N/W Theory

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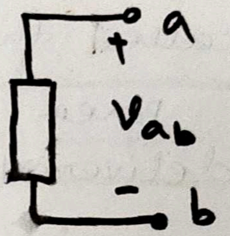
VOLTAGE:

Voltage or potential difference is the energy required to move a unit charge through an element, measured in volts (V).

The voltage 'V_{ab}' between two points 'a' & 'b' in an electric circuit is the energy (or work) needed to move a unit charge from a, b; mathematically,



$$\boxed{V_{ab} = \frac{dw}{dq}}$$

where,

w = energy in joules (J)

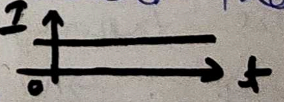
q = charge in coulombs (C)

CURRENT:

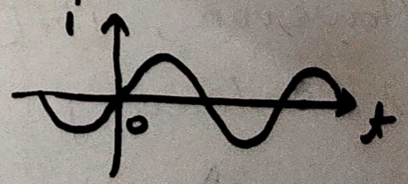
Electric current is the time rate of change of charge, measured in amperes (A).

mathematically, $\boxed{i = \frac{dq}{dt}}$

→ A direct current (dc) is a current that remains constant with time.



→ An alternating current (ac) is a current that varies sinusoidally with time.



Power and Energy:

Power is the time rate of expanding or absorbing energy, measured in watts (W).

mathematically, $\boxed{P = \frac{\Delta w}{dt}}$

$$P = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$$

$$\therefore \boxed{P = vi} \quad \text{——— (1)}$$

The power in eqn (1) is time varying quantity and is called the instantaneous power.

→ If the power has a '+' sign, power is being delivered to or absorbed by the element.

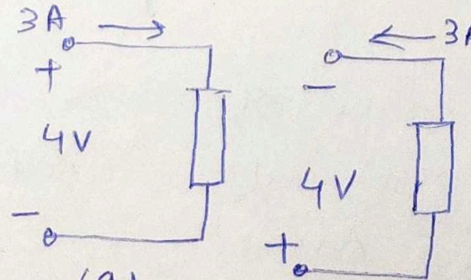
→ If the power has a '−' sign, power is being supplied by the element.

Passive sign convention:

✱ Passive sign convention is satisfied when the current enters through the positive terminal of an element and $P = +vi$. If the current enters through the negative terminal, $P = -vi$.

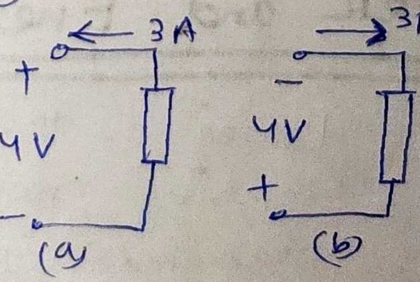
By passive sign convention, current enters through the positive polarity of the voltage. In this case $\boxed{P = +vi}$ or $vi > 0$ implies that the element is absorbing power. However, if $\boxed{P = -vi}$ or $vi < 0$, the element is releasing or supplying power.

eg1.



(a) (b)

Two cases of an element with an absorbing power of 12W (a) $P = 4 \times 3 = 12W$ (b) $P = 4 \times 3 = 12W$



(a) (b)

Two cases of an element with a supplying power of 12 W. (a) $P = -4 \times 3 = -12W$, (b) $P = -4 \times 3 = -12W$.

According to law of conservation of energy, which must be obeyed in any electric circuit, the algebraic sum of power in a circuit, at any instant of time, must be zero.

$$\boxed{\sum P = 0}$$

or, $\boxed{+\text{ Power absorbed} = -\text{ power supplied}}$

The energy is the capacity to do work, measured in joules (J).

The energy absorbed or supplied by an element from time 't_0' to time 't' is,

$$\boxed{W = \int_{t_0}^{t} p \, dt = \int_{t_0}^{t} v i \, dt}$$

* The electric power ~~utit~~ utility companies measure energy in <u>Watt-hours (Wh)</u>, where,

$$\boxed{1 \text{ Wh} = 3600 \text{ joules}}$$

Ohm's Law:

Ohm's law states that the voltage 'V' across a resistor is directly proportional to the current 'i' flowing through the resistor.

$$\boxed{v \propto i}$$

The resistance ~~of~~ 'R' of an element denotes its ability to resist the flow of electric current, it is measured in ohms (Ω).

