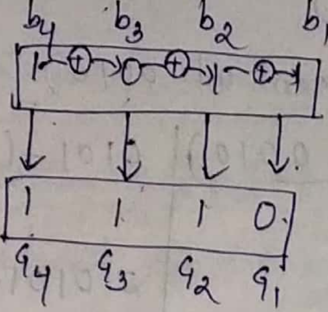
Dt. 23/10/22.

Code conversion: Prepared By - Lipsa Panigrahi Lect. (Electrical)

(a) Binary → Gray



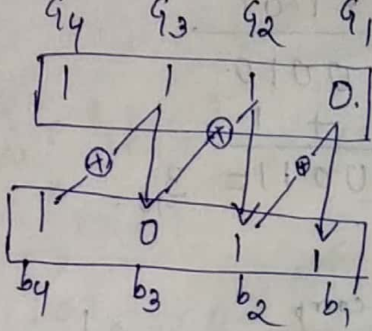
$g_4 = b_4$

$g_3 = b_4 \oplus b_3$

$g_2 = b_3 \oplus b_2$

$g_1 = b_2 \oplus b_1$

$(\cancel{36})_{10}$ (b) Gray → binary :-



$b_4 = g_4$

$b_3 = b_4 \oplus g_3$

$b_2 = b_3 \oplus g_2$

$b_1 = b_2 \oplus g_1$

↓

modified form

$b_4 = g_4$, $b_3 = g_4 \oplus g_3$

$b_2 = g_4 \oplus g_3 \oplus g_2$,

$b_1 = g_4 \oplus g_3 \oplus g_2 \oplus g_1$

Imp EX-OR gate also named as modulo - 2 addn.

$0+0=0$

$0+1=1$

$1+0=1$

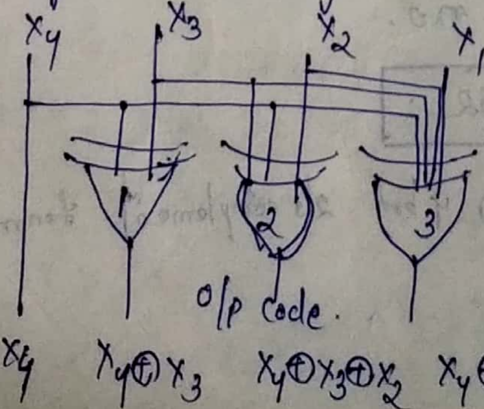
$1+1=0$

pb:- Represent $(56)_8$ in gray code.

$(56)_8 \rightarrow (1\ 0\ 1\ 1\ 1\ 0)_2$

gray code $= (1\ 1\ 1\ 0\ 0\ 1)$

pb:- Identify the following code converter.



x_4 $\quad x_4 \oplus x_3$ $\quad x_4 \oplus x_3 \oplus x_2$ $\quad x_4 \oplus x_3 \oplus x_2 \oplus x_1$

Gray to binary code converter.

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Boolean Algebra :-

'AND' law and 'OR' law.

Identity elements. AND → '1', OR → '0'.

$A \cdot 0 = 0$ $\qquad A + 0 = A$

$A \cdot 1 = A$ $\qquad A + 1 = 1$

① **Commutative law :-** $A+B = B+A$

Dual: $A \cdot B = B \cdot A$

$A \oplus B = B \oplus A$ $\qquad A \uparrow B = B \uparrow A$

Dual: $A \odot B = B \odot A$ $\qquad = \overline{A \cdot B} = \overline{B \cdot A}$

$A \downarrow B = B \downarrow A$.

$\uparrow$ = NAND

$\downarrow$ = NOR

② **Associative law :-** $(A+B)+C = A+(B+C)$

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

Drop $(A \uparrow B) \uparrow C \neq A \uparrow (B \uparrow C)$

*(NAND and NOR operations are not associative)

$(A \oplus B) \oplus C = A \oplus (B \oplus C)$

$(A \odot B) \odot C = A \odot (B \odot C)$

③ **Consensus law :-** (i) $A \cdot B + \bar{A} \cdot C + \cancel{BC} = A \cdot B + \bar{A} \cdot C$.

