3

Inductance:-

It is the property of a conductor by virtue of which it opposes any change in magnitude & direction of electric current passing through the conductor.

Let us consider a coil of 'N' turns carrying a current 'I' which produces a flux 'ϕ' linking with the coil. The flux linkage (ϕ) is proportional to the current (I) through the coil.

$$\therefore \phi \propto I$$

$$\Rightarrow \phi = LI$$

$$\Rightarrow \boxed{L = \frac{\phi}{I}}$$

As the coil has 'N' turns, so $\boxed{L = \frac{N\phi}{I}}$.

Unit :- henry.

$V_L \propto \frac{di}{dt}$; $\boxed{V_L = L\frac{di}{dt}}$

$\therefore L$ = Inductance.

V_L = Induced emf

Self Inductance:-

When a current changes in a circuit, the magnetic flux which is linked in that circuit changes & vice-versa and due to this, an emf is induced in that circuit. This emf is proportional to the rate of change of current.

i.e, $e_L \propto \frac{di}{dt}$

or, $e_L = L\frac{di}{dt}$ ——— (1)

where L = a constant of proportionality called as self inductance.

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$$\Rightarrow L = \frac{e_L}{(di/dt)}$$

where, e_L = self induced emf.

Also 'L' i.e. self inductance may be expressed as,

$$L = \frac{N\phi}{i} \quad \text{——— (2)}$$

Where, N = no. of turns in the circuit.

or, $i = \frac{N\phi}{L} = \frac{1}{L}(N\phi)$

ϕ = Flux linkage.

From eq^n (1), $e_L = L\frac{di}{dt}$

$$= L\frac{d}{dt}\left[\frac{N\phi}{L}\right]$$

$$= L \times \frac{1}{L}\frac{d}{dt}[N\phi]$$

$$\boxed{e_L = N\frac{d\phi}{dt}} \quad \text{——— (3)}$$

Comparing eq^n (1) & eq^n (3)

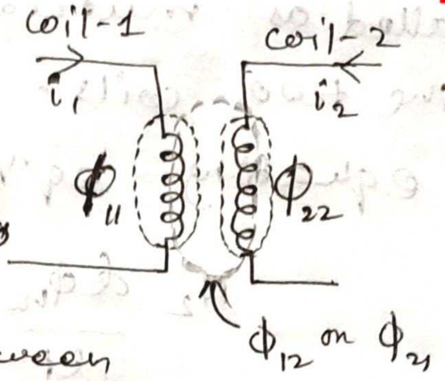
$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$\Rightarrow \boxed{L = N\frac{d\phi}{di}} \quad \text{——— (4)}$$

This is the expression for the self inductance.

Mutual Inductance:-

→ When two coils are kept in such a way that they are electrically isolated but magnetically coupled there will be mutual inductance between these two coils.



→ Hence the portion of magnetic flux which links with the other coil as well as itself is due the magnetically coupled or inductively.

→ Let us consider two coils which are mutually coupled. The currents of i_1 & i_2 which are alternating in nature are carried by two coils.

→ Due to these two currents i_1 & i_2 leakage fluxes of ϕ_{11} & ϕ_{22} are set up.

As coil-2 is linked magnetically with coil-1 & vice-versa, then two fluxes of ϕ_{21} & ϕ_{12} which are called as linkage fluxes, ~~with~~ will be set up.

Thus, the voltage induced in coil-2 is given by,

$$e_2 = N_2 \frac{d\phi_{12}}{dt} \quad \text{——— (1)}$$

Here the flux ϕ_{12} is due to the current i_1 in ~~the~~ coil-1.

N_2 = No. of turns in coil-2.

Again $e_2 \propto \frac{di_1}{dt}$

or, $$e_2 = M \frac{di_1}{dt} \quad \text{——— (2)}$$

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Where 'M' is a constant of proportionality called as mutual inductance between the two coils.

equating eq^n ① & eq^n ②,

$$N_2 \frac{d\phi_{12}}{dt} = M \frac{di_1}{dt}$$

$$\Rightarrow M = N_2 \left[\frac{d\phi_{12}}{di_1}\right] \longrightarrow ③$$

Similarly, $M = N_1 \left[\frac{d\phi_{21}}{di_2}\right] \longrightarrow ④$

N_1 = No. of turns in coil-1

N_2 = No. of turns in coil-2

→ When air medium is present in between two coils, the flux and currents are linearly related.

