Fluid Mechanics & Hydraulic Machine

4th Sem. Mechanical

Prepared By
Alok Bara
Lect. Mechanical

FMHM

Concept of Fluid Flow — shear / deform, it well flow

↳ Substance capable of flowing.

→ most vital all forms of life → (water, blood, milk, air, etc)

Analysis of when fluid is static or dynamic

Fluid → A substance that deforms cont., when subjected to a shear stress - gas or liquid

Fluid mechanics → Branch of Applied mechanics concerned with the static & dynamic of fluid

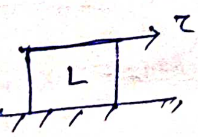
water reservoir — flowing water, (river, pipe flow)

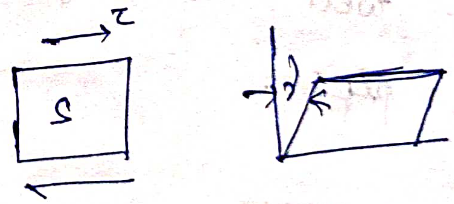
* The analysis of fluid behavior is based on fundamental laws of mechanics - conservation of mass, momentum, energy & laws of thermodynamics.

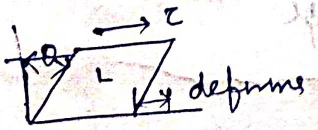
Fluids & solids

Fluids: undergoes strain θ due to shear stress τ

Solids: resists shear by static deformation (up to elastic limit of material)





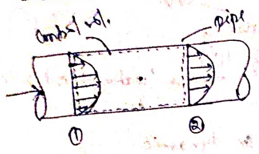


1. deals with cont. stream of fluid without begin or end
2. loosely spaced molecule
3. Intermolecular forces are smaller than solids
4. Fluid deforms cont. when acted shearing stress

1. we consider individual elements in solid.
2. Densely spaced molecule
3. Large intermolecular cohesive force
4. Solid will not deform cont.

Control vol. → A finite region in space has definite vol.

useful in fluid analysis



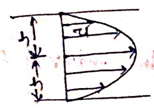
* Lagrangian Description → Describe history of a particle

" time to time analyse to particle → study of fluid particle of fixed identity

Eulerian Description → spatial location $P(x,y,z)$ in flow field at given instant time.

Shearing forces

1. when a fluid in motion, shear stress are developed of the particle of the fluid move relative to one another.
2. adjacent particle have different velocities
3. pipe wall velocity will be zero.
4. velocity will increase as we move towards the center of the pipe..



velocity profile in a pipe

Properties of Fluid

Fluid at rest → statics

Fluid in motion, where pressure force are not considered, fluid Kinematics

Fluid in motion, 'p' ~~not~~ considered ., Fluid Dynamics

Density → (mass density) → Ratio of the mass of fluid to its volume.

$$\rho = \frac{\text{mass of fluid}}{\text{vol. of fluid}} , \quad Kg/m^3$$

$$\rho_{water} = 1\ gm/cm^3 \text{ or } 1000\ Kg/m^3$$

Sp. weight / weight density

$$w = \frac{\text{Weight of fluid}}{\text{Vol. of fluid}} = \frac{\text{mass of fluid} \times \text{Acc. due to gravity}}{\text{vol. of fluid}} = \rho \times g$$

$$\boxed{w = \rho g}$$

$$\boxed{W = mg}$$

Sp. volume → defined as the vol. of a fluid occupied by a unit mass of a fluid or vol. per unit mass of a fluid is called sp. volume.

$$\text{Sp. vol} = \frac{\text{vol. of fluid}}{\text{mass of fluid}} = \frac{1}{\frac{\text{mass of fluid}}{\text{vol. of fluid}}} = \frac{1}{\rho}$$

Sp. Gravity → defined as the ratio of weight density of a fluid to the weight density of a standard fluid.

$$S = \frac{\text{weight density of liquid}}{\text{weight density of water}}$$

$$S(\text{gas}) = \frac{\text{weight density of gases}}{\text{weight density of air}}$$

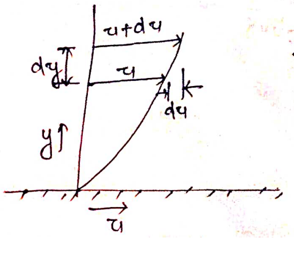
weight density of liquid = S × weight density of water
= S×1000×9.81 N/m³

Density of liquid = S× Density of water
= S×1000 Kg/m³

$\rho_{mercury} = 13.6 \times 1000 = 13600$ kg/m³

Sp. $S_{mercury} = 13.6$

Viscosity → defined as the property of a fluid which offers resistance to the movement of one layer of fluid over adjacent layer of the fluid



→ 2 layer of fluid, distance 'dy'

→ move one & another velocity u, u+du

* viscosity together with relative velocity causes a shear stress acting betⁿ 2 fluid layer

Top layer causes shear stress on adjacent layer & vice versa

→ This shear stress is proportional to the rate of change of velocity w.r.t. y

$$\tau \propto \frac{du}{dy} \Rightarrow \boxed{\tau = \mu \frac{du}{dy}}$$

μ: ~~proportionality~~ constant of proportionality & known as Co-efficient of dynamic Viscosity

$\frac{du}{dy}$ = Rate of shear strain/shear deformation/velocity gradient

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

MKS

$$\mu = \frac{F/A \; (\text{length})^2}{\left(\frac{\text{Length}}{t}\right) \times \frac{1}{\text{length}}} = \frac{\text{Force}/(\text{Length})^2}{1/\text{time}} = \frac{\text{Force} \times \text{Time}}{(\text{length})^2} = \frac{\text{Kgf-Sec}}{m^2}$$

SI unit $= \frac{\text{Newton-sec}}{m^2} = \frac{Ns}{m^2}$

1 Kgf = 9.81 Newtons

One poise $= \frac{1}{10} \frac{Kg}{Sm}$

Kinematic viscosity → Ratio between the dynamic viscosity & density of fluid, ν (nu)

g = 9.81 m/s²

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} = \frac{\text{Force} \times \text{time}}{(\text{length})^2 \times \frac{\text{mass}}{(\text{length})^3}} = \frac{\text{Force} \times \text{time}}{\frac{\text{mass}}{\text{length}}}$$

$$= \frac{\text{mass} \times \frac{\text{length}}{\text{time}^2} \times \text{time}}{\text{mass}/\text{length}} = \frac{(\text{length})^2}{\text{Time}}$$

1 stoke $= \frac{cm^2}{s}$

$= \left(\frac{1}{100}\right)^2 \frac{m^2}{s}$

$= 10^{-4}\, m^2/s$

Newtons law of viscosity → The shear stress(τ) on a fluid element layer is directly proportional to the rate of shear strain.