(ii) $xy + \bar{y} \cdot z + \cancel{v\bar{w}xz} = xy + \bar{y}z$

Proof :- $AB + \bar{A}C + BC(A+\bar{A})$

$= AB + \bar{A}C + BCA + BC\bar{A} = AB(1+C) + \bar{A}C(1+B)$

$= A \cdot B + \bar{A} \cdot C$. (R.H.S)

(4) **Transposition law :-** $A \cdot B + \bar{A} \cdot C = (A+C) \cdot (\bar{A}+B)$

(5) **Distributive law :-** $A \cdot (B+C) = A \cdot B + A \cdot C$

Dual: $A + B \cdot C = (A+B) \cdot (A+C)$

(i) $\bar{A} + AB = \bar{A} + B$

(ii) $\underbrace{AB}_{x} + \underbrace{\overline{AB}}_{\bar{x}}C \Rightarrow x + \bar{x}C$

$= x + C$

$= AB + C$

⑥ De Morgan's law: (i) NOR gate = bubbled AND gate.

$\overline{A+B+C+\cdots} = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdots$

(ii) NAND gate = bubbled OR gate

$\overline{A\cdot B\cdot C\cdots} = \bar{A}+\bar{B}+\bar{C}+\cdots$

Additional law:- To find the complement of a boolean function 'f' (i) find dual of 'f'. i.e. 'f_D'

(ii) Complement all variables

eg:- $F = \bar{A}\cdot(B+\bar{C})(\bar{D}+E)$

$\bar{F} = ?$

Solⁿ (i) $f = \bar{A} + B\bar{C} + \bar{D}E$

(ii) $\boxed{\bar{F} = A + \bar{B}C + D\bar{E}}$

→ Simplify the following boolean expressions.

(a) $A \oplus B \oplus AB = ?$

Solⁿ:- $\underbrace{A \oplus B}_{x} \oplus AB = \bar{x}\cdot AB + x\cdot\overline{AB}$ $A\odot B = \bar{A}\bar{B} + AB$

$= \overline{A\oplus B}\cdot AB + (A\oplus B)\cdot\overline{AB}$

$= (A\odot B)\cdot AB +$ ''

$= (\bar{A}\bar{B}+AB)\cdot AB + (\bar{A}B + A\bar{B})\cdot(\bar{A}+\bar{B})$

$= 0 + AB + \bar{A}B + 0 + 0 + A\bar{B}$

$= AB + \bar{A}B + A\bar{B}$

$= A + B$

(b) If $A\oplus B\oplus C = D$ then find $A\oplus B\oplus D = ?$

Solⁿ:- $\underbrace{A\oplus B}_{x}\oplus \underbrace{A\oplus B}_{x}\oplus C$

$\Rightarrow x \oplus x \oplus C$

$\Rightarrow 0 \oplus C$

$\Rightarrow C$

Q11 Determine the no. of two i/p NAND gates required to implement the following functions.

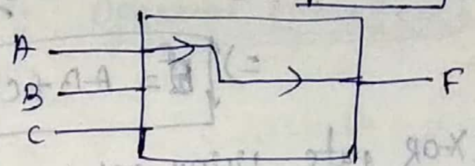
(i) $F = A + \bar{A}\bar{B} + AB\bar{C}$

Sol: $F = \overline{\overline{A + \bar{A}\bar{B} + AB\bar{C}}}$ $= \overline{\bar{A} \cdot \overline{\bar{A}\bar{B}} \cdot \overline{AB\bar{C}}}$

$F = A + A\bar{B} + AB\bar{C}$

$= A\left[1 + \underbrace{\bar{B} + B\bar{C}}_{x}\right]$

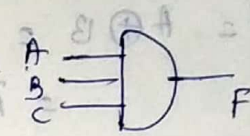
$\boxed{F = A}$ (as $1 + x = 1$)



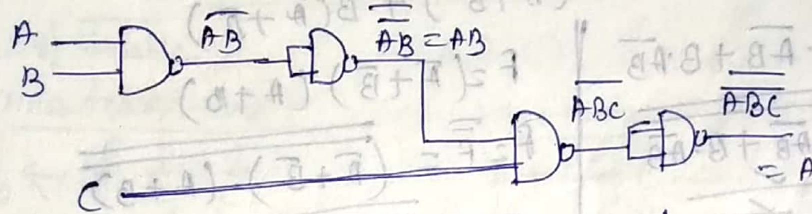
'0' NAND gates.