~~R~~ $\boxed{R = \frac{v}{i}}$

so, $1\ \Omega = 1\ V/A$

→ For a short circuit,

$v = iR = 0$,

showing that the voltage is zero but the current could be anything.

so, A short circuit is a circuit element with resistance approaching zero.

→ Similarly, an element with $R = \infty$ is known as an open circuit.

For an open circuit,

$$i = \lim_{R \to \infty} \frac{v}{R} = 0$$

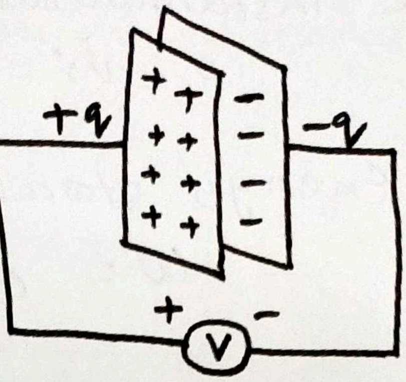
An open circuit is a circuit element with resistance approaching infinity.

Conductance: (G)

$$\boxed{G = \frac{1}{R} = \frac{i}{V}}$$

Conductance is the ability of an element to conduct electric current, it is measured in mhos ($\mho$) or siemens (S).

CAPACITORS :



→ A capacitor consists of two conducting plates separated by an insulator (or) dielectric.

→ The amount of charge stored, represented by 'q', is directly proportional to the applied voltage 'V' so that,

$$\boxed{q = Cv}$$

where, 'C', the constant of proportionality, is known as the capacitance of the capacitor.

* Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).

$$\boxed{1 \text{ farad} = 1 \text{ coulomb}/\text{volt}}$$

* Though $C = q/v$, but it does not depend on 'q' or 'V'. It depends on the physical dimensions of the capacitor.

$C = \frac{\epsilon A}{d}$, where, A = surface Area

ϵ = permitivity of the dielectric material between the plates.

d = distance between the plates.

current, $i = \frac{dq}{dt}$,

$$\boxed{i = C\frac{dv}{dt}}$$ $(\because q = Cv)$

The instantaneous power delivered to the capacitor is,

$$p = vi = Cv\frac{dv}{dt}$$

Energy stored in capacitor is therefore,

$$w = C\int_{-\infty}^{t} v\frac{dv}{dt} = \int_{v(-\infty)}^{v(t)} v\,dv = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

we note that $v(-\infty) = 0$, because the capacitor was unchanged at $t = -\infty$. Thus

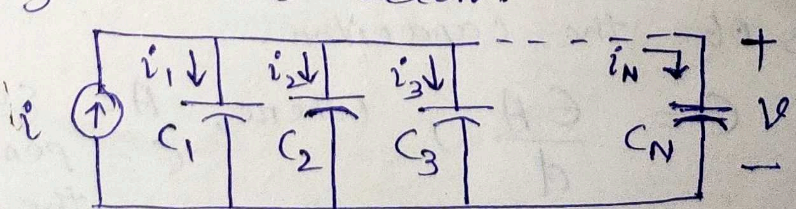
$$\boxed{w = \frac{1}{2}Cv^2}$$

NOTE :

* A capacitor is an open circuit to dc.
* The voltage on a capacitor cannot change abruptly.

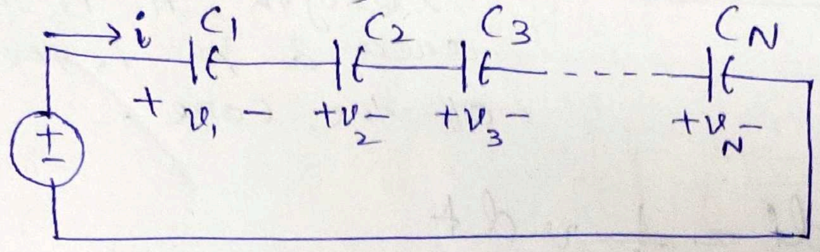
series & parallel capacitors :

→ In order to obtain the equivalent capacitor C_{eq} of 'N' capacitors in parallel, consider the following circuit below.



