shaded area) in which smaller circle of Diameter R is removed from bigger circle of radius R.



* Geometry is symmetric about X-axis
* $\bar{Y} = 0$

(Sol^n) $A_1 = \pi R^2$ $\qquad A_2 = \pi (R/2)^2 = \frac{\pi R^2}{4}$

$x_1 = R$ (⊥^{r} dis. from y-axis) $\qquad x_2 = R + \frac{R}{2} = \frac{3R}{2}$

$y_1 = 0$ [⊥^{r} for point @ centre from x-axis] $\qquad y_2 = 0$

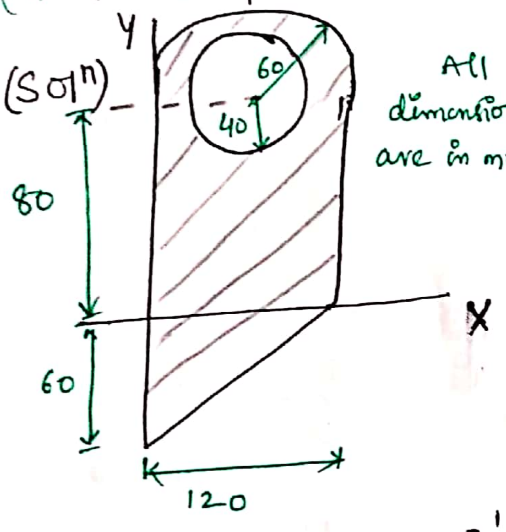
$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{(\pi R^2) R - (\pi R^2/4)(3R/2)}{\pi R^2 - (\pi R^2/4)} = \frac{5}{6} R \quad \text{Ans}$$

$\bar{y} = 0$

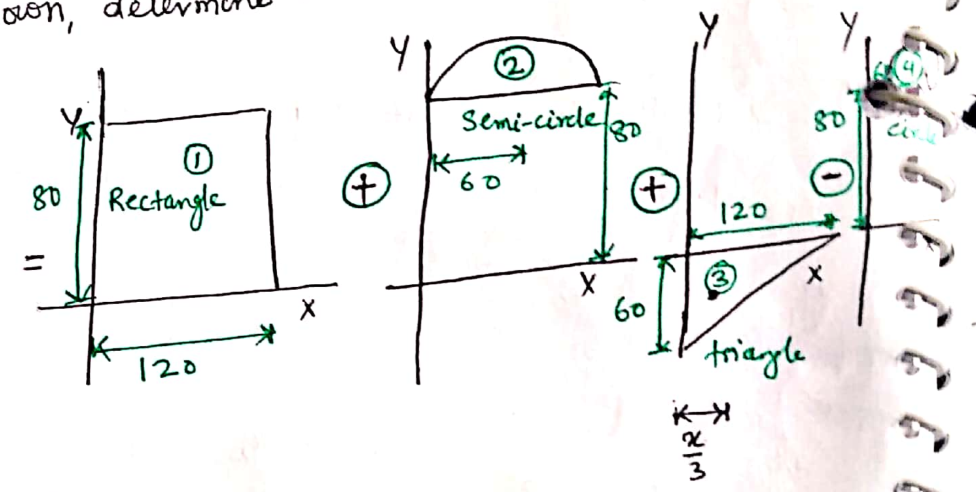
—o—o—

(Q) For the plane area shown, determine the centroid w.r.t. x and y axi...

(Sol^n)



All dimension are in mm



$A_1 = 120 \times 80 = 9600\ mm^2$ $\qquad A_2 = \frac{\pi (60)^2}{2} = 5654.87\ mm^2$

$x_1 = \frac{120}{2} = 60$ $\qquad x_2 = R = 60mm$

$y_1 = \frac{80}{2} = 40$ $\qquad y_2 = 80 + \frac{4R}{3\pi} = 80 + \frac{4 \times 60}{3\pi} = 105.46\ mm$

$A_3 = \frac{1}{2} \times 120 \times 60 = 3600\ mm^2$ $\qquad A_4 = \pi (40)^2 = 5026.55\ mm^2$

$x_3 = \frac{120}{3} = 40mm$ $\qquad x_4 = 60\ mm$

$y_3 = (-)\frac{60}{3} = -20mm$ $\qquad y_4 = 80\ mm$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 - A_4 x_4}{A_1 + A_2 + A_3 - A_4} = 54.79\ mm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 - A_4 y_4}{A_1 + A_2 + A_3 - A_4} = 36.60\ mm$$

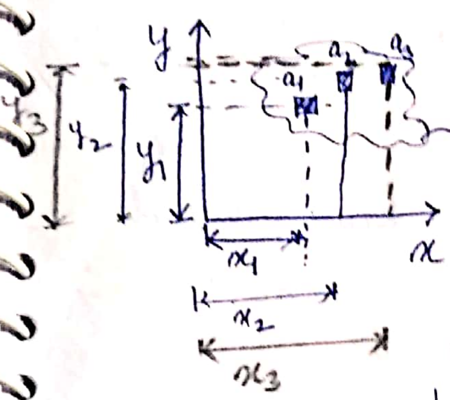
(Ans)

—o—o—

Moment of Inertia

It is defined as the sum of second moment of area of individual sections about an axis.

* Assume small elemental areas (a_1, a_2, a_3) in total area A



→ Taking moments (1^{st}) of all areas about x-axis once

$$I_{xx} = a_1 y_1 + a_2 y_2 + a_3 y_3$$

* ||ly Taking moments (2^{nd}) of area again about x-axis.

$$I_{xx} = a_1 y_1^2 + a_2 y_2^2 + a_3 y_3^2$$

or, $I_{xx} = \Sigma a y^2$ — ①

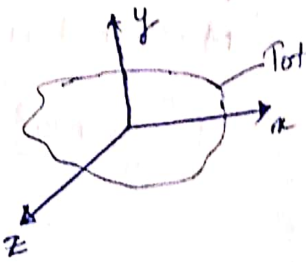
* ||ly $I_{yy} = \Sigma a x^2$ — ②

} moment of inertia about x- & y-axis.

units :- mm^4, cm^4, m^4

Perpendicular axis theorem :-

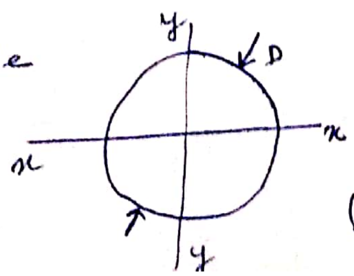
If I_{xx} & I_{yy} are the MOI of a given plane figure, then there is also an MOI which is ⊥r to the plane figure & both I_{xx} & I_{yy}.



* It is also called as Polar axis theorem.

* $I_{zz} = I_{xx} + I_{yy}$ [I_{zz} = polar MOI]

Ex: Circle



$$I_{xx} = I_{yy} = \pi D^4/64$$

$$I_{zz} = I_{xx} + I_{yy} = \frac{\pi D^4}{32}$$

(⊥r to paper)

$$\boxed{I_{zz} = I_p = J = \frac{\pi D^4}{32}}$$

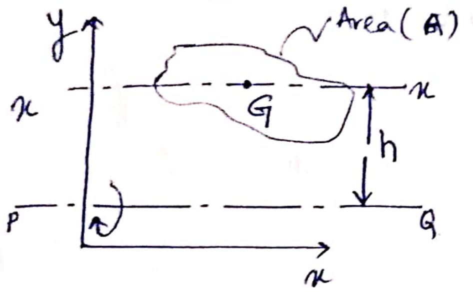
axes.

(4) circle

February	26.01.2018	Tuesday	Subash Bose Jayanti /	01
	14.02.2018	Friday	Veer Surendra Sai Jayanti	01
			Republic Day	

Parallel axis theorem :

The moment of inertia of any given figure about an axis parallel to the centroidal axis is taken as the sum of MOI about the centroidal axis & the product of area and distance square b/w the two axes.