$$\boxed{\tau = \mu \frac{du}{dy}}$$

obey Relation → Newtonian fluid

Not obey → Non-Newtonian fluid

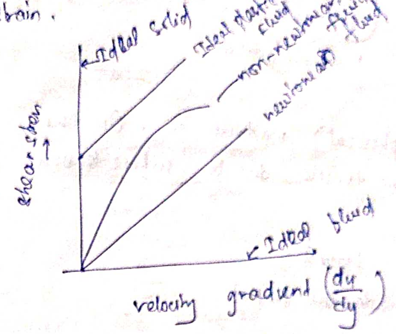
* Temp affect viscosity

fluid → $\mu \downarrow$ $T \uparrow$

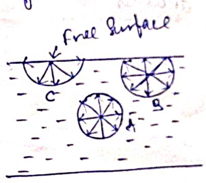
gas → $\mu \uparrow$ $T \uparrow$

Types of fluid

1. ~~Ideal~~ Ideal Fluid → A fluid which is incompressible & is having no viscosity, (imaginary fluid)
2. Real Fluid → A fluid, which posses viscosity actual practice, are real Fluid
3. Newtonian Fluid → , A Real fluid, in which shear stress is directly proportional to shear strain
4. Non-Newtonian fluid → A real fluid, in which shear stress is not proportional to the rate of shear strain
5. Ideal plastic fluid → a fluid, in which shear stress is more than yield value & shear stress is proportional to the rate of shear strain.



Surface tension :→ defined as the tensile force acting on the surface of a liquid in contact with a gas or the surface between 2 immiscible liquid such that contact surface behave like a membrane under tension



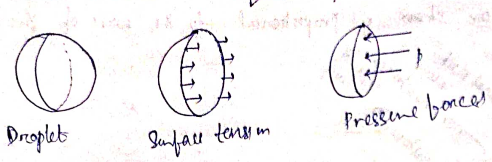
Consider 3 molecule A, B, C, of liquid in a mass of liquid. molecule A attracted in all direction equally by the surrounding molecule of liquid ., resultant force acting on a molecule A is zero

'B' situated near the free surface, upward & downward forced unbalanced , net resultant force on molecule 'B' acting downward dir.

molecule 'C', situated on the free surface of liquid, does resultant downward force.

Thus the free surface of the liquid acts like a very thin film under tension of the free surface of the liquid act as though it is an elastic membrane under tension.

Surface Tension on Liquid droplet :-



Consider small droplet (spherical), radius 'r'. On the entire surface of the droplet, the tensile due to surface tension will be acting

let σ = Surface tension of Liquid

P = Pressure intensity inside the droplet

d = Dia of droplet

• droplet cut in 2 halves, force acting one half will be

(1) tensile force ~~acting~~ due to surface tension around the circumference of the cut portion.

$= \sigma \times$ Circumference

$= \sigma \times \pi d$

(2) Pressure force on the are $\frac{\pi}{4} d^2$ and $= p \times \frac{\pi}{4} d^2$

2 forces will be equal & opposite under equilibrium cond^n

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$p = \frac{\sigma \times \cancel{\pi} \cancel{d} \times 4}{\cancel{\pi} d^{\cancel{2}}}$$

$$\boxed{p = \frac{4\sigma}{d}}$$

Surface tension on a hollow bubble → It is like a soap bubble in air has 2 surface contact with air, one inside & other outside. Thus 2 surface are subjected to surface tension. In such case

$$p \times \frac{\pi}{4} d^2 = 2(\sigma \times \pi d)$$

$$p = \frac{2(\sigma \times \cancel{\pi} \cancel{d}) \times 4}{\cancel{\pi} d^{\cancel{2}}}$$

$$\boxed{p = \frac{8\sigma}{d}}$$

1)) Find the surface tension in a soap bubble of 40mm diameter, when the inside pressure is 2.5 N/mm^2 above atm. Pressure.

Sol^n Data Given, $d = 40mm = 40 \times 10^{-3} m$

$P = 2.5 \, N/m^2$

For soap bubble $= P = \frac{8\sigma}{d} \Rightarrow 2.5 = \frac{8\sigma}{40 \times 10^{-3}} \Rightarrow \sigma = 0.0125 \, N/m$

2)) The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2 atm pressure. Calculate pressure outlet if surface tension is given as 0.0725 N/m of water

Sol^n Dia of droplet $d = 0.04 mm = 0.04 \times 10^{-3} m$

Press outside the droplet $= 10.32 \, N/cm^2 = 10.32 \times 10^4 \, N/m^2$

Surface tension $= \sigma = 0.0725 \, N/m$

Pressure inside droplet $= P = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{0.04 \times 10^{-3}} = 7250 \, N/m^2$

$= \frac{7250}{10^4} \frac{N}{cm^2} = 0.725 \, N/cm^2$

Pressure inside droplet $= P +$ Pressure outside the droplet

$= 0.725 + 10.32 = 11.045 \, N/cm^2$

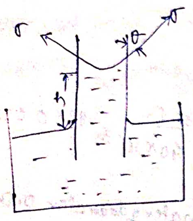
Capilarity is defined as the phenomenon of rise ... small tube relative to the adjacent liquid surface in a small tube relative to the general level of liquid. when the tube is held vertically in the liquid.

→ The rise of liquid surface → Capillary rise

Fall of liquid surface → Capillary depression

* expressed in mm or cm, value depends upon sp. wt of liquid, dia of tube, surface tension tube.

Expression for Capillary rise :-



d = glass tube diameter

θ = Angle of contact betn liquid & glass tube

weight of liquid of height 'h' in the tube = (Area of tube × h) × ρ × g

$= \frac{\pi}{4} d^2 \times h \times \rho \times g$ ——(1)

vertical component of the surface tensile force

$= (\sigma \times \text{Circumference}) \times \cos\theta$

$= \sigma \times \pi d \times \cos\theta$ ——(2)

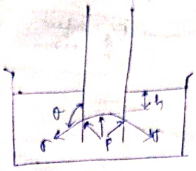
for equilibrium eqn (1) & (2) equating

$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$

$h = \frac{\sigma \cancel{\pi} \cancel{d} \cos\theta \; 4}{\cancel{\pi} d^{\cancel{2}} \rho g}$

$$\boxed{h = \frac{4\sigma\cos\theta}{\rho g d}}$$

Expression for Capillary fall :- If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside of the liquid



h = height of depression in tube.

In equilibrium, 2 forces are acting on the mercury inside the tube.