$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$

If 'N' capacitors are connected in series ~~the~~ as shown below,



$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$

eg: If N=2,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

INDUCTORS:

→ An inductor consists of a coil of conducting wire.

→ If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.

$$\boxed{v = L\frac{di}{dt}}$$

* Inductance is the property whereby an inductor exhibits opposition to the ~~change~~ change of current flowing through it, measured in ~~henes~~ henrys (H).

$$\boxed{L = \frac{N^2 \mu A}{l}}$$

where, 'N' is the number of turns, 'l' is the length, 'A' is the cross-sectional area & 'μ' is the permeability of the core.

$$di = \frac{1}{L} v\,dt$$

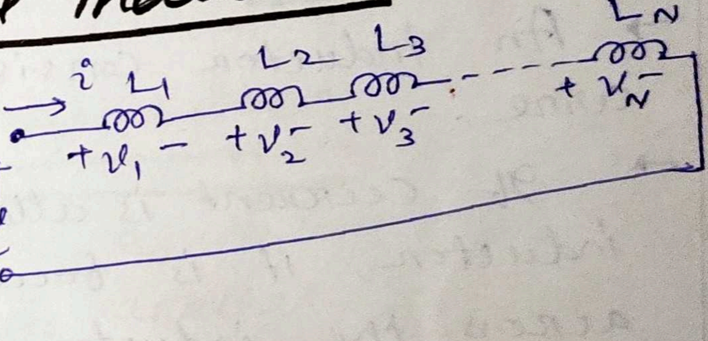
$$\boxed{i = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau}$$

Power, $P = vi = \left(L\frac{di}{dt}\right) i$

$$\boxed{w = \frac{1}{2} L i^2}$$

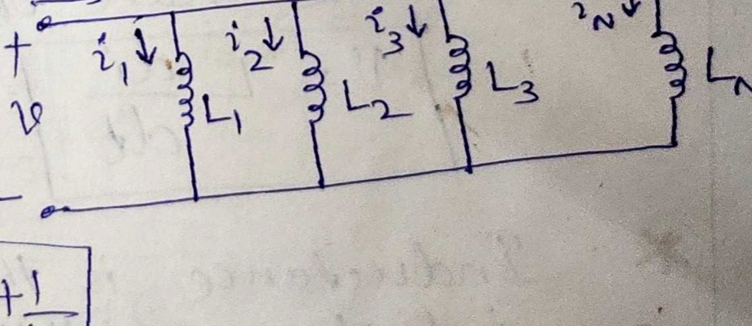
Series & parallel inductors:

series:



$$\boxed{L_{eq} = L_1+L_2+L_3+\dots+L_N}$$

parallel:



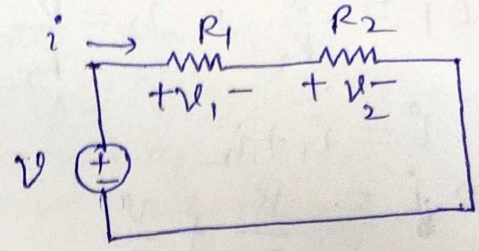
~~L_{eq}~~

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}+\dots+\frac{1}{L_N}}$$

eg: N = 2, ~~L_{eq}~~ $\frac{1}{L_{eq}} = \frac{1}{L_1}+\frac{1}{L_2} \Rightarrow \boxed{L_{eq} = \frac{L_1 L_2}{L_1+L_2}}$

Resistors :

① Series Resistors & Voltage Division:



$R_{eq} = R_1 + R_2$

$v_1 = iR_1$, $v_2 = iR_2$

If KVL is applied in clockwise direction,

$$-v + v_1 + v_2 = 0$$

$$\Rightarrow v = v_1 + v_2 = iR_1 + iR_2$$

$$\Rightarrow v = i(R_1 + R_2)$$

$$\Rightarrow \boxed{i = \frac{v}{R_1 + R_2}} \Rightarrow i = \frac{v}{R_{eq}} \Rightarrow \boxed{v = iR_{eq}}$$

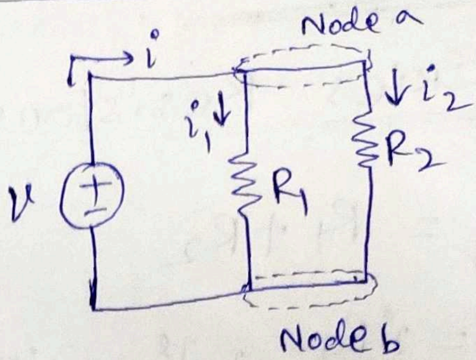
$$v_1 = iR_1 = \left(\frac{v}{R_1 + R_2}\right) R_1 \; ; \; v_2 = iR_2 = \left(\frac{v}{R_1 + R_2}\right) R_2$$

$$\Rightarrow \boxed{v_1 = \left(\frac{R_1}{R_1 + R_2}\right) v} \text{ — ⓐ} \; ; \; \boxed{v_2 = \left(\frac{R_2}{R_1 + R_2}\right) v} \text{ — ⓑ}$$

Similarly, $$\boxed{v_n = \left(\frac{R_n}{R_1 + R_2 + \cdots + R_N}\right) v} \text{ — ⓒ}$$

eqⁿ ⓐ, ⓑ & ⓒ are known as principle of voltage division.

Parallel Resistors and Current Division:



$v = i_1 R_1 = i_2 R_2$

$i_1 = \frac{v}{R_1}$, $i_2 = \frac{v}{R_2}$

$i = i_1 + i_2$

$\Rightarrow i = \frac{v}{R_1} + \frac{v}{R_2}$

$= v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}}$ $\Rightarrow i = \frac{v}{R_{eq}} \Rightarrow \boxed{v = iR_{eq}}$

Where, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$

$\therefore \boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$

$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2}$

$i_1 = \frac{v}{R_1} = \frac{iR_1R_2}{R_1(R_1+R_2)} = \frac{iR_2}{R_1+R_2}$

$\Rightarrow \boxed{i_1 = \left(\frac{R_2}{R_1+R_2}\right) i}$

$i_2 = \frac{v}{R_2}$

$= \frac{iR_1R_2}{R_2(R_1+R_2)}$

$= \frac{iR_1}{R_1+R_2}$

$\Rightarrow \boxed{i_2 = \left(\frac{R_1}{R_1+R_2}\right) i}$

Types of Elements:

1. Active and passive Elements:

→ The elements that are capable of generating a signal or amplifying a signal or rectify a signal is known as active element otherwise passive element.

eg) Active

→ Voltage source
→ Current source
→ Transistor
→ Op-amp
→ Diode
→ TRIAC
→ SCR
→ DIAC

Passive

→ Resistor
→ Inductor
→ Capacitor
→ Transformer

2. Linear & Non linear Element:

→ Any element's characteristic is straight line is linear.

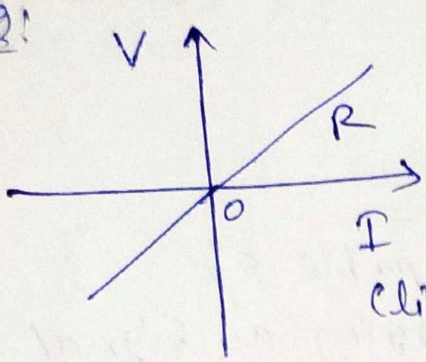
→ The element that obeys superposition and homogeneous property is known as linear elements otherwise Non-linear.

$$f(x_1 + x_2) = f(x_1) + f(x_2) \longrightarrow \text{superposition}$$

$$f(kx) = k f(x) \longrightarrow \text{Homogeneous}$$

OR, The element that characteristics are straight line and passing through the origin is known as linear otherwise Non-linear.