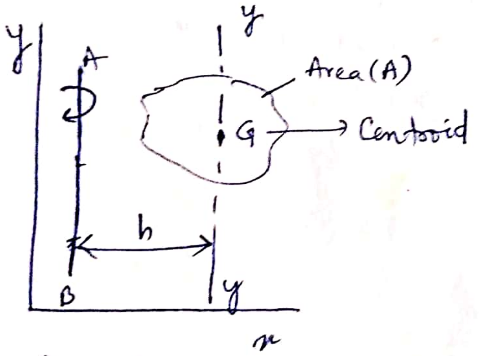


x-x (centroidal axis) → (passing through centre of mass)

> MOI about PQ :-

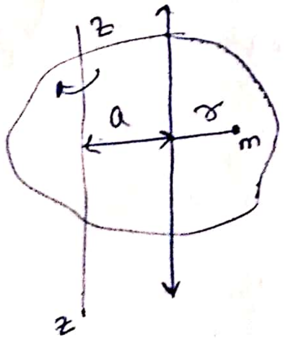
$$\boxed{I_{PQ} = \underbrace{I_{xx}}_{(I_{CM})} + A\cdot h^2}$$

Ex:-



$$\boxed{I_{AB} = I_{yy} + Ah^2}$$

(parallel to each other)



Proof of parallel axis theorem :

$I_{zz'} = m(a+r)^2$

$I_{zz'} = \Sigma m a^2 + \Sigma m r^2 + \Sigma 2 m a r$

$I_{zz'} = Ma^2 + I_{CM} + 2a\Sigma M r$

$$\boxed{I_{zz'} = I_{cm} + Ma^2}$$

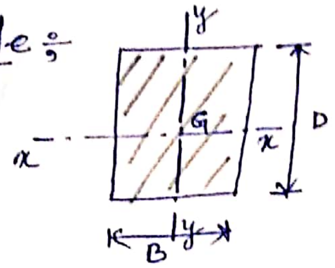
moment of mass

$\Sigma M r = 0$

$I_{zz'}$ for complete body

MOI for plane figures :-

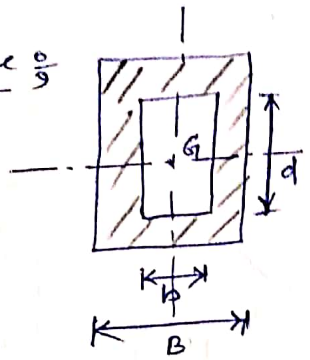
1) Rectangle :-



$$I_{xx} = \frac{BD^3}{12} \quad ; \quad I_{yy} = \frac{DB^3}{12}$$

G → Centroid

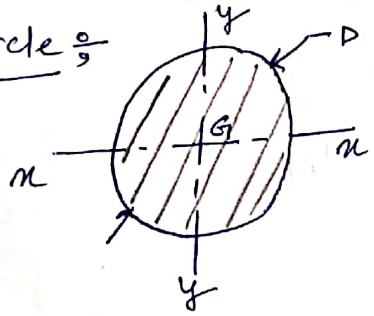
2) Hollow Rectangle :-



$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{BD^3 - bd^3}{12}$$

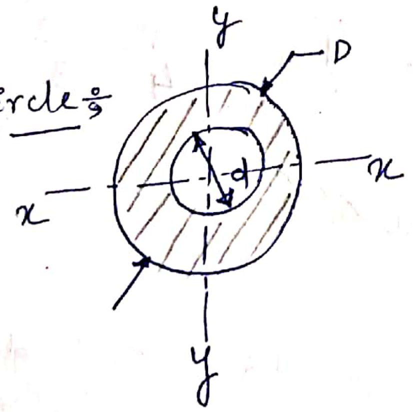
$$I_{yy} = \frac{DB^3}{12} - \frac{db^3}{12} = \frac{DB^3 - db^3}{12}$$

3) Circle :-



$$I_{xx} = \frac{\pi D^4}{64} \quad ; \quad I_{yy} = \frac{\pi D^4}{64}$$

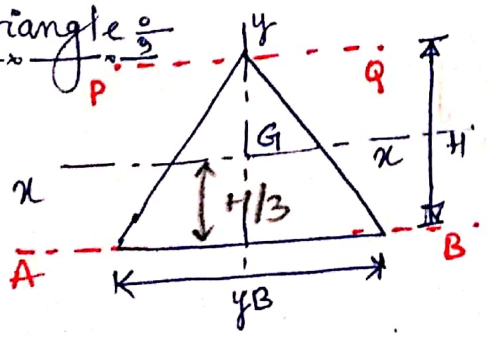
4) Hollow circle :-



$$I_{xx} = \frac{\pi D^4}{64} - \frac{\pi}{64} d^4 = \frac{\pi}{64}(D^4 - d^4)$$

$$I_{yy} = I_{xx}$$

5) Triangle :-

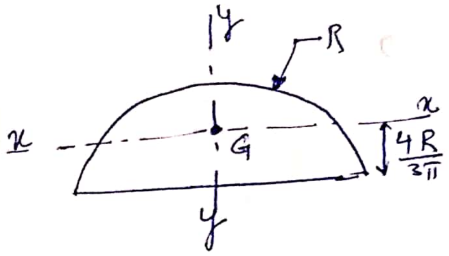


$$I_{AB} = BH^3/12 = I_{cm} + A\cdot[H/3]^2$$

$$I_{PQ} = BH^3/4 = I_{cm} + A\cdot[2H/3]^2$$

$$\boxed{I_{CM} = I_{xx} = \frac{BH^3}{36}}$$

6) Semi-circle :-

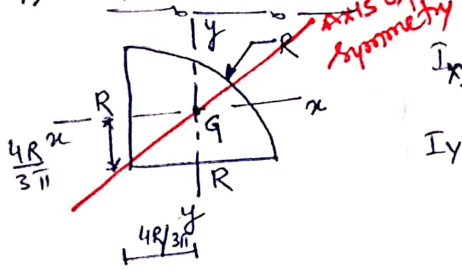


G → Centroid

$I_{xx} = 0.11 R^4$

$I_{yy} = \frac{\pi}{8} R^4$

7) Quarter circle :-

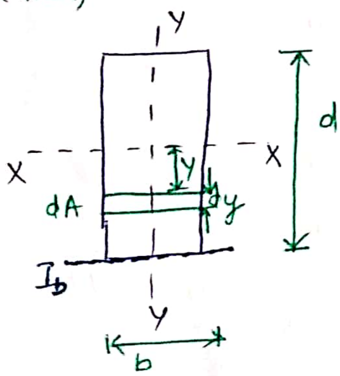


$I_{xx} = \frac{0.11}{2} R^4$

$I_{yy} = \frac{0.11}{2} R^4$

—o—o—

MOI of Rectangle :-
(Area)



For entire rectangle

$I_{xx} = \int y^2 dA = \int_{-d/2}^{+d/2} y^2 . (b \times dy) = b \int_{-d/2}^{+d/2} y^2 dy$

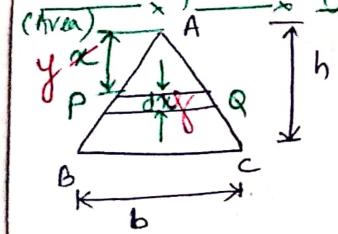
(for elemental strip) $\rightarrow I_{xx}$

or, $I_{xx} = \frac{bd^3}{12}$ ly $I_{yy} = \frac{db^3}{12}$

* x-x (Reference)

$I_b = \frac{bd^3}{12} + A\left(\frac{d}{2}\right)^2 = \frac{bd^3}{12} + bd.\frac{d^2}{4} = \frac{bd^3}{3}$

MOI of Triangle :-
(Area)



MOI of strip PQ about Base (BC)

$I_{BC} = (h - \cancel{x}\,y)^2 \, dA$ —— ①

(elemental strip PQ)

Assume PQ as rectangle :- ~~dA=~~

ΔAPQ & ΔABC are similar

$\frac{PQ}{BC} = \frac{\cancel{x}\,y}{h}$

or, $PQ = \frac{y\cancel{x}}{h} * b$

* BC (Reference)

$$dA = PQ \cdot dx = \frac{\cancel{x}\,y}{h} \cdot b \cdot dx$$

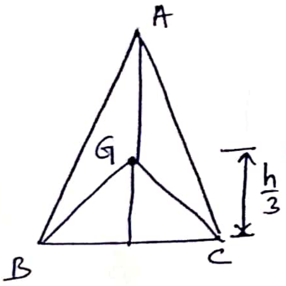
Substitude dA in eqn ① ; $I_{BC} = \int_0^h (h-\cancel{x}\,y)^2 \cdot \frac{\cancel{x}\,y}{h} \cdot b\, \cancel{dx}\, dy$

or, $I_{BC} = \frac{b}{h}\int_0^h (h^2 - 2h\cancel{x}\,y + \cancel{x^2}\,y^2)\cancel{x}\,y\, \cancel{dx}\,dy = \frac{b}{h}\int_0^h (h^2 \cancel{x}\,y - 2h\cancel{x^2}\,y^2 + \cancel{x^3}\,y^3)\,\cancel{dx}\,dy$

or, $I_{BC} = \frac{b}{h}\left[\frac{h^2\cancel{x^2}\,y^2}{2} - 2h\frac{\cancel{x^3}\,y^3}{3} + \frac{\cancel{x^4}\,y^4}{4}\right]_0^h = \frac{b}{h}\left[\frac{h^4}{2} - \frac{2h^4}{3} + \frac{h^4}{4}\right]$

or, $I_{BC} = bh^3\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right] = bh^3\left[\frac{6-8+3}{12}\right]$

or, $\boxed{I_{BC} = \frac{bh^3}{12}}$

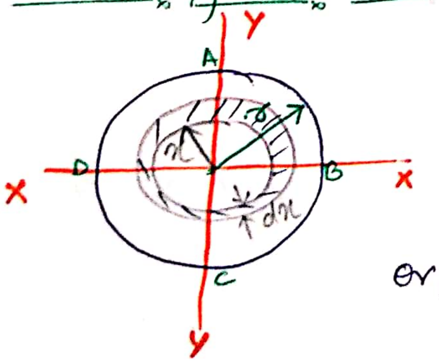


$$I_{BC} = I_G + Ad^2$$

or, $I_G = \frac{bh^3}{12} - \frac{1}{2} \times b \times h \left(\frac{h}{3}\right)^2$