1. Surface tension acting in the downward direction & is equal to $\sigma \times \pi d \times \cos\theta$

2. hydrostatic force acting upward and is equal to intensity of pressure at a depth h × Area

$= P \times \frac{\pi}{4} d^2$

$= \rho g h \times \frac{\pi}{4} d^2$

$\sigma \pi d \cos\theta = \rho g h \frac{\pi}{4} d^2$

$h = \frac{\sigma \cancel{\pi} \cancel{d} \cos\theta \; 4}{\rho g \cancel{\pi} d^{\cancel{2}}}$

$$\boxed{h = \frac{4\sigma\cos\theta}{\rho g d}}$$

Pressure Measurement

When a fluid contained in a vessel, it exerts force at all points on the sides and bottom and top of the container. The force per unit area is called pressure.

F = force
A = Area on which the force acts

then intensity of pressure = $\boxed{P = \frac{F}{A}}$

Pressure of fluid on a surface always act normal to the surface.

unit → kgf/m^2 & kgf/cm^2

KPa = Kilo Pascal = $1000\ N/m^2$

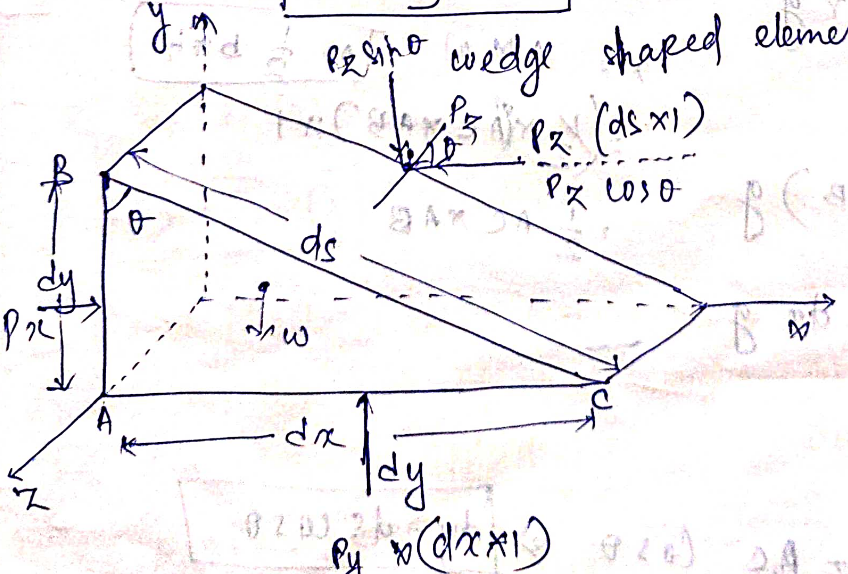
1 bar = 100 KPa = $10^5\ N/m^2$

Pascal's law → It states that the pressure or intensity of pressure at a point in a static limit are equal to all direction.

→ The fluid element is of very small dimension dx, dy & dz

$\boxed{P_x = P_y = P_z}$

wedge shaped element,



∠ABC = θ

Apply pressure on each face

$P = F/A \Rightarrow F = P \times A$

$P_x \times AB = F_x$

$P_x (dy \times 1)$ = Pressure force acting on AB face (F_x)

$P_y \times (dx \times 1)$

$P_z\ (ds \times 1)$

Consider arbitary fluid element of wedge shaped fluid mass at rest, width of element is unity

Press. acting on the face AB : P_x

AC : P_y

BC : P_z

$P = F/A$ = Forces acting on the element

$F = (P \times A)$

Force on face AB = Pressure in x-dir × Area of face AB

$= P_x \times (dy \times 1)$

$AC = P_y \times (dx \times 1)$

$BC = P_z \times (ds \times 1)$

weight of element $= mg$ ← act center of gravity

$(\rho = m/V)$

$m = \rho V$

$= \rho V g$

$V = A \times w$ $\boxed{A = \frac{1}{2} b \times h}$ triangular shape

$= (\frac{1}{2} \times AC \times AB) \times 1$

$= \frac{1}{2} AC \times AB$

$= \rho (\frac{1}{2} AC\, AB) g$

$= \rho \frac{1}{2}\, dy\, dx\, g$

Consider triangle ABC

$\cos\theta = \frac{AB}{BC} \Rightarrow AB = BC \cos\theta \Rightarrow \boxed{dy = ds \cos\theta}$

$\sin\theta = \frac{AC}{BC} \Rightarrow AC = BC \sin\theta$

$\boxed{dx = ds \sin\theta}$

$\left.\begin{array}{l}\Sigma H = 0 \\ \Sigma V = 0\end{array}\right\}$ fluid is at rest

Resolve the forces in x-direction

$P_x (dy \times 1) = P_z \cos\theta (ds \times 1)$

$P_x (dy \times 1) = P_z \cos\theta (ds \times 1) = 0$

$P_x\, dy = P_z\, ds \cos\theta$

$P_x\, dy = P_z\, dy$

$\boxed{P_x = P_z}$ —— (1)

Resolve forces in y direction

$P_y (dx \times 1) = P_z \sin\theta (ds \times 1) + W$

$= P_z \sin\theta (ds \times 1) + \frac{1}{2} \rho g\, dx\, dy$

$P_y (dx \times 1) - P_z \sin (ds \times 1) - \frac{1}{2} \rho g\, dx\, dy = 0$

element is very small & hence weight is negligible

$P_y\, dx = P_z \sin ds$

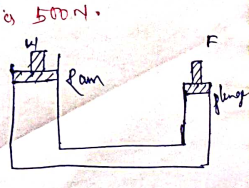
$P_y\, dx = P_z\, dx$

$\boxed{P_y = P_z}$ —— (2)

Eqn (1) & (2)

$\boxed{P_x = P_y = P_z}$

Prob. A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm dia. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500 N.



Soln $D = 30$ cm $= 0.3$ m

$d = 4.5$ cm $= 0.045$ m

$F = 500$ N

$W = ?$

Area of ram $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.3)^2 = 0.07068 \, m^2$

plunger $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.045)^2 = 0.00159 \, m^2$

Pressure intensity due to plunger $= \frac{\text{Force of plunger}}{\text{Area of plunger}} = \frac{500}{0.00159} \, N/m^2$

$= 314465.4 \, N/m^2$

$P_{ram} = \frac{\text{weight}}{\text{Area of Ram}} = \frac{W}{A} = \frac{W}{0.07068} \, N/m^2$