(ii) How many NAND gates are required

$F = ABC \longrightarrow$ 3-i/p AND gate



Sol: $F = \overline{\overline{\overline{\overline{AB}}\,C}}$



(iii) How many NAND gates are required

$F = ABCD$

Sol: $\overline{\overline{\overline{\overline{AB}}\;\overline{\overline{CD}}}} \longrightarrow$ (6) 2-i/p NAND gates.

(iv) $F = \overline{ABCD} \longrightarrow$ 4 i/p NAND gate.

Sol: $F = \overline{\overline{\overline{\overline{AB}}\;\overline{\overline{CD}}}} = \overline{\overline{\overline{AB}} \cdot \overline{\overline{CD}}} \longrightarrow$ (5) NAND gates.

Imp Q(V) simplify $F(A,B,C) = \bar{A}C + \bar{C}D + \bar{B}C + AB$ to 4 literals

$\bar{A}C = \bar{A}C(B+\bar{B}) = \bar{A}CB + \bar{A}C\bar{B}$

$= \bar{A}CB(D+\bar{D}) + \bar{A}C\bar{B}(D+\bar{D})$

$= \bar{A}CBD + \bar{A}CB\bar{D} + \bar{A}C\bar{B}D + \bar{A}C\bar{B}\bar{D}$

$\bar{C}D = \bar{C}D(A+\bar{A}) = \bar{C}DA + \bar{C}D\bar{A}$

$= \bar{C}DA(B+\bar{B}) + \bar{C}D\bar{A}(B+\bar{B})$

$= \bar{C}DAB + \bar{C}DA\bar{B} + \bar{C}D\bar{A}B + \bar{C}D\bar{A}\bar{B}$

$\bar{B}C = \bar{B}C(A+\bar{A}) = \bar{B}CA + \bar{B}C\bar{A}$

$= \cancel{\bar{B}CA(D+\bar{D}) + \bar{B}C\bar{A}(D+\bar{D})}$

$= \bar{B}CAD + \bar{B}CA\bar{D} + \bar{B}C\bar{A}D + \bar{B}C\bar{A}\bar{D}$

Literal = Variables (or) complement of a variable.

$F(A,B) \rightarrow A, \bar{A}, B, \bar{B} \leftarrow$ 4 literals

$F(A,B,C) = A, \bar{A}, B, \bar{B}, C, \bar{C} \leftarrow$ 6 literals.

Sol?ⁿ:- $F(A,B,C) = \bar{A}C + \bar{C}D + \bar{B}C + AB$

$= (\bar{A}+\bar{B})C + \bar{C}D + AB$

$\cancel{= AB + C}$

$= \overline{AB}C + \bar{C}D + AB = AB + C + \bar{C}D$

$\Rightarrow \boxed{F = AB + C + D} \rightarrow$ 4 literals.

Q// Implement EX-OR gate using minimum no. of (a) NAND gates (b) NOR gates.

Sol?:- $F = A \oplus B = \bar{A}B + A\cdot\bar{B}$

$= \bar{A}B + A\bar{B} + A\cdot\bar{A} + B\cdot\bar{B}$

$= A(\bar{A}+\bar{B}) + B(\bar{A}+\bar{B})$

$F = A\cdot\overline{AB} + B\cdot\overline{AB}$

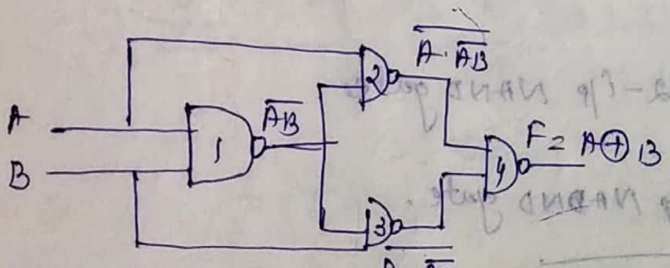
$F = \overline{\overline{F}} = \overline{\overline{A\cdot\overline{AB} + B\cdot\overline{AB}}}$

$F = \overline{\overline{A\cdot\overline{AB}}\cdot\overline{B\cdot\overline{AB}}}$

$F = (\bar{A}+\bar{B})(A+B)$

$F = \overline{\overline{F}} = \overline{\overline{(\bar{A}+\bar{B})\cdot(A+B)}}$

$F = \overline{\overline{\bar{A}+\bar{B}} + \overline{A+B}}$



Fig(1):- EX-OR using min no. of NAND gates.