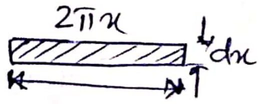
or, $\boxed{I_G = \frac{bh^3}{36}}$

(Area) MOI of Circle :



MOI of elemental strip about z axis :

$$I_{zz} = \int x^2\, dA = \int_0^r x^2 \cdot 2\pi x \cdot dx$$



or, $\boxed{I_{zz} = \frac{\pi r^4}{2} = \frac{\pi D^4}{32}}$

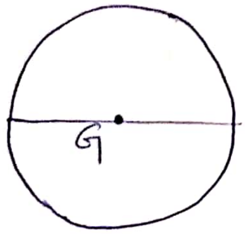
$I_{xx} = I_{yy}$ [Symmetry]

from $\perp^r$ axis theorem;

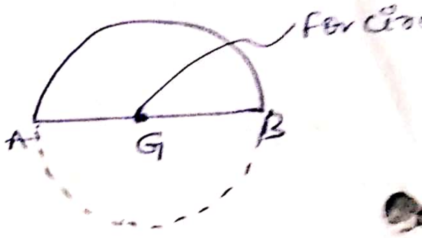
$$I_{zz} = I_{xx} + I_{yy}$$

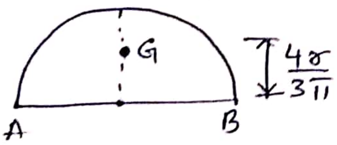
or, $I_{zz} = 2I_{xx} \Rightarrow \boxed{I_{xx} = \frac{\pi D^4}{64}}$

MOI of Semi-Circle :-



$I_{semi\text{-}circle}$ about G (AB line) $= \frac{1}{2}\left[\frac{\pi D^4}{64}\right]$





From $\parallel$ axis theorem,

$$I_{AB} = I_G + A\bar{y}^2$$

or, $I_G = \frac{1}{2}\left[\frac{\pi D^4}{64}\right] - \frac{\pi D^2}{4 \times 2} \cdot \left[\frac{2D}{3\pi}\right]^2$

or, $I_G = \frac{1}{2}\left[\frac{\pi D^4}{64}\right] - \frac{D^4}{18\pi}$

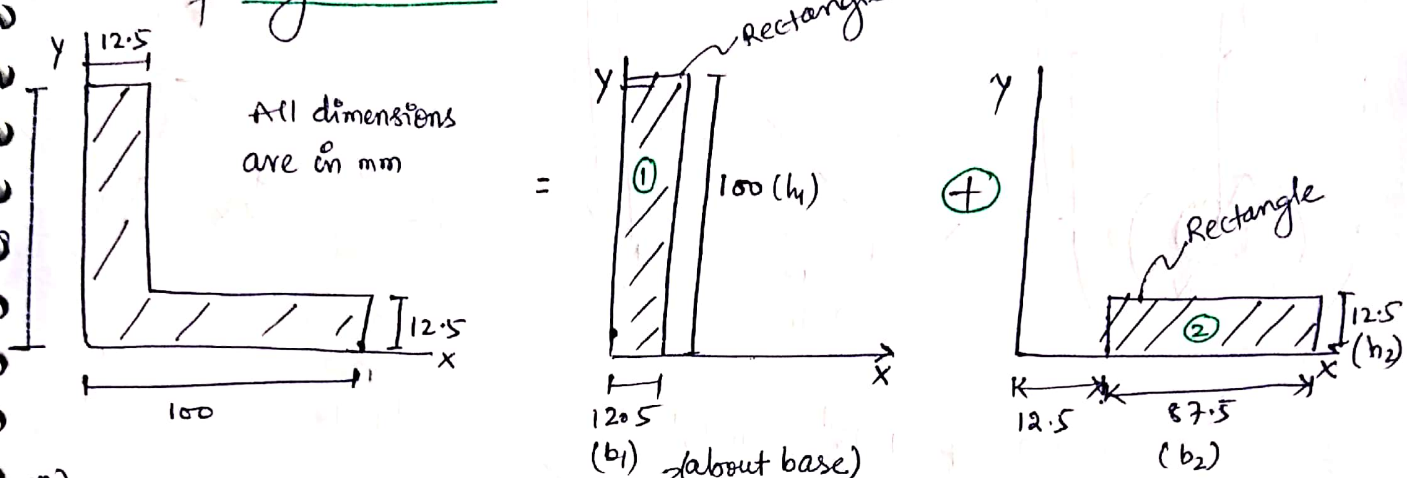
or, $\boxed{I_G = 0.11 r^4}$ or $\cancel{\left(\frac{0.11}{24}\right)} D^4$ $0.006875 D^4$

$$I_G = \frac{1}{2}\left[\frac{\pi R^4}{4}\right] - \frac{\pi R^2}{2} \times \frac{16R^2}{9\pi^2}$$

or, $\boxed{I_G = 0.11 R^4}$

Moment of Inertia (Area)

(Q.) MI of angle Section about x-axis



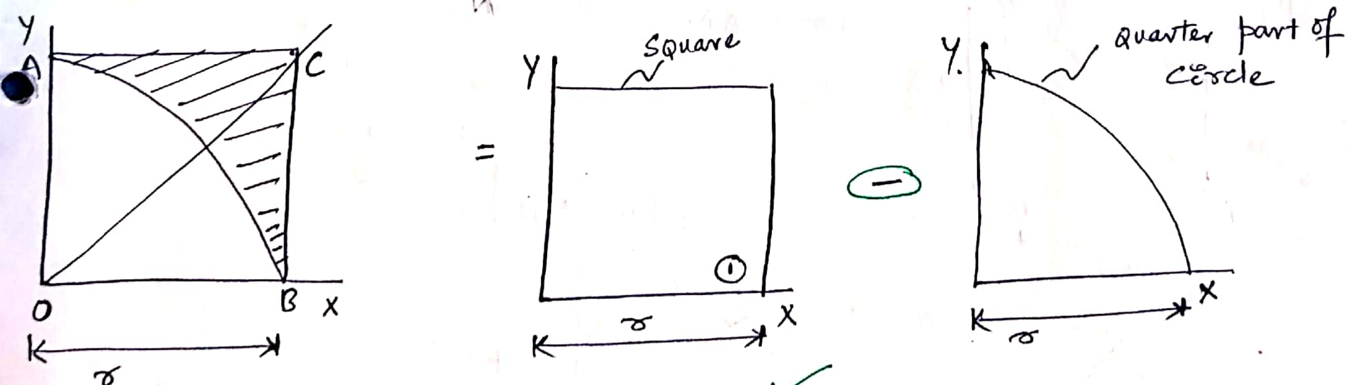
(Soln)

$$I_x = \frac{b_1 h_1^3}{3} + \frac{b_2 h_2^3}{3} = \frac{12.5 \times (100)^3}{3} + \frac{87.5 \times (12.5)^3}{3}$$

or, $I_x = 4.167 \times 10^6 + 0.0569 \times 10^6$

or, $\boxed{I_x = 4.223 \times 10^6 \ mm^4}$ (Ans)

(Q) MOI of shaded portion about x-axis ;- $I_{OB} = I_c + Ay^2$



(Soln) $I_x = bh^3/3 + (-\frac{\pi r^4}{16})$

$= \frac{r^4}{3} - \frac{\pi r^4}{16}$

$= r^4 (1/3 - \pi/16)$

$\boxed{I_x = 0.14 r^4}$ (Ans)

$= bh^3/3 - \pi D^4/(64 \times 4)$

$= \frac{D^4}{48} - \frac{\pi D^4}{64 \times 4}$ $[b = h = r]$

$= D^4 (1/48 - \pi/64 \times 4)$

$\boxed{I_x = 0.0086 D^4}$ (Ans)

Scanned with CamScanner

Mechanics [Branch of physics]

Statics : object @ Rest [deals with study of object @ Rest]

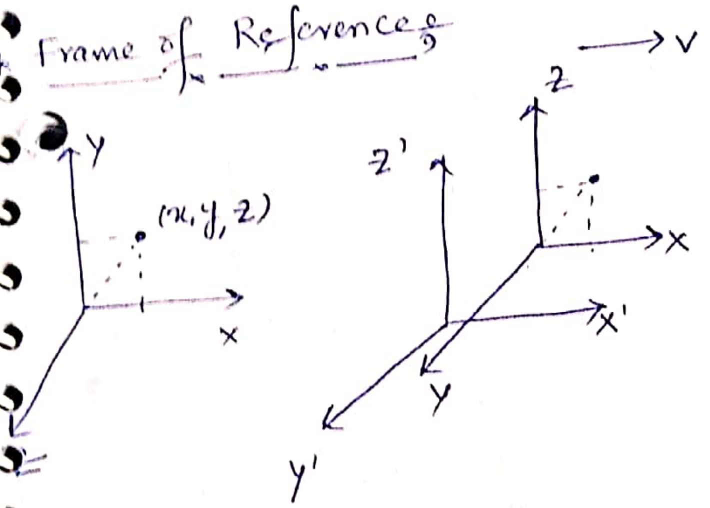
Kinematics : $\boxed{\text{Cause}}$ of motion is not considered (u, v, t, a, s) [Related to body in motion]
(w/o mass) (Branch of dynamics)

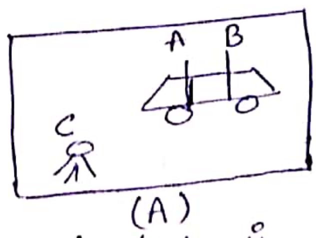
Dynamics : Force is considered

Object :–
- Rest (with respect to time, position is constant) – Reference is impo.
- Motion

Rest ⟷ motion [Relative terms]

Frame of Reference :





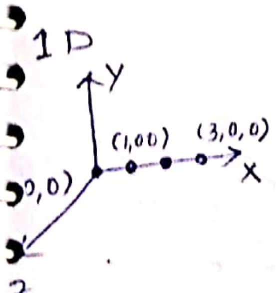
B(object) = ?