$\frac{W}{0.07068} = 314465.4$

$= 314465.4 \times 0.7068$

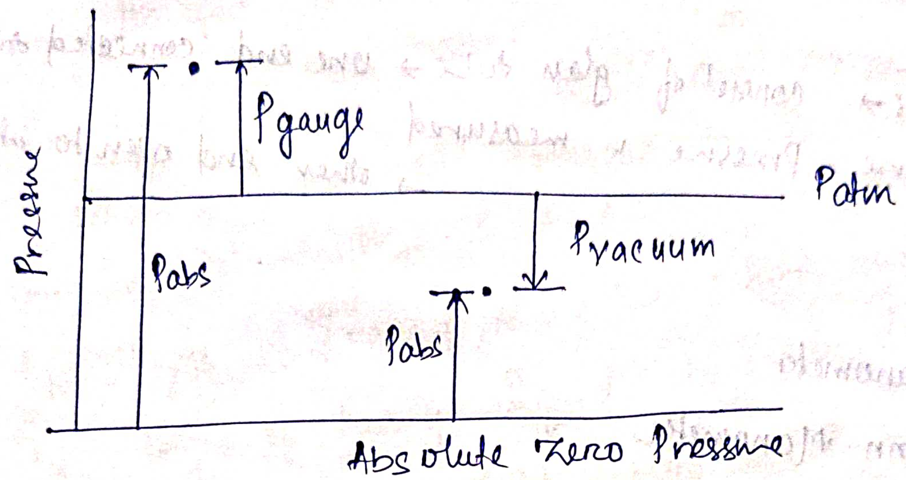
$= 22222 \, N$

$= 22.222 \, kN$

Absolute, Gauge, Atmospheric and Vaccum Pressure

Pressure of fluid measured in 2 diff system (1) It is measured above the abs. zero or complete vaccum. and it is called the abs. pressure.

(2) Pressure is measured above the atm Pressure and it is called gauge Pressure.



1. Pabs → the pressure which is measured with ref. to absolute vaccume Pressure.

2. Pgauge → measure with the help of pressure measurement instrument in which the atmospheric pressure is taken as datum.

3. Pvacuum → the pressure below the atmospheric

Mathmatically

1. $P_{abs} = P_{atm} + P_{gauge}$

2. $P_{vacuum} = P_{atm} - P_{abs}$

* Patm at sea level at 15° → $101.3\ KN/m^2$ or $10.13\ N/cm^2$

Problem

Measurement of Pressure :- measure fluid pressure

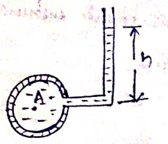
1. Manometer
2. Mechanical gauge } device

Manometer → It is defined as the device used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.

Simple manometer :→ consist of glass tube → one end connected to a point where pressure is measured → other end open to atm.

1. Piezometer
2. U-Tube manometer
3. Single column Manometer

1. Piezometer → measure the gauge pressure
one end → connected pt where pressure measured
other end → open to atm.

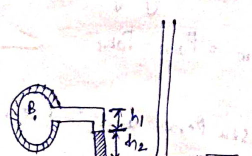


→ The rise of ~~pressure~~ liquid gives pressure head at a point

Pressure at A = $\rho g h$, N/m2

U-tube Manometer →

(i) for P gauge



(ii) for Pvacuum

U-shaped, - one end → connected to a point at which pressure to be measured
- other end → remain open to atmosphere

→ tube is generally contain mercury or any other liquid, whose sp. gravity is greater than the specific gravity of the liquid whose pressure to be maintained.

(i) For P gauge
Let B is point where pressure to be measured, whose value is P. The datum line is A-A

h_1 = Height of light liquid above the datum line
h_2 = Height of heavy liquid above the datum line
S_1 = Sp. gravity of light liquid
ρ_1 = Density of light liquid = 1000 × S_1
S_2 = Sp. gravity of heavy liquid
ρ_2 = Density of heavy liquid = 1000 × S_2

→ horizontal surface, Pressure is same, hence pressure above the horizontal datum line A-A in the left column & in the right column of U-tube manometer should be same.

Press at left column = $P + \rho g h_1$
right column = $\rho_2 g h_2$

∴ $P_1 + \rho g h_1 = \rho_2 g h_2$

$$\boxed{P_1 = \rho_2 g h_2 - \rho_1 g h_1}$$

(b) For Vaccum Pressure

Pressure above A-A in the left column

$= \rho_2 g h_2 + \rho_1 g h_1 + P$

Right. column A-A = 0

$\rho_2 g h_2 + \rho_1 g h_1 + P$

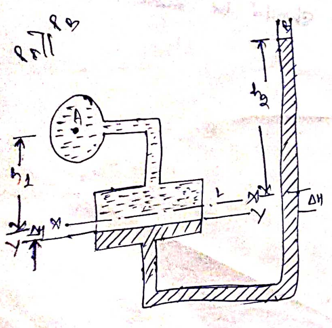
$$\boxed{P = -(\rho_2 g h_2 + \rho_1 g h_1)}$$

Single column manometer → modified from of a U-tube manometer in which large cross-sectional area (about 100 times) as compared to the area of the tube connected to one of the limbs. (Left limbs) manometer

2 types → 1. Vertical Single column manometer

2. Inclined Single column manometer

Vertical Single column manometer :-



X-X be the datum line in the reservoir

When manometer is connected to the Pipe, due to high Pressure at 'A', the heavy liquid in the reservoir will be pushed downward & will rise in the right limb.

Let Δh = Fall in heavy liquid in reservoir

h_2 = Rise in heavy liquid in right limb

h_1 = Height of center of Pipe above X-X

P_A = Pressure at A, which is to be measured

A = Cross-sectional Area of reservoir

a = Area of cross-sectional of right limb

S_1 = Sp. gravity of liquid in pipe

S_2 = Sp. gravity of heavy liquid in reservoir & right limb

ρ_1 = density of liquid in pipe

ρ_2 = density of liquid in reservoir

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in right limb.

$A \times \Delta h = a h_2 \Rightarrow \boxed{\Delta H = \frac{a h_2}{A}}$

→ area × displaced

Now consider datum line Y-Y. Then pressure in the right limb above Y-Y

$= \rho_2 \times g \times (\Delta h + h_2)$

Pressure of left limb Y-Y $= \rho_1 \times g \times (\Delta h + h_1) + P_A$

Equating $\rho_2 g(\Delta h + h_2) = \rho_1 g(\Delta h + h_1) + P_A$

$P_A = \rho_2 g(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$

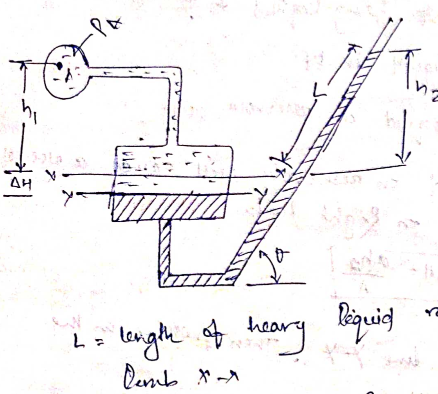
$= \Delta h(\rho_2 g - \rho_1 g) + h_2 \rho_2 g - \rho_1 g h_1$

$$\boxed{P_A = \frac{a h_2}{A}(\rho_2 g - \rho_1 g) + h_2 \rho_2 g - \rho_1 g h_1}$$