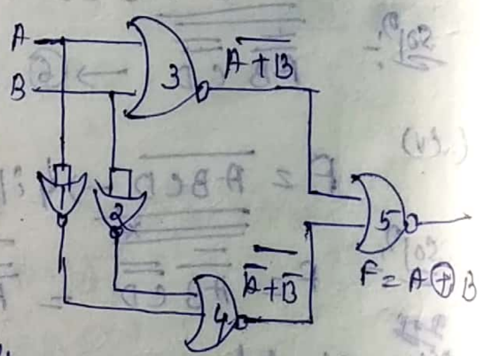
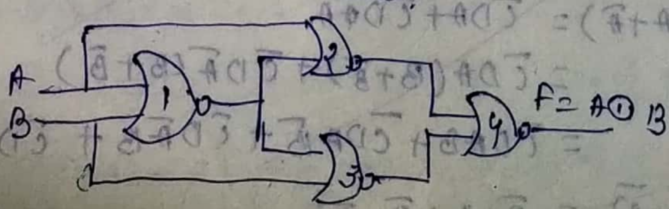


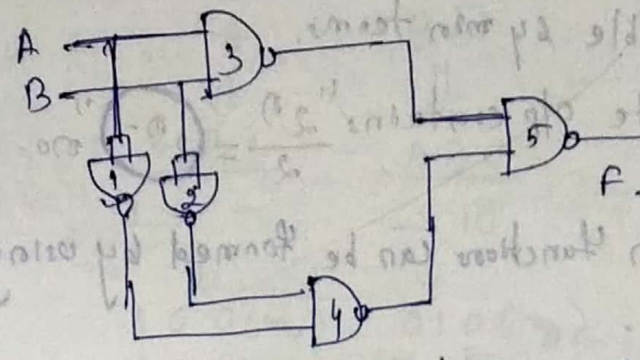
Fig 2:- EX-OR using min no. of NOR gates.

The above ckts are converted to EX-NOR gate as follows:-

(1) In fig(1) replace all NAND gates with NOR gates as shown below:-



(ii) In Fig(2) replace all NOR gates by NAND gates



$F = A \odot B$

Operator precedence:-

(1) parantheeis ( )

(2) NOT

(3) AND

(4) OR → least priority.

Min terms, Max terms:-

Variable = 1, $\overline{\text{Variable}} = 0$ F(A, B, C)   Variable = 0, $\overline{\text{Variable}} = 1$

(m_0 to m_7) min terms → 8   Max terms → 8 (M_0 to M_7)

$m_0 \leftarrow \overset{0}{\bar A}\cdot\overset{0}{\bar B}\cdot\overset{0}{\bar C}$ $\overset{1}{\bar A}+\overset{1}{\bar B}+\overset{1}{\bar C} \longrightarrow M_7$

$m_1 \leftarrow \overset{0}{\bar A}\cdot\overset{0}{\bar B}\cdot\overset{1}{C}$ $\overset{1}{\bar A}+\overset{1}{\bar B}+\overset{0}{C} \longrightarrow M_6$

1 1 1   1 1 1

$m_6 \leftarrow \overset{1}{A}\cdot\overset{1}{B}\cdot\overset{0}{\bar C}$ $\overset{0}{A}+\overset{0}{B}+\overset{1}{\bar C} \longrightarrow M_1$

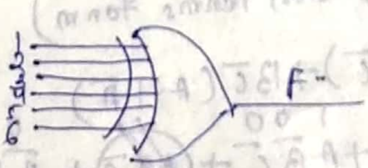
$m_7 \leftarrow \overset{1}{A}\cdot\overset{1}{B}\cdot\overset{1}{C}$ $\overset{0}{A}+\overset{0}{B}+\overset{0}{C} \longrightarrow M_0$

Property:- (i) $\overline{M_j} = m_j$ and vice versa.