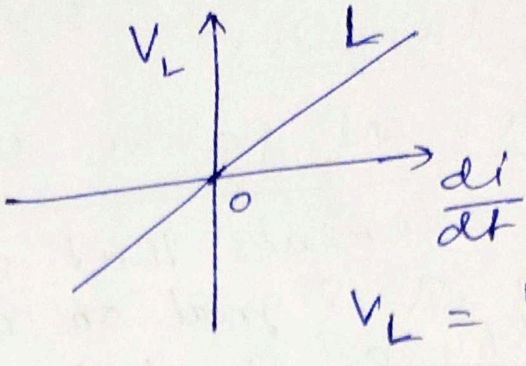
eg:



(linear)

$V = RI$

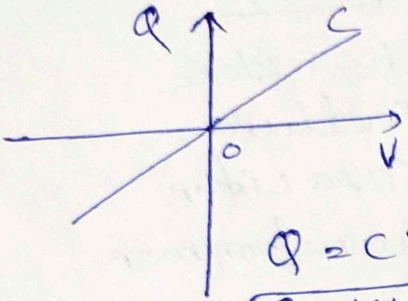
$\boxed{V \propto I}$



(linear)

$V_L = L\frac{di}{dt}$

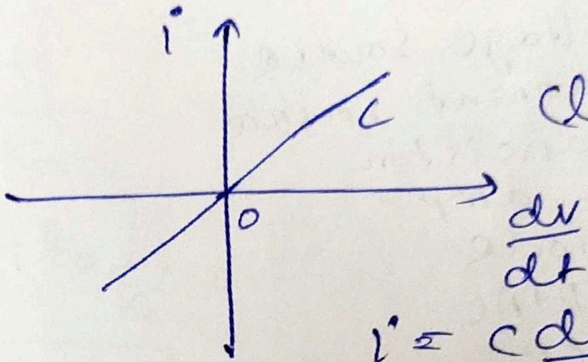
$\boxed{V_L \propto \frac{di}{dt}}$



(linear)

$Q = CV$

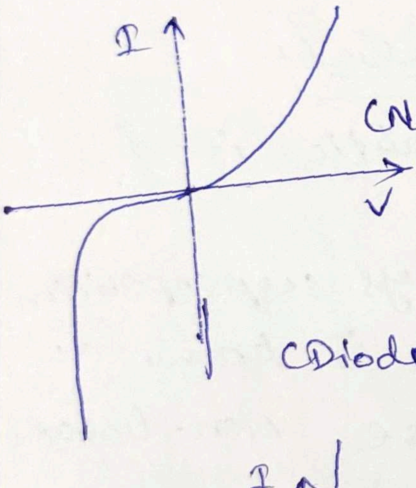
$\boxed{Q \propto V}$



(linear)

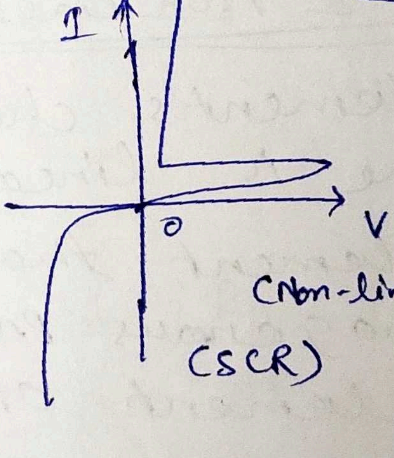
$i = C\frac{dv}{dt}$

$\boxed{i \propto \frac{dv}{dt}}$



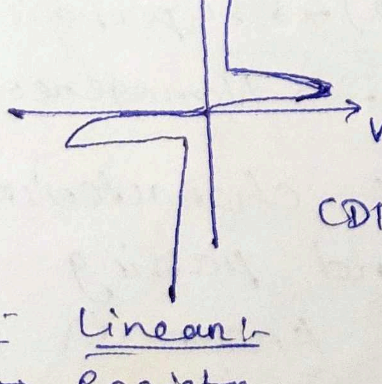
(Nonlinear)

(Diode)



(Non-linear)

(SCR)



(Non-linear)

(DIAC)

eg:- Linear:-

- → Resistor,
- → Inductor
- → Capacitor

Non linear

→ Diode, Transistor, SCR, DIAC, TRIAC, Opamp etc.