Area A is very large as compared to a, hence a/A becomes very small, can be neglected

$$\boxed{P_A = h_2 \rho_2 g - h_1 \rho_1 g}$$

Inclined Single column Manometer :- → more sensitively
→ Due to inclination the distance moved by heavy liquid in the right limb is more



L = length of heavy liquid moved in the right limb X-X

θ = Inclination of right limb with horizontal

h_2 = vertical rise of heavy liquid in right limb

X-X = L × Sin θ

From $P_A = h_2 \rho_2 g - h_1 \rho_1 g$

Substitute the value of h_2, we get

$$\boxed{P_A = L \sin\theta \, \rho_2 g - h_1 \rho_1 g}$$

$A \times \Delta H = a \times h_2 \Rightarrow \boxed{\Delta H = \frac{a h_2}{A}}$

$\rho_1 g(\Delta H + h_1) + P_A \rightarrow$ left

$h_2 = \sin\theta L$

$\sin\theta = \frac{h_2}{L} \Rightarrow h_2 = \sin\theta L$

$\Delta H = \frac{a \sin\theta L}{A}$

$\rho_2 g(\Delta H + L\sin\theta) \leftarrow$ right

$\rho_2 g(\Delta H + L\sin\theta) = \rho_1 g(\Delta H + h_1) + P_A$

$P_A = \Delta H(\rho_2 g - \rho_1 g) + L\sin\theta \rho_2 g - h_1 \rho_1 g$

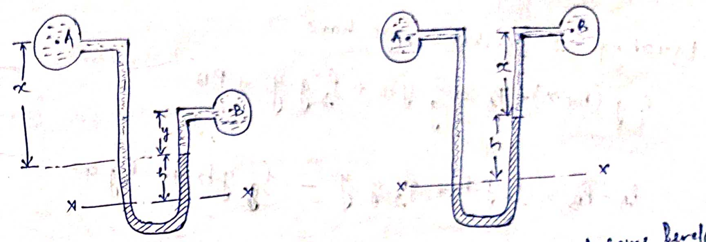
$= \underbrace{\frac{a \sin\theta L}{A}}_{small}(\rho_2 g - \rho_1 g) + L\sin\theta \rho_2 g - \rho_1 g h_1$

$\boxed{P_A = L\sin\theta \rho_2 g - \rho g h_1}$

Deffrential Manometer → device used for measuring the deffrence of Pressure betn 2 points in 2 deffrent pipes

→ It is U-shaped → It contain a heavy liquid, who ends are connected to the points, whose deffrence pressure is measured.

U-tubed Deffrential manometer :-



2 pt deffrent level (a)

A & B are at same level (b)

(a) Let 2 points A & B at deffrent level and also contain liquid of deffrent sp. gr. These points are connected to the U-tube deffrential manometer

→ Let Pressure A & B → P_A & P_B

h = Deffrence of mercury level in the U-tube

y = Distance of Center of B, from the mercury level in the right limb.

x = Distance of center of A, from the mercury level in the right limb.

ρ_1 = density of liquid at A

ρ_2 = density of liquid at B

ρ_g = Density of heavy liquid or mercury

Taking datum line at X-X'

Pressure above X-X' in the left limb $= \rho_1 g (h+x) + P_A$

P_A = Pressure at A.

~~Equating 2 pressure, we have~~

Pressure above X-X' Right limb $= \rho_g g h + \rho_2 g y + P_B$

P_B = Pressure at B

Equating 2 pressure, we have

$$\rho_1 g (h+x) + P_A = \rho_g g h + \rho_2 g y + P_B$$

$$P_A - P_B = \rho_g g h + \rho_2 g y - \rho_1 g h - \rho_1 g x$$

diff of press. $P_A - P_B = (\rho_g - \rho_1) g h + \rho_2 g y - \rho_1 g x$

(b) A & B are in same level and contain the same liquid density ρ_1, then

Pressure above X-X in Right limb $= \rho_g g h + \rho_1 g x + P_B$

left limb $= \rho_1 g (h+x) + P_A$

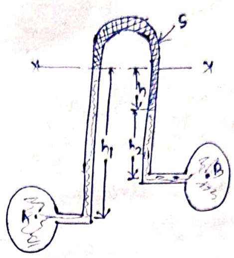
Equating 2 press

$$\rho_g g h + \rho_1 g x + P_B = \rho_1 g (h+x) + P_A$$

$$P_A - P_B = \rho_g g h + \rho_1 g x - \rho_1 g (h+x)$$

$$= \rho_g g h - \rho_1 g h = \boxed{g h (\rho_g - \rho_1) = P_A - P_B}$$

Inverted U-tube manometer → consist of inverted U-tube, containing light liquid



→ 2 ends of the tube are connected to the points whose pressure difference is measured.

→ measuring diff of low Pressure.

→ Inverted U-tube differential manometer connected to the 2 points A & B.

Let the Pressure at A is more that that of Pressure at B.

h_1 = height of liquid in left limb below the datum line X-X

h_2 = " " " Right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of the ~~liquid~~ light liquid

P_A = Pressure at A

P_B = Pressure at B

Taking X-X as datum line, then pressure in the left limb below X-X

$= P_A - \rho_1 g h_1$

Pressure in the Right limb below X-X

$= P_B - \rho_2 g h_2 - \rho_s g h$

Equating $P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_s g h$

$$\boxed{P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_s g h}$$

Fluid Kinematics

Fluid Flow

→ deals with motion of particle without considering the forces.

→ velocity at a point on flow field at any time

Types of Flow → 1) steady & unsteady flow → $\left(\frac{\partial v}{\partial t}\right) \neq 0$, $\left(\frac{\partial p}{\partial t}\right) \neq 0$, $\left(\frac{\partial \rho}{\partial t}\right) \neq 0$

→ flow parameter (v, ρ, p etc)

→ parameter not change w.r.t time

$\left(\frac{\partial v}{\partial t}\right) = 0$ $\quad \partial p/\partial t = 0$ $\quad \frac{\partial \rho}{\partial t} = 0$

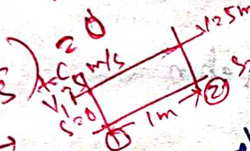
$dv = v_2 - v_1 = 5 - 5 = \frac{1 - 0}{} \quad \frac{0}{1} = 0$

$dt = t_2 - t_1$

2) Uniform & Non-uniform ~~turbulent~~ flow

→ flow in which the velocity at given time does not change w.r.t space (length of dir of flow) ← unit distance