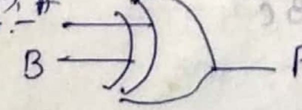
(ii) (a) sum of all min terms = 1 i.e. $\sum_{i=0}^{(2^n-1)} m_i = 1$

(b) The product of all max terms = 0 i.e. $\prod_{j=0}^{(2^n-1)} M_j = 0$.

pb//:- How many min terms are present at the o/p of 6 i/p EX-OR gate.



Soln:-



$F = \overset{0}{\bar A}\overset{1}{B} + \overset{1}{A}\overset{0}{\bar B}$ (2 out of 4 min terms)

$= m_1 + m_2$

$F = m_1 + m_2 + m_3 + m_4$ (4 out of 8 min terms) for 6 i/p EX-OR gate the total no. of min terms are 32 out of possible 64 min terms.

→ n i/p EX-OR gate o/p contains $\frac{2^n}{2} = 2^{n-1}$ no. of min terms.

→ How many Boolean functions can be formed by using 'n' boolean variables.

n-Boolean variables ⟶ Boolean function

$x = 2^n$ min terms ⟶ $2^x = 2^{2^n}$ ✓

F (A, B)

	A	B	F_0	F_1	F_2	F_3	—	F_{15}
m_0	0	0	0	0	0	0		1
m_1	0	1	0	0	0	0		1
m_2	1	0	0	0	1	0		1
m_3	1	1	0	1	0	1		1
			ϕ(Null)	AND	inhibition			identity

$F_{15} \rightarrow m_0 + m_1 + m_2 + \cdots$

$m_2 = A \cdot \bar{B}$

$= A/B$ (Inhibition)

Forms of Boolean functions:

① SOP (sum of products) form

② POS (product of sums) form

③ Cannonical form ⟶ Cannonical sop form / sum of min terms form
⟶ Cannonical pos form / product of maxterms form

→ Convert $F(A,B,C) =$ ~~A·(B+C̄) into Can~~ $A + B\bar{C}$ into Cannonical SOP form. (sum of min terms form)

Soln.:

$F = A(B+\bar{B})(C+\bar{C}) + B\bar{C}(A+\bar{A})$

$F = \overset{111}{ABC} + \overset{110}{AB\bar{C}} + \overset{101}{A\bar{B}C} + \overset{100}{A\bar{B}\bar{C}} + \cancel{AB\bar{C}} + \overset{010}{\bar{A}B\bar{C}}$

$= m_7 + m_6 + m_5 + m_4 + m_2$

$\boxed{F = \Sigma m(2,4,5,6,7)}$ → cannonical sop form.

$\boxed{F = \Pi M(0,1,3)}$ ← cannonical pos form.

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Method-2:

$\underset{A}{1}$ $\Downarrow$

A	B	C
1	−	−

$\downarrow$

100 (m_4)
101 (m_5)
110 (m_6)
111 (m_7)

$\underset{B\ \bar{C}}{1\ 0}$ $\Downarrow$

A	B	C
−	1	0

$\downarrow$

010 (m_2)
~~110~~ (repetation)

$F = \Sigma(2,4,5,6,7)$

→ convert the following expression into cannonical pos form.

$F(x,y,z) = \bar{x}\cdot(\bar{x}+\bar{y}+\bar{z})(y+\bar{z})$ → cannonical pos form (product of max terms form)

Sol^n:-

$\frac{1}{x}$

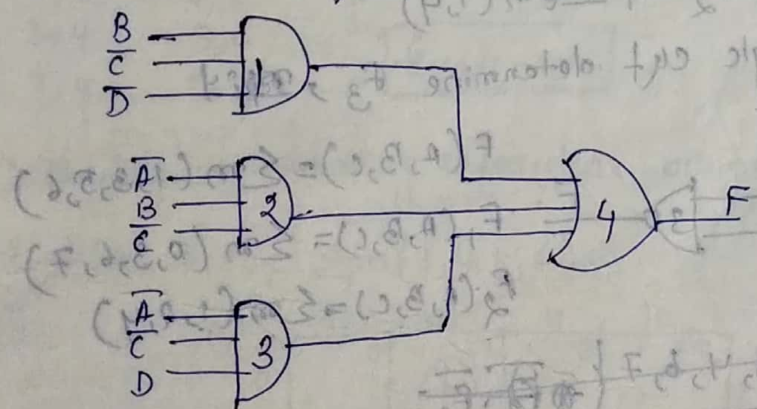
x+y+z
1 0 0
1 0 1
1 1 0
1 1 1

$\underset{y+\bar{z}}{0\ 1}$

x+y+z
0 0 1
~~1 0 1~~

$$\boxed{F = \Pi M(\cancel{0,4,2,3,5,7})(1,4,5,6,7)}$$

Imp → In the following logic cut identify the redudant gate?