3: Unilateral & Bilateral:

Uni → one
Bi → Two.

→ The element that offers same impedance in both the direction of current is known as bilateral elements otherwise unilateral elements.

Or, The elements that characteristics are same in both the direction of current is known as bilateral ~~other~~ otherwise unilateral.

eg:

Unilateral	Bilateral
→ Diode	→ Resistor
→ SCR	→ Inductor
→ Transistor	→ Capacitor
	→ DIAC

4: Lumped & Distributed:

→ If the separation of R, L, C is possible is known as lumped elements otherwise distributed.

eg:

lumped	distributed
→ parameters used in lab experiments.	→ Transmission line
	→ Antenna.

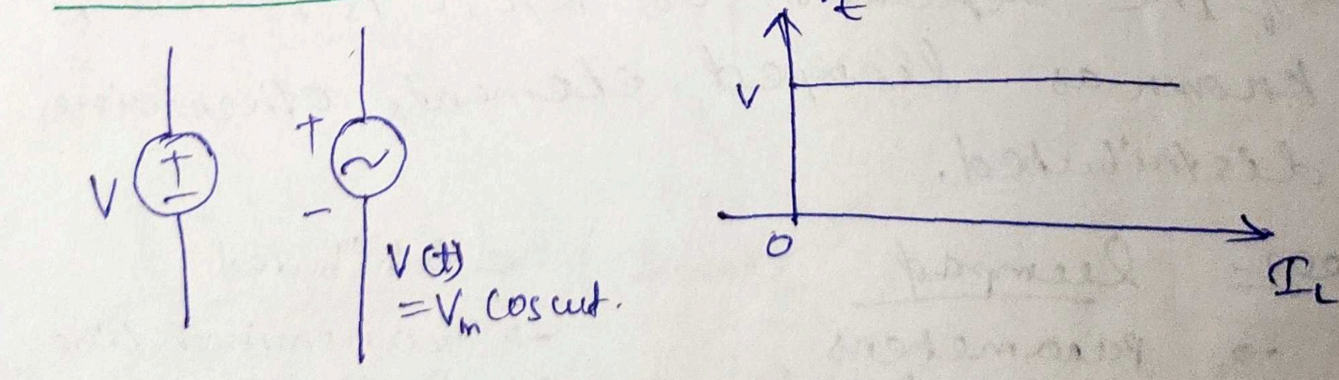
5: Time Variant and Time Invariant:

→ The element's characteristics depending upon time is called time variant Otherwise time invariant.

TYPES OF SOURCES:

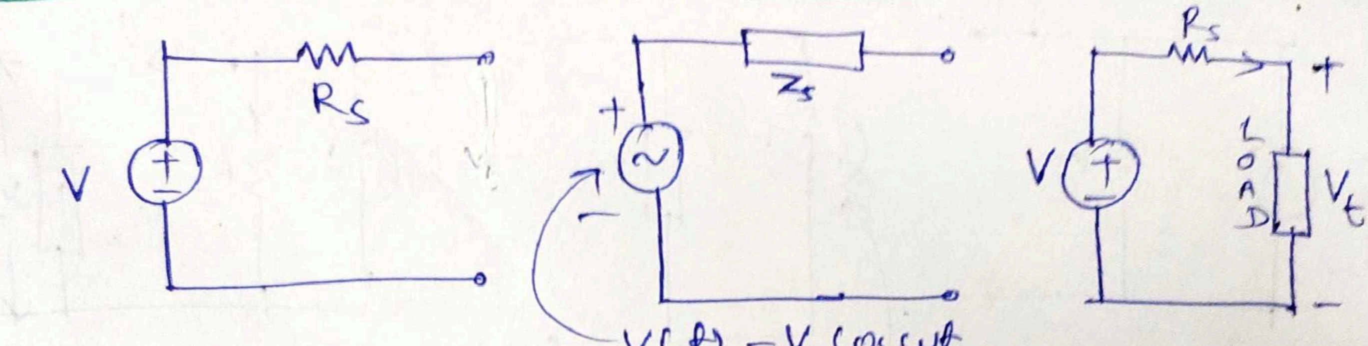
- Independent Source
  - Voltage source
    - Ideal
    - Practical
  - Current source
    - Ideal
    - Practical
- Dependent Source
  - VDVS
  - CDVS
  - CDCS
  - VDCS

Ideal Voltage source



→ The terminal voltage is independent of load current or, the terminal voltage is not a function of load current is known as ideal voltage source.