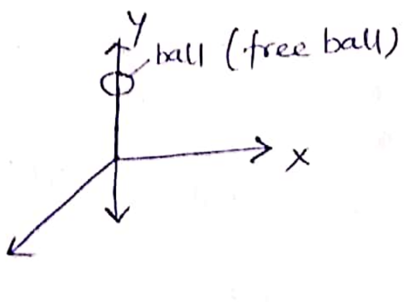
> If vehicle is frame of reference, B is @ Rest.

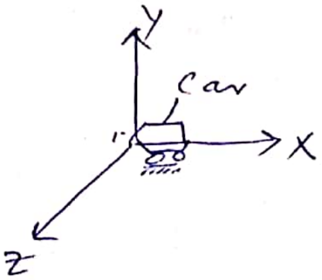
> If C is frame of reference, B is @ motion

Motion (w.r.t coordinate axis)
- 1D (St. line motion) Imp
- 2D Imp
- 3D

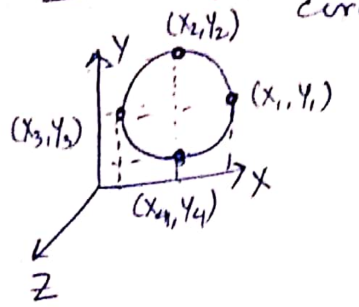
1D





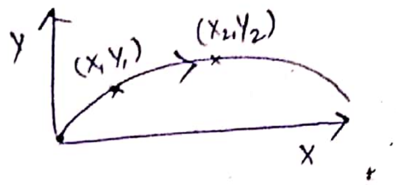


2D



circular path

* 2 coordinates are specified
* Circular motion
* projectile motion



3D

motion of honey bee in a Room.

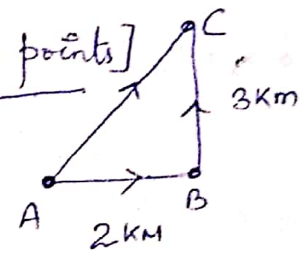
---o---o---

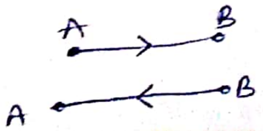
Kinematics :
- Projectile motion
- Relative motion
- Imp Terms
- Straight line motion

Terms : 1. distance and displacement
2. speed and velocity and their types
3. acceleration and its types
4. Kinematics equation [eqn of motion for uniform accn]

1. distance and displacement :-

distance	displacement
(AB + BC)	AC [locate final & initial points]
* length of path	* shortest path b/w initial & final position of an object.
* can't be zero	* can be zero
* scalar	* vector
* no need of direction	* |Displacement| ≤ Distance





2. Speed and velocity.

Speed	velocity
distance / time	displacement / time
scalar	vector
can never be 0	can be 0

$$|velocity| \leq speed$$

3. Acceleration

$$O \xrightarrow{\vec{u}} \cdots\cdots \overset{\vec{v}}{O}$$

t

(accelerate / braking)

acc^n = Rate of change of velocity

= $\dfrac{\text{change in velocity}}{\text{time}}$

$$\boxed{a = \frac{\vec{v} - \vec{u}}{t}}$$

Types / Physical Quantity	Average	Instantaneous
Speed	$S_{avg} = \dfrac{\text{Total dist.}}{\text{Total time}}$ $S_{avg} = S/t$	$v = \dfrac{ds}{dt}$
Velocity	$\vec{V}_{avg} = \dfrac{\vec{s}}{t}$	$\vec{V} = \dfrac{d\vec{s}}{dt}$
Acc^n	$\vec{a} = \dfrac{\vec{v} - \vec{u}}{t}$	$\vec{a} = \dfrac{d\vec{v}}{dt}$

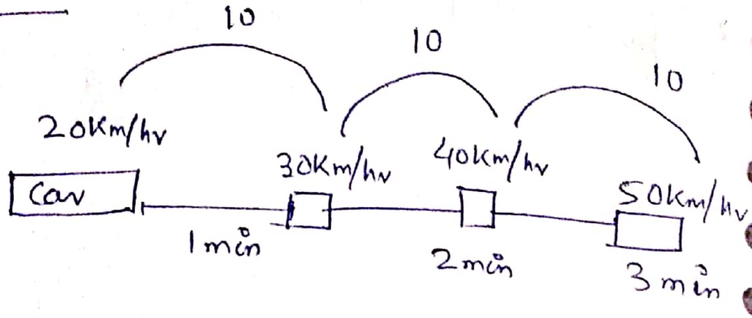
Home v_1 v_2 v_3 office

instantaneous (@ a time)

4. Eqn of motion with uniform motion

$\vec{a} = \frac{\vec{v} - \vec{u}}{t}$ → [constant]

t → [equal]



Equal change in velocity in equal interval of time

Ex:- free fall of ball

Eqn of motion's

i) $a = \frac{dv}{dt} \Rightarrow \int_u^v dv = a\int_0^t dt \Rightarrow v - u = at$

$\boxed{\vec{v} = \vec{u} + at}$ —— (1)

ii) $v = u + at$

$\frac{ds}{dt} = u + at \Rightarrow \int_0^s ds = \int_0^t (u + at)\, dt$

$\boxed{s = ut + \frac{1}{2}at^2}$ —— (2)

displacement

iii) $v = u + at$

$v^2 = (u+at)^2 = u^2 + 2uat + a^2t^2 = u^2 + 2a(\underbrace{ut + \frac{1}{2}at^2}_{s})$

or, $\boxed{v^2 = u^2 + 2as}$ —— (3)

1, 2, 3 ⟶ Kinematic eqn's

Straight line Motion (1D-motion)

ball $u = 0$

↓ 1s, 10 m/s

↓ 2s, 20 m/s

↓ 3s, 30 m/s

$v = u + at$

$g = a = 10\ m/s^2$

- Free Fall
- Throwing a ball downward/upward
- In equal time interval, change in velocity is constant → uniform accⁿ
- Eqⁿ of motion can be applied

$accⁿ = \frac{dv}{dt}$

i) $u = 0$

↓ downward dirⁿ (motion) ⟶ $v = u + gt,\ s = ut + \frac{1}{2}gt^2,\ v^2 = u^2 + 2gs$

$a = +g$ (motion of ball and dirⁿ of gravity same)

= + accⁿ [per unit time, a↑]

ii) — $v = 0$

↓ g

ball thrown upwards — motion ↑, gravity ↓ — $a = -g$

⟶ $v = u - gt,\ s = ut - \frac{1}{2}gt^2,\ v^2 = u^2 - 2gs$

Imp

distance travelled in nth secs ?

S_{n-1}

$t = n-1$

nth $t = n$

?

$$S_{nth} = S_n - S_{n-1}$$

$$= \left(un + \frac{1}{2}an^2\right) - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= un + \frac{1}{2}an^2 - \left[un - u + \frac{1}{2}a(n^2 - 2n + 1)\right]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n^2 - 2n + 1)$$

$$= \cancel{un} + \cancel{\frac{1}{2}an^2} - \cancel{un} + u - \cancel{\frac{1}{2}an^2} + an - \frac{a}{2}$$

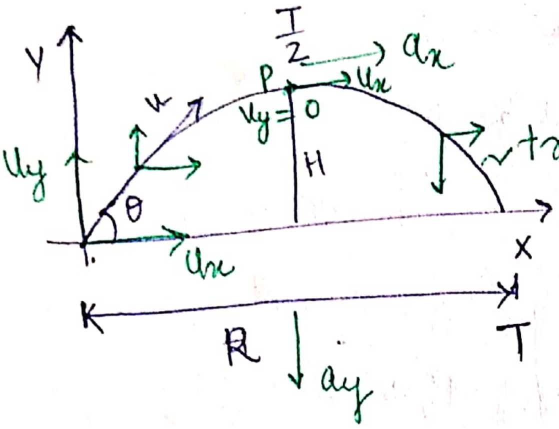
$$\boxed{S_n = u + \frac{a}{2}(2n - 1)}$$

Projectile Motion (2-D motion)

[* Ground to Ground
* Height to Ground]

- Ground to Ground
- Javeline throw

* cricket ball



Horizontal range (R)

Maxm height (H)

Time (T)