→ $\left(\frac{\partial v}{\partial s}\right)_{t=c} = 0$



Non-uniform → the flow in which the velocity at give space change w.r.t space

→ $\left(\frac{\partial v}{\partial s}\right)_{t=c} \neq 0$

3) laminar & turbulent flow

fluid particle move along straight line & parallel

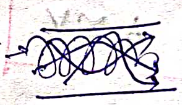
→ Re < 2000 → losses are less

→ Flow due to gravity

turbulent → zig-zag way

→ high energy loss

→ Re > 4000



4) Incompressible & Compressible flow

→ ρ = constant

→ liquid are incompressible

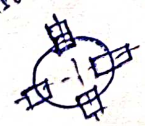
where gases are compressible

Compressible → Flow in which density of fluid changes point to point

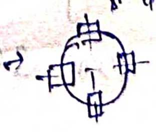
$\rho \neq$ constant

5) Rotational & Irr-rotational flow

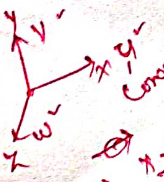
→ fluid particals flow along stream lines & also rotate their own axis



Irr-rotational → fluid particles flow along stream lines & do not rotate about their own axis



6) One, two, three dimensional flow



component

→ x-axis, one co-ordinate

velocity $V = f(x)$, $u = f(x)$

x-dir component

→ Flow velocity is function of time & space co-ordinate

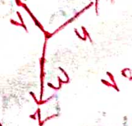
y v x u

$v = f(x)$

$u = f(x, y)$

$v = f(x, y)$

Velocity is function of time & rectangular space coordinate (x, y)



$v = f(x)$

$v = f(x, y, z)$

$u = f(x, y, z)$

$v = f(x, y, z)$

$w = f(x, y, z)$

→ velocity is function of time & 3-mutually dir directions

Rate of flow or Discharge (Q)

→ The quantity of fluid flowing per sec through a section of a pipe or a channel

→ Incompressible fluid

→ The rate of discharge is expressed as the volume of fluid flowing across the ~~cro~~ section per sec.

(1) For liquid the unit of Q are m^3/s or liter/s

(2) For gases the unit of Q is kgf/s or N/s

Consider liquid flow through pipe line

A = Cross sectional area of pipe line

V = Avg. velocity of fluid across the section

The discharge $\boxed{Q = A \times V}$

Continuity Equation :-

→ The eq^n^ based on the principle of conservation of mass is called continuity equation.

→ A fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

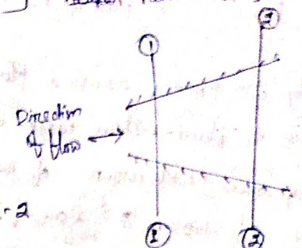
→ Consider 2 cross-section of a pipe

let V_1 = Avg Velocity at cross-section 1-1

ρ_1 = Density at section 1-1

A_1 = Area of pipe at section 1-1

& V_2, ρ_2, A_2 are corresponding ~~Values~~ values at section 2-2



The rate of flow at section 1-1

$= \rho_1 A_1 V_1$

The rate of flow at section 2-2

$= \rho_2 A_2 V_2$

According to Law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$ ← It is applicable to the compressible as well as incompressible fluid and is called Continuity equation.

If fluid is incompressible $\rho_1 = \rho_2$

then continuity eq^n^ . $\boxed{A_1 V_1 = A_2 V_2}$

Dynamics of fluid Flow

→ Study of forces causing flow

→ study of fluid motion with the forces causing flow.

→ The dynamics behaviour of the fluid flow is analyzed by the Newtons 2nd law of motion, which relates acceleration with forces.

→ The fluid is assumed to be incompressible & non-viscous

Equation of motion

Acc. to Newtons 2nd law of motion, the net Force F_x acting on fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-direction. Thus

mathmatically $\boxed{F_x = m\, a_x}$

In fluid flow, the following forces are present

1 - F_g - gravity force
2 - F_p - Pressure force
3 - F_v - Force due to viscosity
4 - F_t - Force due to turbulence
5 - F_c - Force due to compressibility

Thus in equation, the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

(1) If the force due to compressibility, ... is negligible, the

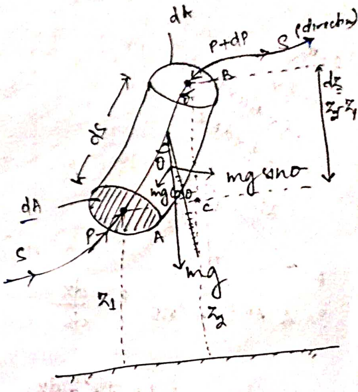
Net force $\boxed{F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x}$

Reynolds equation of motion

(2) For flow, (F_t) negligible, the resulting equation of motion are known as Navier-stokes eqn.

(3) If Flow assumed to be Ideal, viscous force (F_v) is zero, the eqn of motion is known as Euler's equation of motion

Euler's equation of Motion :-



→ flow S (direction)

→ Cylindrical section element

→ flow is steady $\frac{dv}{dt} = 0$

→ The fluid is incompressible, $\rho = K$

→ Fluid is non-viscous (Ideal) layer, no viscous

→ flow is irrotational.

dp = change in pressure due to pressure diff, the fluid is flow

1) Pressure force (P, dP+P)

2) Gravity force (mg,)

$P = F/A \Rightarrow F = PA$

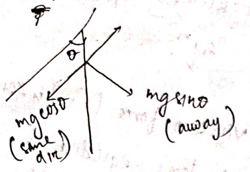
$F_{entry} = P\,dA \qquad F_{Exit} = (P + dP)\, dA$

$F_{pressure} = F_{entry} - F_{exit}$

$F_P = pdA - (p+dp)dA$

$F_P = \cancel{pdA} - \cancel{pdA} - dpdA$

$F_P = -dpdA$



Weight $= mg\cos\theta$

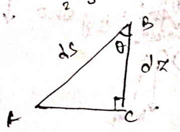
$= \rho dA \times ds\, g\cos\theta$

$\rho = m/V$

$m = \rho V$

$= \rho \times \text{Area} \times \text{length}$

$= \rho \times dA \times ds$



In Δ ABC → $\cos\theta = \frac{B}{H} = \frac{dz}{ds}$

$= F_g = \rho\, dA\, \cancel{ds}\, g \frac{dz}{\cancel{ds}}$

$\boxed{\text{Weight} = \rho \cdot dA\, g\, dz}$

Force due to gravity

$\boxed{F_G = \rho g\, dA\, dz}$

Pressure force F_P ↗ dire

F_G ↙

flow ↗

Net force $= F_P - F_G$

responsible for travel fluid upper side

$\boxed{= -dpdA - \rho g\, dA\, dz}$ —(1)