Practical voltage source:



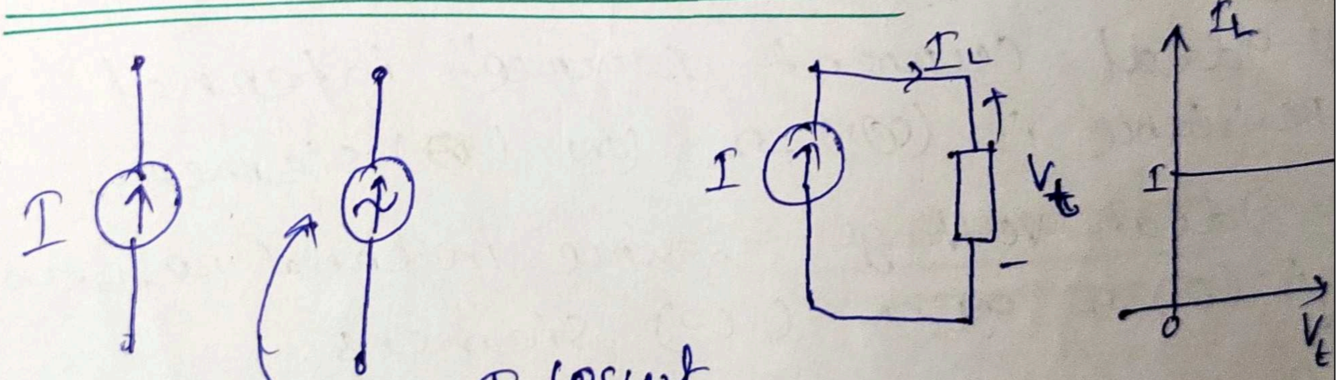
$V(t) = V_m \cos \omega t$

or

$V_m \sin \omega t$

→ The terminal voltage is the function of load current or the terminal voltage is dependent upon load current is known as practical voltage ~~so~~ source.

Ideal current source:



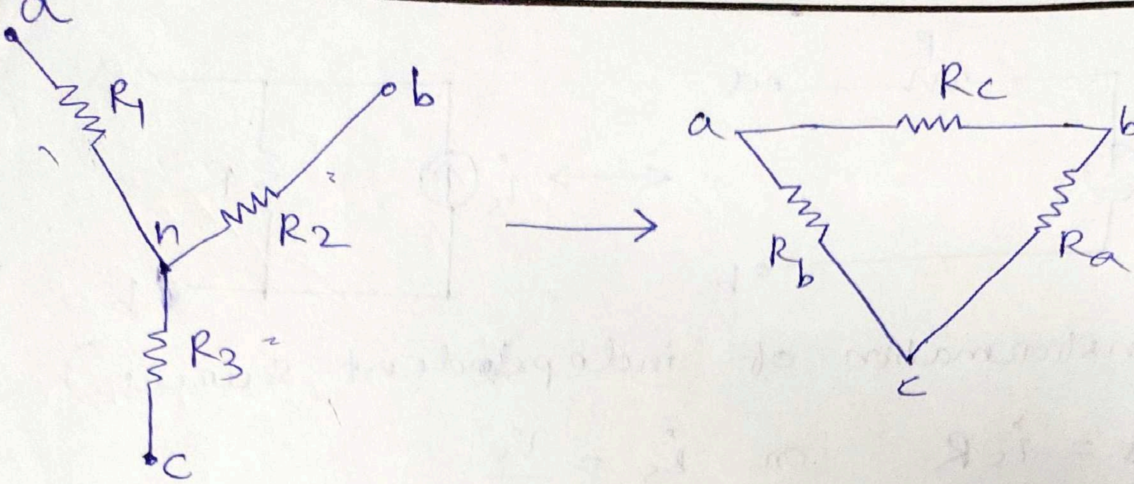
$i(t) = I_m \cos \omega t$

or

$I_m \sin \omega t$.