~~equating eq^n ③ & eq^n ④~~

~~$M = N_2 \frac{d\phi_{12}}{di_1} = N_1 \frac{d\phi_{21}}{di_2}$~~

So,

$$M = N_2 \frac{\phi_{12}}{i_1} \longrightarrow ⑤$$

$$M = N_1 \frac{\phi_{21}}{i_2} \longrightarrow ⑥$$

7

Coefficient of coupling (K):

The ratio of linkage flux to total flux is called as co-efficient of coupling.

$$\therefore K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2} \quad \text{—— (7)}$$

(8) $\phi_{12} < \phi_1$ & $\phi_{21} < \phi_2$

Thus $K < 1$ (always) & max^m value of $K = 1$

Multiplying the two eq^ns (5) & (6),

$$\therefore M^2 = N_1 N_2 \frac{\phi_{12} \cdot \phi_{21}}{i_1 \cdot i_2}$$

$$= N_1 N_2 \frac{K\phi_1 \cdot K\phi_2}{i_1 \cdot i_2}$$

$$= K^2 N_1 N_2 \frac{\phi_1}{i_1} \times \frac{\phi_2}{i_2}$$

$$\therefore M^2 = K^2 \left(\frac{N_1 \phi_1}{i_1}\right) \times \left(\frac{N_2 \phi_2}{i_2}\right)$$

$$\Rightarrow M^2 = K^2 L_1 L_2$$

where, $L_1 = \frac{N_1 \phi_1}{i_1}$

$L_2 = \frac{N_2 \phi_2}{i_2}$

ϕ_1 & ϕ_2 are total fluxes.

$$\therefore \phi_1 = \phi_{12} + \phi_{11}$$

$$\phi_2 = \phi_{21} + \phi_{22}$$

8 'ϕ_{22}' links with the turns of coil-2 only & 'ϕ_{11}' with coil-1 only.

~~So $M^2 = K^2 \sqrt{L_1}$~~

$$M^2 = K^2 L_1 L_2$$

$$\Rightarrow M = K\sqrt{L_1 L_2}$$

$$\Rightarrow \boxed{K = \frac{M}{\sqrt{L_1 L_2}}} \quad \text{—— (8)}$$

7.16 Leakage flux

2

Magnetic flux is classified into two groups. (I) Useful flux (ii) Leakage flux.

The total magnetic flux produced is equal to the sum of the useful flux and the leakage flux.

The part of the total magnetic flux which has its path wholly within the magnetic circuit is called useful flux. The magnetic flux having its path partly in magnetic circuit and partly in air is called leakage flux. These fluxes are shown in fig 7.9

The ratio of total flux produced to the useful flux is called leakage factor or leakage coefficient.

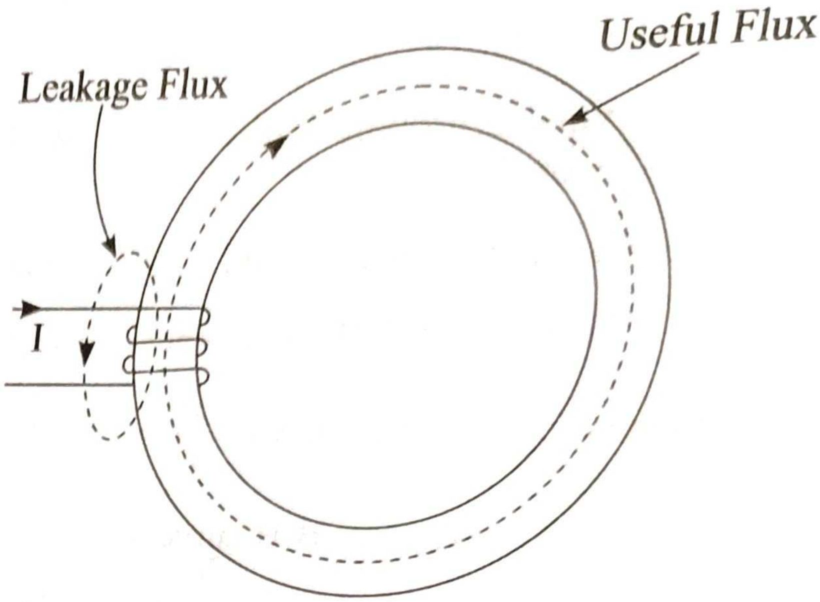

Fig 7.9

7.1 INTRODUCTION :

Generally coupled circuits are associated with magnetic circuits i.e. they are electrically isolated but magnetically coupled. The example of coupled circuit is transformer which has two separate electrical circuits but magnetically the two windings are coupled. Both self-inductance and mutual inductance are present in this circuit. Self inductance is associated with single coil but mutual inductance is between two coils and it is due to a common magnetic flux. Both kvL and loop analysis can be applied to coupled circuits. Also DOT convention may be applied for the loop analysis which is a most accurate method to analysis the coupled circuits. Some of the concepts like linkage flux, leakage flux are also coming into picture in this circuit. The coefficient of coupling between two coils may be considered in this circuit.