Newtons law of motion

$F = ma$ ($F = F_{net}$ force act on fluid element)

$F = \rho\, dA\, ds \times a$ $\rho = \frac{m}{V} \Rightarrow m = \rho V$ $\boxed{m = \rho\, dA\, ds}$

$a = \frac{dV}{dt}$

$= \frac{dV}{ds} \times \frac{ds}{dt} = \frac{dV}{ds} \times V$

$F = \rho\, dA\, ds \frac{dV}{ds} V$

$F_{net} = \rho\, dA\, dV\, V$ —(2)

From (1) & (2)

$-dpdA - \rho g\, dA\, dz = \rho V dA\, dV$

$\frac{-dpdA - \rho g\, dA\, dz}{\rho g\, dA} = \frac{\rho dA\, V dV}{\rho g\, dA}$

$-\frac{dp}{\rho g} - dz = \frac{V dV}{g}$

$\Rightarrow \boxed{\frac{V dV}{g} + \frac{dp}{\rho g} + dz = 0}$ ← Eulers eqn of motion

by Integrating

$\int \frac{V}{g} dV + \int \frac{dp}{\rho g} + \int dz = \int 0$ ← Indefinite Integral (no limit)

$\frac{V^2}{2g} - C_1 + \frac{p}{\rho g} + C_2 + z + C_3 = C_4$

$\frac{V^2}{2g} + \frac{p}{\rho g} + z = C_4 - C_1 - C_2 - C_3$

$= C_5$

$\boxed{\frac{V^2}{2g} + \frac{p}{\rho g} + z = C}$ ← bernoullis eqn

Bernoullis eqn from Eulers equation

$$\frac{v\,dv}{g} + \frac{dp}{\rho g} + dz = 0$$

Integrating

$$\int \frac{v}{g}\,dv + \int \frac{dp}{\rho g} + \int dz = \text{Constant}$$

$$\boxed{\frac{v^2}{2g} + \frac{p}{\rho g} + z = \text{Constant}}$$

If flow is incompressible, ρ = Constant

$\frac{v^2}{2g}$ = Kinetic energy per unit weight or Kinetic head

$p/\rho g$ = Pressure energy per unit weight of fluid or Pressure head

z = Potential energy per unit weight or potential head

Practical Application of Bernoullis equation :-

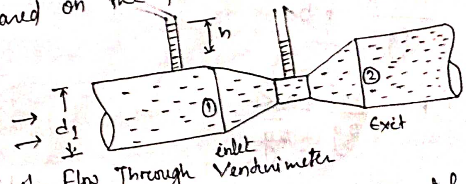
→ It is applied in all problems of incompressible fluid flow, where energy consideration are involved. Its application are

1. Venturimeter
2. Orifice meter
3. Pitot tube

Venturimeter → A device used for measuring the rate of a flow of a fluid flowing through a pipe. It has 3-parts

1. A short converging part
2. Throat
3. Deverging part

It is based on the principle of bernollis equation.



Expression of Rate of Flow Through Venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing

let d_1 = diameter at inlet or at section (1)

p_1 = Pressure at section (1)

v_1 = velocity at section (1)

a = area of Cross-section (1) = $\frac{\pi}{4} d_1^2$

d_2, p_2, v_2, a_2 are corresponding values at section (2)

Applying Bernoulli's equation at section (1) & (2), we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As pipe is horizontal, hence $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

difference of pressure head

$$\frac{P_1 - P_2}{\rho g} = h$$

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

Now appling continuity eq^n (1) & (2) section

$$a_1 V_1 = a_2 V_2$$

$$\boxed{V_1 = \frac{a_2 V_2}{a_1}}$$

Substitute this value of V_1 in equation

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$$h = \frac{V_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$\Rightarrow V_2^2 = 2gh\left[\frac{a_1^2}{a_1^2 - a_2^2}\right]$$

$$\Rightarrow V_2 = \sqrt{2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right)}$$

$$\boxed{V_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}}$$

Discharge $Q = a_2 V_2$ ✓

$$Q = a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

$$\boxed{Q = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}}$$

← Eq^n gives the discharge under ideal cond^n called, theoretical discharge

∴ Actual discharge is less than theoretical discharge

$$\boxed{Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}}$$

C_d = Co-efficent of Venturimeter and its value is less than 1.

Value of 'h' given by differential U-tube manometer

Case-I → Let the differential manometer contain a liquid which is heavier than the liquid flowing through the pipe.

Let S_h = Sp. gravity of the heavier liquid
S_0 = Sp. gravity of the liquid flowing through pipe
x = Deffrence of the heavier liquid column in U-tube.

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

Case-II → If the defferential manometer contain a Liquid which is lighter than the liquid flowing through the pipe, the value 'h' is as given by

$$h = x\left[1 - \frac{S_l}{S_0}\right]$$

Where S_l = Sp. gravity of lighter liquid in U-tube
S_0 = Sp. gravity of fluid flowing through pipe.
x = Deffrence of lighter liquid column in U-tube.

Case-III → Inclined ventumeter with Deffrential U-tube manometer

Let the defferential manometer contain heavier liquid the height 'h' is given as

$$h = \left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = x\left[\frac{S_h}{S_0} - 1\right]$$

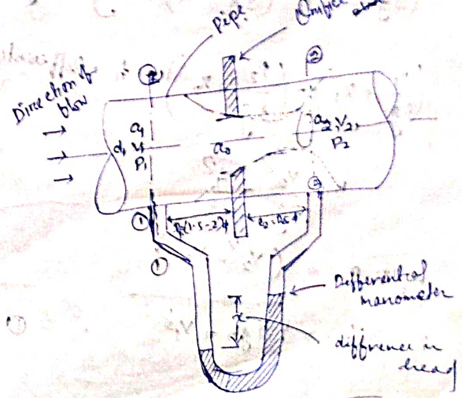
Case-IV → Inclined ~~manometer~~ venturimeter in which differential defferential manometer contain a liquid which is lighter than the liquid flowing through the pipe, the value h is given by

$$h = \left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = x\left[1 - \frac{S_l}{S_0}\right]$$

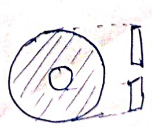
Orifice Meter or Orifice plate :-

→ It is a device used for measuring the rate of flow of a liquid through the pipe

→ cheaper as compared to venturimeter
→ work same principle of manometer
→ It consist of a flat circular plate, which has sharp circular edge hole called orifice, which is concentric with pipe
→ The orific diameter is kept generally 0.5 times the diameter of pipe, through it may vary from 0.4 - 0.8 times the pipe diameter.