Sol^n:- $F = (B\bar{C}\bar{D}) + (\bar{A}B\bar{C}) + (\bar{A}\bar{C}D)$

~~$B\bar{C}\bar{D}$~~

	ABCD
m_4 ←	0100
m_{12} ←	1100

ABCD
0100
0101

m_4 m_5

ABCD
0001
0101

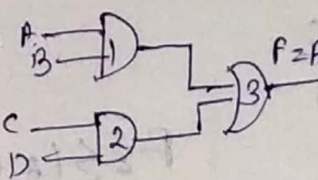
m_1 m_5

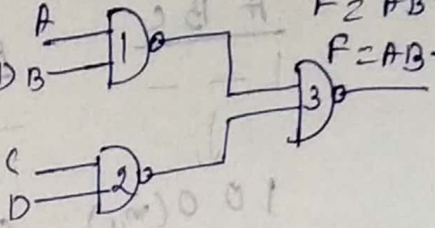
gate-2 is redudant gate.

Two-level logic:-

① SOP form :- ⇒ $\boxed{\text{AND-OR}}$ logic. $\equiv$ $\boxed{\text{NAND-NAND}}$ logic.

$F = A\cdot B + C\cdot D$

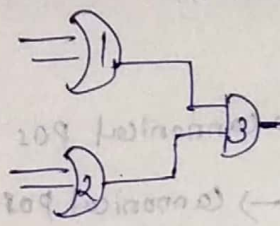




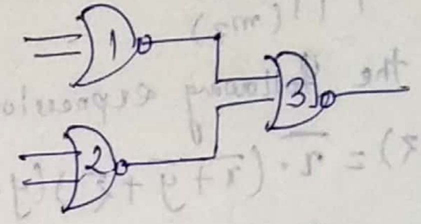
$F = \overline{\overline{AB}\cdot\overline{CD}}$

$F = AB + CD.$

② POS form :- $\boxed{\text{OR-AND}}$ $\equiv$ $\boxed{\text{NOR-NOR}}$ logic



$\equiv$



Q. $F_1(A,B,C) = \sum m(0,2,3,5,6)$

$F_2(A,B,C) = \sum m(1,2,3,4,6)$

find (a) $F_1\cdot F_2$ (b) $F_1 + F_2$ (c) $F_1 - F_2$

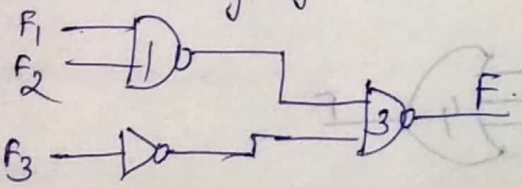
Soln:- all minterms common to both F_1 and F_2

$\sum m(2,3,6)$

$F_1 + F_2 = \sum(0,1,2,3,4,5,6)$

$F_1 - F_2 = \sum m(0,5)$, $F_2 - F_1 = \sum m(1,4)$

Pb// In the following logic ckt determine F_3, if



$F(A,B,C) = \sum m(1,3,5,6)$

$F_1(A,B,C) = \sum m(0,3,6,7)$

$F_2(A,B,C) = \sum m(1,2,4)$

Soln:- ~~$\overline{F_1\cdot F_2} = \{0,1,2,3,4,6,7\}\cdot\overline{F_3}$~~

$F = \overline{\overline{F_1\cdot F_2}\cdot\overline{F_3}} \Rightarrow \boxed{F = F_1\cdot F_2 + F_3}$

$= \overline{(\overline{F_1}+\overline{F_2})\cdot\overline{F_3}}$

$= \overline{\overline{F_1}\cdot\overline{F_3} + \overline{F_2}\cdot\overline{F_3}}$

$= (\overline{\overline{F_1}\cdot\overline{F_3}})\cdot(\overline{\overline{F_2}\cdot\overline{F_3}}) = (F_1+F_3)(F_2+F_3)$

$= F_1\cdot F_2$