* moves along a path determined by gravity and air Resistance. → Projectiles

$\boxed{R, H, T} = ?$

$u_x = u\cos\theta$, $u_y = u\sin\theta$

$a_y = -g$ [oppo. to u_y]

[negligible air friction]

$a_x = 0$

* velocity changes @ each point

* u_x = Remains constant (uniform motion)

* v_y continuously changes due to force of gravity. (uniformly accelerated motion)

$V = u + at$

In y-dirn ; $v_y = u_y + gt$

at maxm height $v_y = 0$ =) $0 = u_y - g\left(\frac{T}{2}\right)$

or, $\boxed{T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}}$

At point 'P', $[v_y = 0]$

$\cancel{s_y = u_y t + \frac{1}{2} a_y}$ $v_y^2 = u_y^2 + 2a_y s_y$

$0 = u_y^2 + 2(-g)(H)$

$\boxed{H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}}$

$R = u_x T = (u\cos\theta)\frac{2u\sin\theta}{g}$

$\boxed{R = \frac{u^2\sin 2\theta}{g}}$

When H_{max} = ?

when $\sin\theta$ is max, $\theta = 90°$

$$\boxed{H_{max} = \frac{u^2}{2g}}$$

when R_{max} = ?

when $\sin 2\theta$ is max, $\theta = 45°$

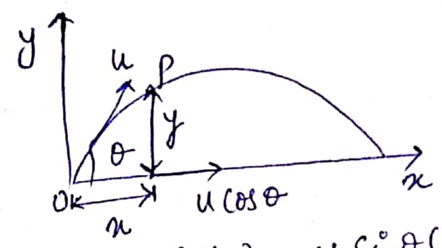
$$\boxed{R_{max} = \frac{u^2}{g}}$$

when T is max^m

when $\sin\theta$ is max^m, $\theta = 90°$

$$\boxed{T = \frac{2u}{g}}$$

Eq^n of the path of a projectile :-



$u\cos\theta$ (u_x) ; $u\sin\theta$ (u_y)

$y = ut + \frac{1}{2}at^2$

$y = (u\sin\theta)\cancel{t} - \frac{1}{2}g\left[\frac{x}{u\cos\theta}\right]^2$

$\left[\frac{x}{u\cos\theta}\right]$

$$\boxed{y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}}$$

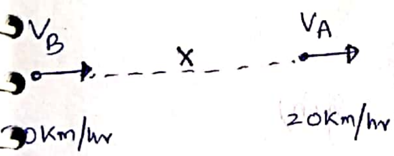
* eq^n of a parabola

*

Relative Motion

Relative Motion < 1D, 2D

1D



When 'B' catches A ?

object — observer

in 1 sec, (distance reduced)

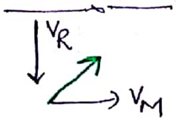
V_B m/s — V_A m/s

$V_{BA} = \underline{\vec{V}_B - \vec{V}_A}$

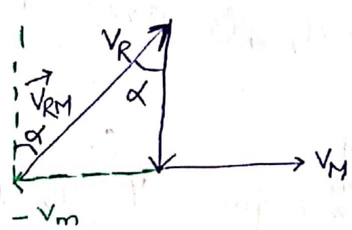
speed of B w.r.t. A

2 D

Rain and Man



$\vec{V}_{RM} = \vec{V}_R + (-\vec{V}_M)$



$\tan\alpha = \frac{V_m}{V_R}$

$\alpha = \tan^{-1}(V_m/V_R)$

Safe angle to prevent Rain inside umbrella

$\vec{V}_{BA} = \vec{V}_B + (-\vec{V}_A)$

velocity

* if B and A in opposite dirⁿ

$\vec{V}_{BA} = \vec{V}_B + \vec{V}_A$

Kinematics of particle

- → Rectilinear motion (along a straight line)
- → Curvilinear motion (curved path)

Rectilinear motion

(i) uniform motion [V = constant, a = 0]

$S = x = y = ut$ —— ①

(ii) velocity varies with time is constant [uniformly accelerated motion]

a = constant [eqⁿ of motion can be applied]

$V = u + at$ —— ①

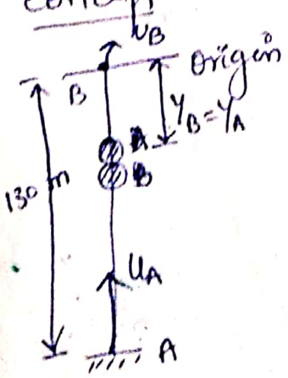
$V^2 = u^2 + 2aS$ —— ② S = displacement

$S = ut + \frac{1}{2}at^2$ —— ③

(iii) Motion with variable acceleration [$a \neq$ constant]

Eqⁿ ①, ②, ③ are not valid

concept



Single body

$y_0 = y$, $y = -130$ m (final position, below origin)

Two bodies

u_B thrown from top after 2 secs A is thrown

B → 1st motion (origin)

$y_0 = 0$

time = t sec

t' = time lag

Body A

$y_0 = -130$

time = (t − 2)

= (t − t')

1

(Q) The accⁿ of a particle is given by $a = t^3 - 3t^2 + 5 \; m/s^2$, where the time t in sec. If the velocity of the particle at $t = 1s$ is $6.25 \; m/s$ and displacement is $8.8 m$, Calculate velocity and displacement at $t = 2s$.

[Rectilinear motion with variable accⁿ]

(solⁿ) $\boxed{a = \frac{dv}{dt}}$; $\int (t^3 - 3t^2 + 5) \, dt = \int dv$

or, $v = \frac{t^4}{4} - t^3 + 5t + c_1$

at $t = 1$, $v = 6.25$, $c_1 = 2$

$$\boxed{v = \frac{t^4}{4} - t^3 + 5t + 2} \quad \text{---- (1)}$$

$\boxed{v = \frac{ds}{dt}}$; $\int \left(\frac{t^4}{4} - t^3 + 5t + 2\right) dt = \int ds$

$S = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5}{2}t^2 + 2t + c_2$

at $t = 1$, $S = 8.8$, $c_2 = 4.5$

$$\boxed{S = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5}{2}t^2 + 2t + 4.5} \quad \text{---- (2)}$$

put $t = 2$ in eqⁿ (1), (2)

$V = 8 \; m/s$

~~S = 16.1 m~~   Imp $S_{@2sec} = S_2 - S_0$ ← start/initial   why not S_1?

$S_{@2sec} = 11.6 \; m$

(Q) The acceleration of a body starting from rest and moving along a straight line is given by $a = t/30 + 2/3$, $a(m/s^2)$, $t(s)$. obtain the velocity at $t = 10s$.

(Soln) $a = \frac{dv}{dt} \Rightarrow \int\left(\frac{t}{30} + \frac{2}{3}\right)dt = dv$?

or, $v = \frac{t^2}{60} + \frac{2t}{3} + C$ ×

$a = \left.\frac{t}{30} + \frac{2}{3}\right]_{\text{at } t=10} = 1\ m/s^2$

$u = 0$ (Rest)

$v = u + at = 0 + 1 \times 10 = 10\ m/s$

(Q) A particle starting from $\boxed{\text{rest}}$ from the origin moves in a straight line whose equation of motion is given by $v = 2t^3 - 3t^2$. what will be the displacement of the particle after 4 seconds?

(Soln) $v = \frac{ds}{dt} = 2t^3 - 3t^2$ or, $\int_0^s ds = \int_0^t (2t^3 - 3t^2)\,dt$

or, $S = \frac{t^4}{2} - \cancel{3}t^3 + C$

$\boxed{\cancel{V = u + at}}$ $a = \frac{dv}{dt} = \left.6t^2 - 6t\right]_{\text{at } t=4} = 72\ m/s^2$

$\cancel{v} = \cancel{u}^{\,0} + at \Rightarrow v = at$

$v^2 - u^2 = 2as$

or, $\left[\left.2t^3 - 3t^2\right]_{t=4}\right]^2 - 0 = 2 \times 72 \times s \Rightarrow s = 44.44\ m$

2

(Q) Accⁿ of a particle is given by $a = 90 - 6x^2$ cm/s², x in cm. If particle starts with zero initial velocity from origin, determine velocity when $x = 5$ cm; position where velocity is again zero, position where velocity is maxᵐ.