(2) → vena contracta
c/s reduces
fluid jets,
$d\downarrow$ $V\uparrow$, $P\downarrow$



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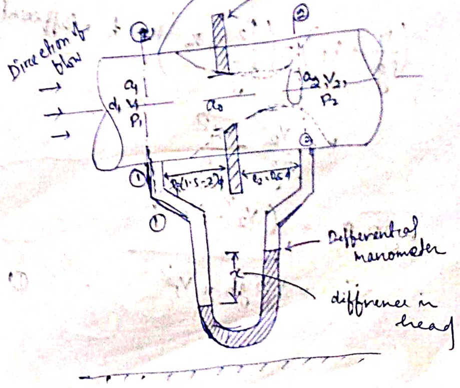
(2) → vena contracta

c/s reduces

fluid jets,

d↓ v↑, P↓



A differential manometer is connected at section (1) which is at a distance about 1.5 to 2.0 times the pipe diameter up stream from orifice plate.

At section (2), which is at a distance about half the diameter of the orifice on the down stream side from orifice plate.

Let P_1 = Pressure at section (1)

v_1 = velocity at section (1)

a_1 = area of pipe at section (1),

and P_2, v_2, a_2 are corresponding values at section (2).

Applying bernoullis equation at section (1) & (2), we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = h = \text{Differential head}$$

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$2gh = (v_2^2 - v_1^2)$$

$$v_2 = \sqrt{2gh + v_1^2} \quad \text{--- (1)}$$

Now section (2) is at the vena contracta and

a_2 = area of the vena contracta

if a_0 = area of the orifice

then, we have

$$C_c = \frac{a_2}{a_0} \qquad C_c = \text{Co-efficient of contraction}$$

$$a_2 = a_0 \times C_c \quad \text{--- (2)}$$

By continuity eq^n

$$\boxed{a_1 v_1 = a_2 v_2} \Rightarrow v_1 = \frac{a_2 v_2}{a_1} = \frac{a_0 C_c v_2}{a_1} \quad \text{--- (3)}$$

Subsitute v_1 value in eq^n (1)

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

$$v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2 \rightarrow v_2^2 - \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2 \Rightarrow v_2^2\left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

$$v_2^2 = 1 + -\left(\frac{a_0}{a_1}\right)^2 C_c^2$$

$$v_2^2\left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

$$v_2 = \frac{\sqrt{2gh}}{\sqrt{\left(1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right)}}$$

The discharge $Q = v_2 a_2 = v_2 a_0 C_c$

$$Q = \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - (a_0/a_1)^2 C_c^2}} \quad \text{--- (4)}$$

thanks..!

The above eq^n simplified

$$C_d = C_c \frac{\sqrt{1-(a_0/a_1)^2}}{\sqrt{1-(a_0/a_1)^2 C_c^2}}$$

Relation of C_d & C_c

$$C_c = C_d \times \frac{\sqrt{1-(a_0/a_1)^2 C_c^2}}{\sqrt{1-(a_0/a_1)^2}}$$

Substitute C_c eq^n in eq^n (4)

$$Q = a_0 \times C_d \frac{\sqrt{1-(a_0/a_1)^2 C_c^2}}{\sqrt{1-(a_0/a_1)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-(a_0/a_1)^2 C_c^2}}$$

$$= \frac{C_d\, a_0 \sqrt{2gh}}{\sqrt{1-(a_0/a_1)^2}}$$

$$\boxed{Q = \frac{C_d\, a_0\, a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

$$C_d = \frac{Q_{ac}}{Q_{the}}$$

< 1 when

$= 0.7 - 0.85$

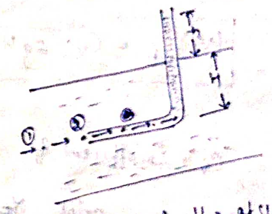
Pitot - tube

A device used for measuring the velocity of flow at any point in a pipe channel. It is based on principle, that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the K.E into P.E

→ The pitot tube consist of a glass tube, bent at right angles

* The lower end bent through 90°. The rise up in the tube due to the conversion of K.E into P.E.

* The velocity is determined by measurement of the rise of liquid in the tube



Consider a point 1 & 2 at same level
Such pt (2) just as inlet of the pitot tube
(1) far away from the tube

P_1 = intensity of pressure at point (1) , v_1 = velocity of flow at (1)

P_2 = " " (2) v_2 = " " (2)

H = Depth of tube in the liquid

h = Rise of liquid in the tube above the free surface

Applying Bernoulli's eq^n at point (1) & (2) we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) & (2) we get

$\frac{P_1}{\rho g}$ = Pressure head at (1) = H

$\frac{P_2}{\rho g}$ = Pressure head at (2) = (h+H)

Substitute these values, we got

$$H + \frac{V_1^2}{2g} = (h+H)$$

$$h = \frac{V^2}{2g} \quad V^2 = 2gh \quad = \boxed{V=\sqrt{2gh}}$$ ← theoretical velocity

$$\boxed{(V_1)_{act} = C_v\sqrt{2gh}}$$ ← Actual velocity

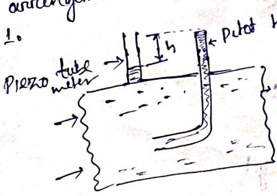
C_v : Co-efficient of pitot tube

Velocity at any point

$$\boxed{V = C_v\sqrt{2gh}}$$

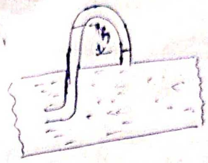
Velocity of flow in a pipe by pitot tube: For finding the velocity at any point in a pipe by pitot tube, the following arrangement ~~provided~~ adopted.

1.



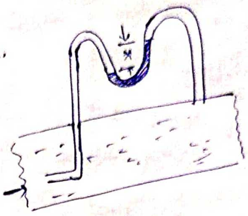
Pitot tube along with a vertical piezometer tube

2.



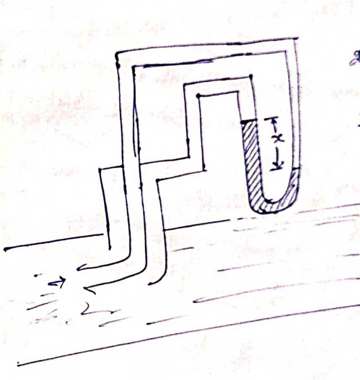
Pitot tube connected with piezometer tube

3.



Pitot tube with vertical piezometer tube connected with a differential U-tube manometer

4.



Pitot-static tube, which consist of 2 circular concentric tube one inside & other with some annular space in between. The outlet of these 2 tubes are connected to the differential manometer, where the difference of pressure head 'h' is measured by knowing the difference of the level of the manometer liquid 'x',

Then $h = x\left[\frac{S_g}{S_o} - 1\right]$