Rectilinear motion with variable accⁿ

(Solⁿ) Given; $v = 0$, $x = 0$, $t = 0$
(Given) origin initial (starts)

$$a = 90 - 6x^2 \text{ cm/s}^2 \quad \text{--- (1)}$$

Imp $$a = \frac{dv}{dt} \cdot \frac{dx}{dx} = v\frac{dv}{dx} = 90 - 6x^2$$

$$\boxed{a = v\frac{dv}{dx}}$$

or, $\int v\,dv = \int (90 - 6x^2)\,dx$

or, $\frac{v^2}{2} = 90x - 2x^3 + C_1$

put at $x = 0$, $v = 0 \Rightarrow C_1 = 0$

$$\therefore \boxed{\frac{v^2}{2} = 90x - 2x^3} \quad \text{--- (2)}$$

(I) Find $v = ?$ @ $x = 5$ cm
In eqⁿ (2) put $x = 5$ cm, $\boxed{v = 20 \text{ cm/sec}}$ (Ans)

(II) Find position where velocity is zero
In eqⁿ (2) put $v = 0$ $\quad 0 = 90x - 2x^3$
or, $\boxed{x = 6.71 \text{ cm}}$ (Ans)

(III) Find position where velocity is maxᵐ,
$a = \frac{dv}{dx} = 0 \Rightarrow$ ~~$90 \times 6x$~~ ~~$\Rightarrow x = 15$ cm (Ans)~~
In eqⁿ (1), $90 - 6x^2 = 0 \Rightarrow \boxed{x = 3.87 \text{ cm}}$ (Ans)

(4) A cage descends a mine shaft with an $a = 0.5\ m/s^2$. (1)
After the cage has travelled 25m, a stone is dropped [free fall]
from the top of the shaft.
Determine (1) the time taken by the stone to hit the cage.
(2) distance travelled by the cage before impact.

(Soln) $v^2 - u^2 = 2as$

or, $v^2 - 0 = 2 \times 0.5 \times 25$

or, $v = 5\ m/s$

$v = u + at$, $\Rightarrow 5 = 0 + 0.5 \times t$ $\Rightarrow t = 10\ sec$

At $t = 10$ sec, the stone is 25m above the cage.

$S_s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 9.8 \times t^2$

$S_s = 4.9t^2$ (distance travelled by stone) —— (1)

Distance travelled by cage
$S_c = ut + \frac{1}{2}at^2 = 5t + \frac{1}{2}(0.5)t^2$ —— (2)

$S_s - S_c = 25$

or, $4.9t^2 - (5t + 0.25t^2) = 25$

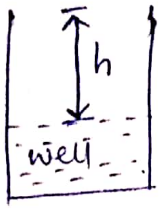
or, $t = (2.91$ or $12.9)$ secs

$S_s = 4.9(2.91)^2 = 41.7\ m$

$S_c = \frac{1}{2} \times 0.5 \times (12.91)^2 = 41.7\ m$ [before impact $u = 0$]

(Q) A stone is dropped into a well, and the sound of a splash is heard after 4 secs. Assuming the velocity of sound to be 350 m/s. What is the depth of the well?

(Soln)



h = level of water below the top of the well

$S = ut + \frac{1}{2} at^2$

or, $h = 0 + \frac{1}{2} \times g \times t_1^2$ or, $h = 5t_1^2$ $(g = 10\ m/s^2)$

or, $t_1 = (h/5)^{1/2}$ — ①

[time taken by the stone to hit the water surface]

time taken by sound to travel 'h' m through air,

$t_2 = \frac{h}{350}$ — ②

Total time (elapsed) $(t_1 + t_2)$ $= t_1 + t_2 = \left(\frac{h}{5}\right)^{1/2} + \frac{h}{350} = 4$ sec

or, $(h/5)^{1/2} = (4 - h/350)$

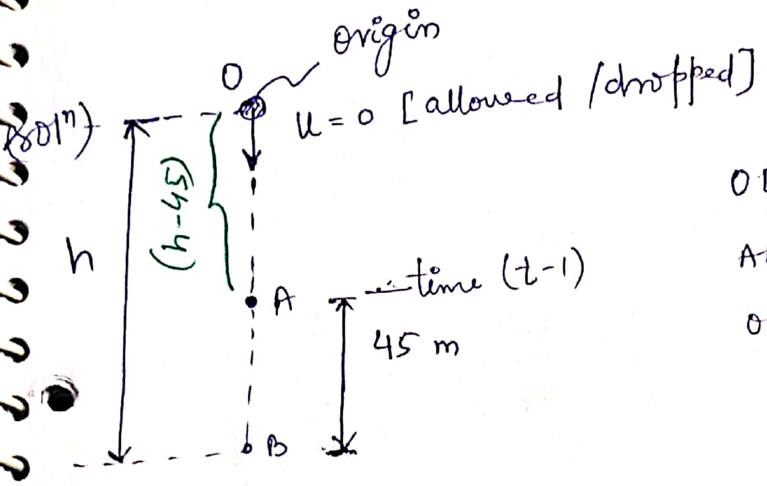
Squaring both sides; or, $h^2 - 27300h + 1960000 = 0$

$h = 72$ m | ~~27228 m~~ unrealistic

The depth of water surface (72 m) below the top of the well is 72 m.

(Q) A body is allowed to fall vertically under the action of gravity. It travels two points in its path placed 45m apart in 1sec. Find from what height above the higher point was the body allowed to fall. (2)

[Motion under gravity]



	time (sec)
OB →	t
AB →	1
OA →	(t-1)

(I) Motion from O to B

$y = +h, \quad u = 0 \quad ; \quad y = ut + \frac{1}{2}gt^2$

$+h = 0 + \frac{1}{2}gt^2 \Rightarrow h = \frac{1}{2}gt^2$ —— (1)

(II) Motion from O to A

$y = +(h-45)$,

$y = \cancel{u}^{0}(t-1) + \frac{1}{2}g(t-1)^2$

or, $+(h-45) = 0 + \frac{1}{2}g(t-1)^2$

or, $h - 45 = \frac{1}{2}g(t^2 - 2t + 1)$ —— (2)

Substitute eqn (1) in eqn (2)

or, $\cancel{\frac{1}{2}gt^2} - 45 = \cancel{\frac{1}{2}gt^2} - gt + \frac{1}{2}g$

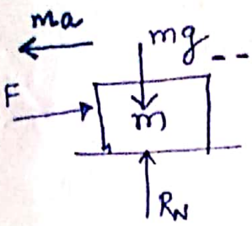
or, $t = 5.087$ sec [time to reach from O to B]

Substitute 't' in eqn (1), $h = 126.94$ m

Body is allowed to fall from or, $(h - 45) = 81.94$ m (Ans)

D' Alembert's Principle

* Assume ;- Smooth Surface ($f=0$)



$R = F = ma$ [Newton's 2nd law] R = resultant force [If more than single force]

$R = \sum F = ma$

If $a=0$; Static problem ; $\sum F = 0$

D'Alembert's Principle ;

$$\sum F + \underbrace{(-ma)}_{\text{inertia force}} = 0$$

(opposes the motion) (accⁿ)

$$\sum F_x + (-ma_x) = 0$$

$$\sum F_y + (-ma_y) = 0$$

$$\left.\begin{aligned} \sum F_t + (-ma_t) &= 0 \\ \sum F_n + (-ma_n) &= 0 \end{aligned}\right\} \text{For curvilinear motion}$$

- In Newton's law ; FBD is required (2)
- In D-Alembert's Principle ;- FBD (single) is sufficient.

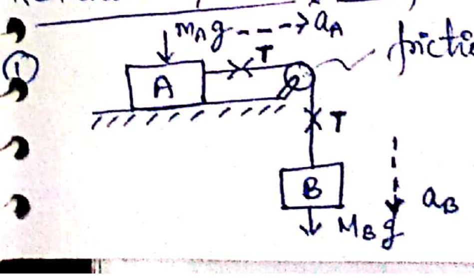
Eqⁿs of dynamic equilibrium ;

$\sum F_x = 0$ $\sum F_y = 0$ $\left.\sum F_t = 0 \quad \sum F_n = 0\right\}$ including inertia forces

- FBD (external forces, weight, normal reacⁿ)
- Resultant forces (dirⁿ of accⁿ ---- line)
- Inertia force (opposite dirⁿ)

Relation b/w accⁿ of connected bodies ;

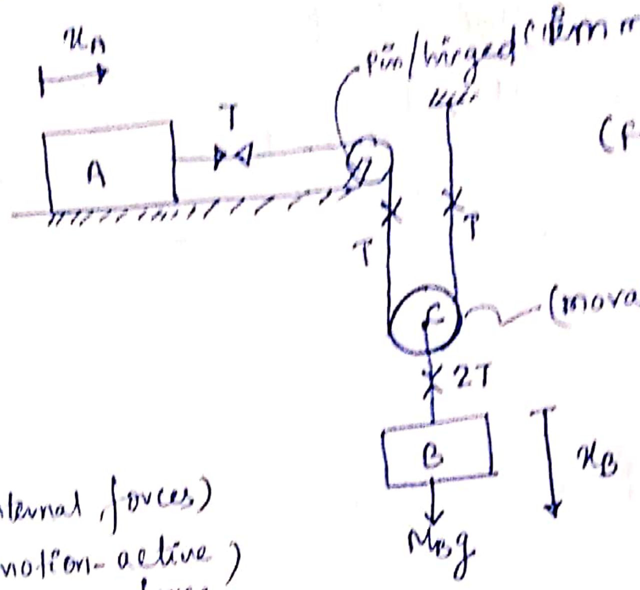
① frictionless, immovable pulley



> If bodies are connected by string passing over immovable pulley then s, a, v are same.

$x_A = x_B$; $v_A = v_B$; $a_A = a_B$

(2)



T (internal forces)

wt. (motion-active force)

> motion is due to wt. of the body

> w/k done by internal (T) forces is zero.

$$\boxed{\text{No. of Tension} \times A(x, v, a) = \text{No. of Tension} \times B(x, v, a)}$$

$+T \times x_A - 2T \times x_B = 0$ (internal forces)

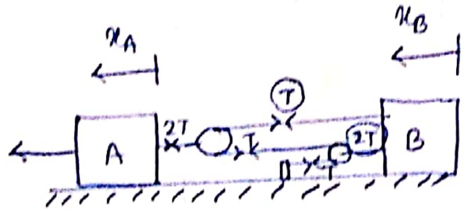
$$\boxed{x_A = 2x_B}$$

$$\frac{dx_A}{dt} = 2\frac{dx_B}{dt}$$

$$\boxed{v_A = 2v_B}$$

$$\boxed{a_A = 2a_B}$$

(3)



$$\boxed{2 \times x_A = 3T x_B}$$

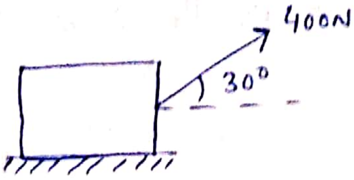
$$x_A = \frac{3}{2} x_B$$

$$\text{ly } v_A = \frac{3}{2} v_B$$

$$\text{ly } a_A = \frac{3}{2} a_B$$

Scanned with CamScanner

(Q) The 350 N box rests on horizontal for which $\mu_k = 0.32$ (Kinetic friction). Find the velocity of the box in 4 secs starting from rest. Use D-Alembert's Principle.



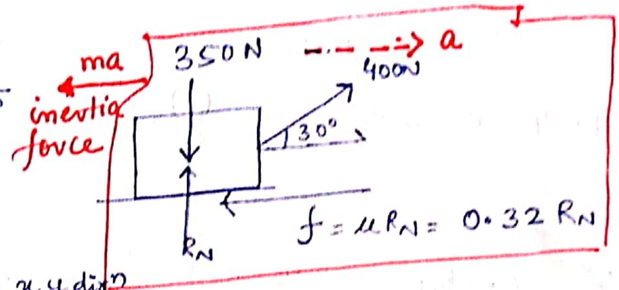
$\boxed{\text{D- Alembert's Principle}}$

(Sol^n) Given;

$u = 0$; $V = ?$; $t = 4\,sec$

$V = u + at \Rightarrow V = 0 + a \times 4 \Rightarrow \boxed{V = 4a}$ —— (1)

Draw FBD of box



Apply D-A Principle in x, y dir^n

$+\sum F_y = 0$ $\quad R_N + 400 \sin 30 - 350 = 0$

($\uparrow +$) (upward) $\quad$ or, $\boxed{R_N = 150\,N}$

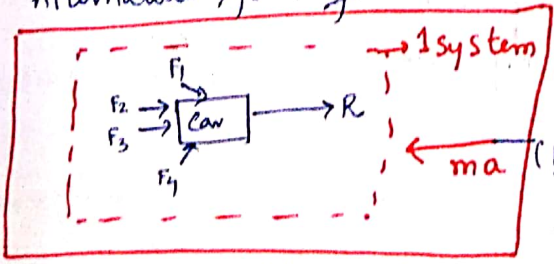
$\sum F_x = 0$ $\quad 400 \cos 30° - 0.32 R_N - ma = 0$ $\quad$ (150) $\quad \frac{350}{9.81}$

($\longrightarrow +$) RHS $\quad$ or, $a = 8.364\,m/s^2$

From eq^n (1) ; $V = 4 \times 8.364 = 33.45\,m/s$

—(Ans)

"Alternative form of Newton's 2nd law of motion"



(If we apply equal force (R) in opposite dir^n, system will come to rest)

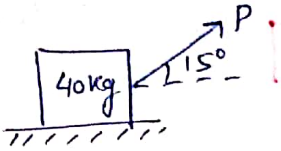
Dynamic equilibrium state

$\sum F - ma = 0$ $\quad$ inertia force

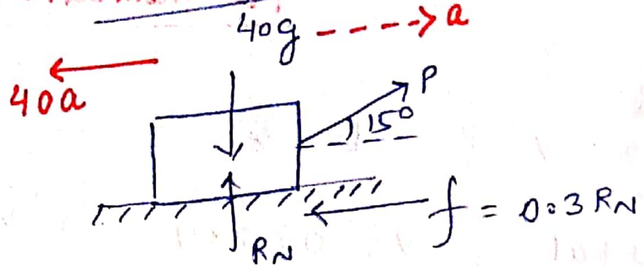
$R - ma = 0$

(Q) Find P required to accelerate the block shown in figure below at $2.5\ m/s^2$, $[\mu = 0.3]$. Use D'Alembert's Principle

(Sol^n)



FBD of block



$\sum F_y = 0$ $(\uparrow +)$ $\quad R_N + P \sin 15° - 40 \times 9.81 = 0$

or, $R_N = 392.4 - P \sin 15°$ ——— (1)

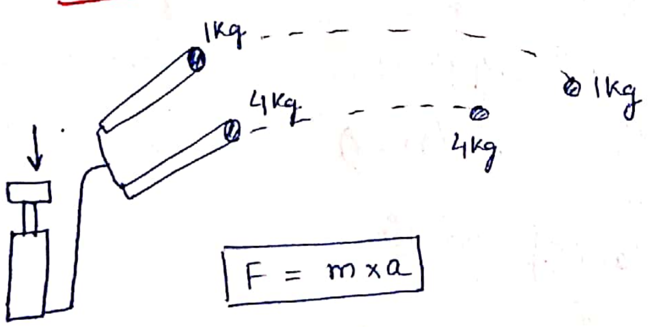
$\sum F_x = 0$ $(\rightarrow +)$ $\quad P \cos 15° - 0.3 R_N - 40a = 0$

From eq^n (1)

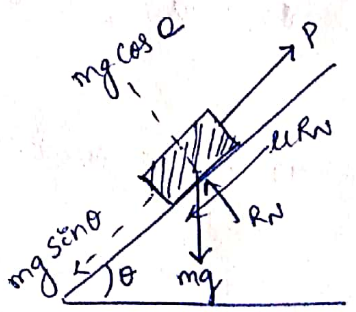
or, $P \cos 15° - 0.3 \left[392.4 - P \sin 15°\right] - 40 \times 2.5 = 0$

or, $\boxed{P = 208.63\ N}$

Newton's 2nd law :-



$\boxed{F = m \times a}$



$P - mg \sin\theta - \mu_k R_N = ma$ ——— (1)

$R_N - mg \cos\theta = 0$ ——— (2)

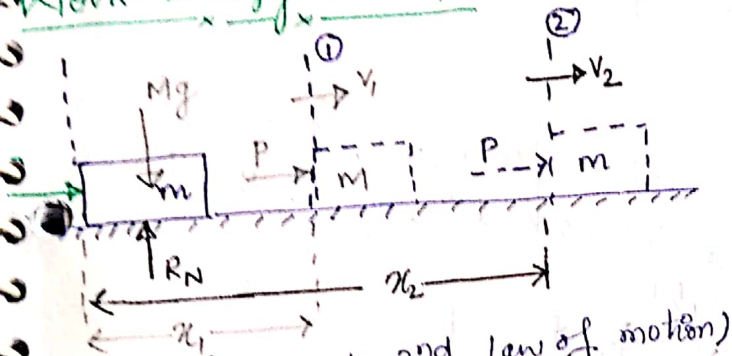
Kinetics of a particle

Three imp. Principles

1. D'Alemberts
2. Work-Energy
3. Impulse-Momentum

• **Kinetics** :- Forces and Mass

Work-Energy Principle :-



$F = ma$ (Newton's 2nd law of motion)

$= m v \frac{dv}{dx}$ $\left[a = \frac{dv}{dt} \cdot \frac{dx}{dx} = v\frac{dv}{dx} \right]$

or, $\int_{x_1}^{x_2} F \cdot dx = \int_{v_1}^{v_2} mv\, dv$

or, $F(x_2 - x_1) = \frac{1}{2} m (v_2^2 - v_1^2)$

or, $F \cdot \Delta x = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

(1) $\boxed{W_{1\to 2} = KE_2 - KE_1}$ $KE_2 = KE$ @ position 2

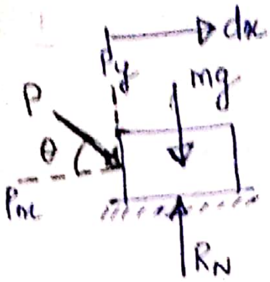
Work-Energy Principle

Workdone = Force × displacement in the dirn of force

= Force in the dirn of of displacement × displacement

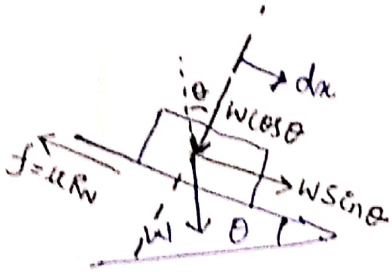
Notes: * If dirn of force & displacement are same, then workdone is +ve

* If dirn of force & displacement are opposite, then workdone is -ve



work done = $P\cos\theta \cdot dx$

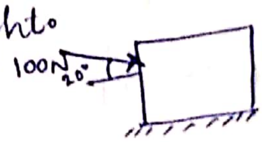
Note ; If forces acts $\perp^r$ to the dirn of motion, then workdone by forces is '0'.



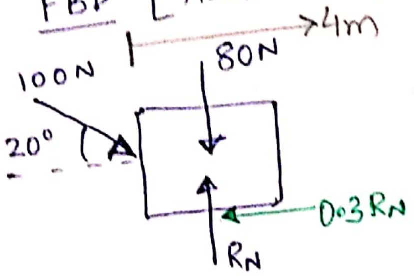
workdone = $W\sin\theta \, dx - \mu R_N \, dx$

Note ; work done by frictional force is Negative.

(Q) A block of weight 80N is acted upon by a force of 100N. If the $\mu_s = 0.30$. Determine the work done by all the forces as the block moves 4m to the right.



(Soln) FBD [Remove support w/o restriction]



$\Sigma F_y = 0$; $R_N - 80 - 100\sin 20^\circ = 0$
$(\uparrow +)$ $R_N = 114.20$ N

Work done = $[(+)100\cos 20 - 0.3 \times R_N] \times 4$ (Δx)

or, $\boxed{W = 238.84 \text{ N}}$

(Q)

Due to weight, R_N ; work done = 0

Wlord done due to applied force (+)

work done due to frictional force (−)

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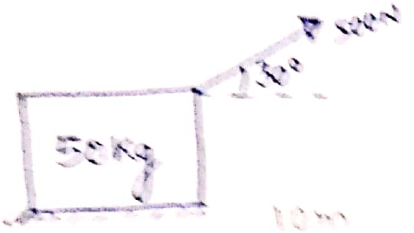
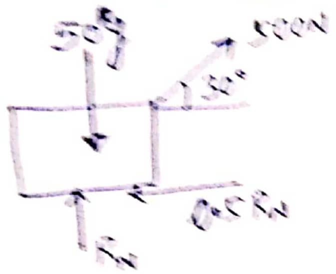
(3) A force of 500N is acting on a block of mass 50 kg resting on a horizontal surface. Determine the velocity after the block has travelled a distance of 10m. [$\mu = 0.5$]



FBD

w/o constraint



$\sum F_y = 0 \quad (+\uparrow)$

or, $R_N - 50\times g + 500\sin 30^\circ = 0$

or, $R_N = 240.5$ N

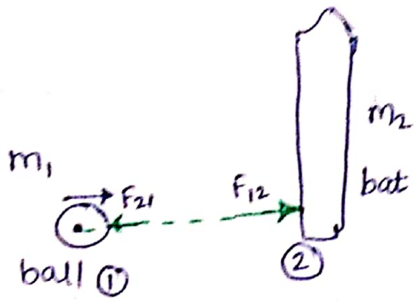
Apply work energy principle:

$W_{1-2} = KE_2 - KE_1$

or, $(500\cos 30 \times 10 - 0.5 \times \underset{(R_N)}{240.5}) \times 10 = \frac{1}{2} m (v_2^2 - \cancel{v_1^2}^{\,0})$

or, $v_2 = 11.185$ m/s

Impulse and Momentum || Impact, Collision b/w Elastic bodies



(Fig 1)

* when ball strikes the bat
- ball exerts force on the bat
- Same amount of force/reaction acts on the ball by the bat.

F_{12} force exerted by body 1 (ball) on body 2 (bat)

* Such large force developed for a short interval of time → Impulse force (F_{12}, F_{21})

① **Impulsive Force:** is defined as a large force which is exerted in a short interval of time during collision.

③ **Impulse of Force:**

I = Impulsive force × time interval

$$\boxed{I = F \times \Delta t}$$

② **Momentum:** = mass × velocity
= m × v

④ **Principle of Impulse and momentum:**

$F = ma$

Resultant force

$F = m\dfrac{dv}{dt}$

or, $\displaystyle\int_{t_1}^{t_2} F \cdot dt = \int_{v_1}^{v_2} m\, dv$ when 'F' acts for dt (time), velocity ↑es by dv

or, $F(t_2 - t_1) = m(v_2 - v_1)$

or, $F\,\Delta t = mv_2 - mv_1$

or, $\boxed{\text{Impulse} = \text{Final momentum} - \text{Initial momentum}}$ = change in momentum

(5) Law of conservation of momentum:

I = Change in momentum

$\boxed{I = m\,\Delta V}$

During impact $F_{12} = F_{21}$
(Fig. 1)

During impact sum of impulses is zero.

$I = 0 = m\Delta V$ = change in momentum

$\boxed{\text{Final momentum} = \text{Initial momentum}}$

$\underbrace{m_1v_1' + m_2v_2'}_{\text{after impact}} = m_1v_1 + m_2v_2$

$v_1, v_2 \rightarrow$ before impact
$v_1', v_2' \rightarrow$ after impact
$m_1, m_2 \rightarrow$ masses of bodies

(6) Coefficient of Restitution: (e)

Body 1 strikes Body 2 $(v_1 > v_2)$

(1) $\xrightarrow{v_1}$ (2) $\xrightarrow{v_2}$

m_1, m_2

$v_1 - v_2$ = velocity of approach

$v_1 > v_2$

Before collision body 1 approaches body 2;
when Body 1 strikes Body 2;

(1) $\xrightarrow{v_1'}$ (2) $\xrightarrow{v_2'}$

$v_2' > v_1$ (then only separation occurs)

$v_2' - v_1'$ = velocity of separation

$$\boxed{e = \frac{v_2' - v_1'}{v_1 - v_2}}$$

$$\boxed{e = - \frac{(v_2' - v_1')}{(v_2 - v_1)}} \qquad 0 \leq e \leq 1$$

7. Types of Impacts

(A) If $e = 1$, then the impact is called as perfectly elastic impact.

(B) If $0 < e < 1$, then the impact is called as semi-elastic impact.

(C) If $e = 0$, then the impact is called as perfectly plastic impact.

$$e = - \frac{(v_2' - v_1')}{(v_2 - v_1)} = 0$$

or, $\boxed{v_2' = v_1'} = v'$

Law of conservation of momentum:

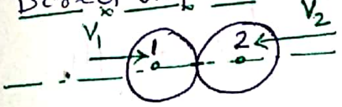
$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

or, $m_1 v_1 + m_2 v_2 = (m_1 + m_2)\, v'$

or,
$$\boxed{v' = \frac{(m_1 v_1 + m_2 v_2)}{(m_1 + m_2)}}$$

8. Types of Impacts

(A) Direct Impact



line of impact (normal to contact surfaces during impact)

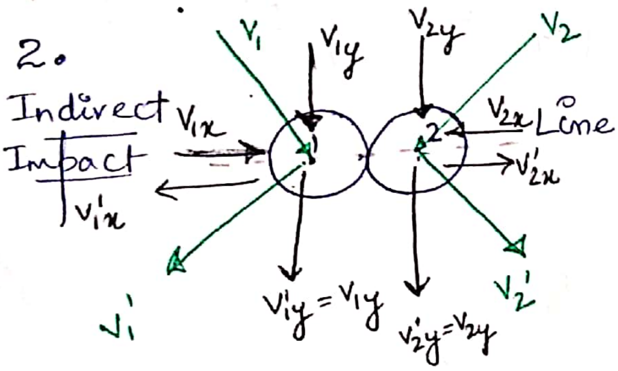
If $v_1 > v_2$ $(m_1 > m_2)$ Right $\rightarrow$

If the velocities of bodies just before and after impact are collinear with line of impact, then the impact is called as direct impact.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \quad \text{——— (1)}$$

$$e = (-)\frac{(v_2' - v_1')}{(v_2 - v_1)} \quad \text{——— (2)}$$

2. Indirect Impact



Line of impact

'Non-collinear with LOI'

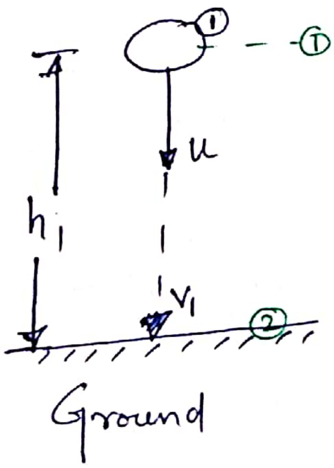
* components of velocities ⊥ to LOI remains same after impact.

$\boxed{v_{1y}' = v_{1y}}$; $\boxed{v_{2y}' = v_{2y}}$

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1x}' + m_2 v_{2x}' \quad \text{——— (1)}$$

$$e = -\frac{v_{2x}' - v_{1x}'}{(v_{2x} - v_{1x})} \quad \text{——— (2)}$$

(9) Collision b/w two bodies when mass of one body is ∞ :-



$$(KE + PE)_1 = (KE + PE)_2$$

or, $\frac{1}{2} m u^2 + m g h_1 = \frac{1}{2} m v_1^2 + 0$

or, $v_1^2 = u^2 + 2gh_1$

or, $\boxed{v_1 = \sqrt{u^2 + 2gh_1}}$ ⟶ If body is dropped

If body is dropped

$v_1 = \sqrt{2gh_1}$