→ The load current is independent of terminal voltage or the load current is not a function of terminal voltage is known as ideal current source.

Star to Delta Transformation:

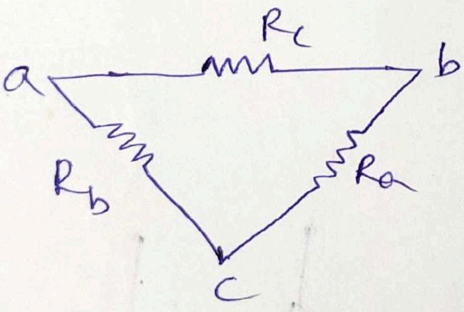


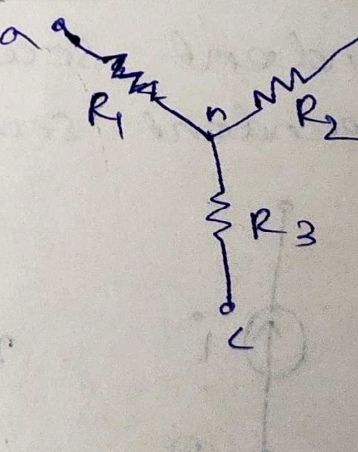
$$R_a = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_b = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_c = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Delta to star transformation:



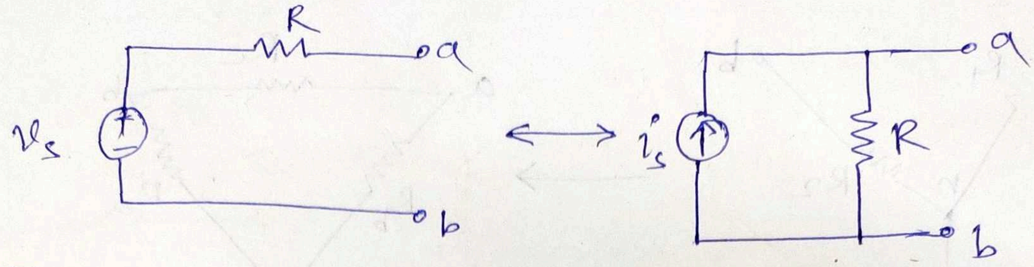


$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

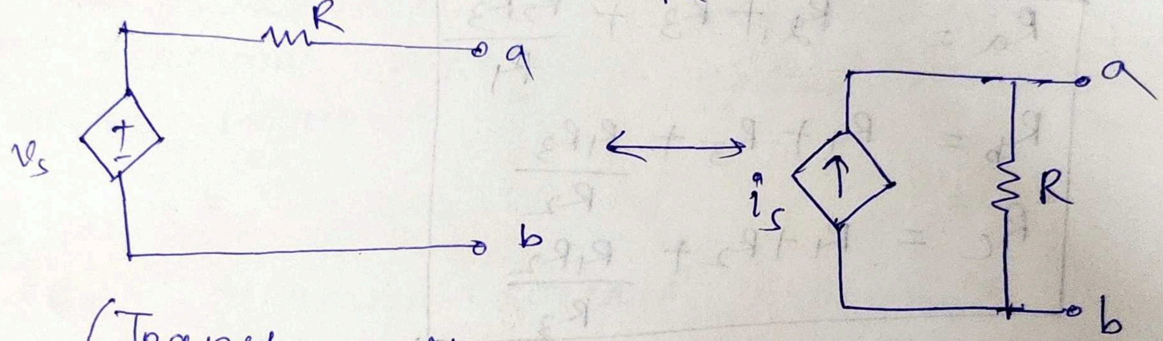
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Source Transformation:



(Transformation of independent sources.)

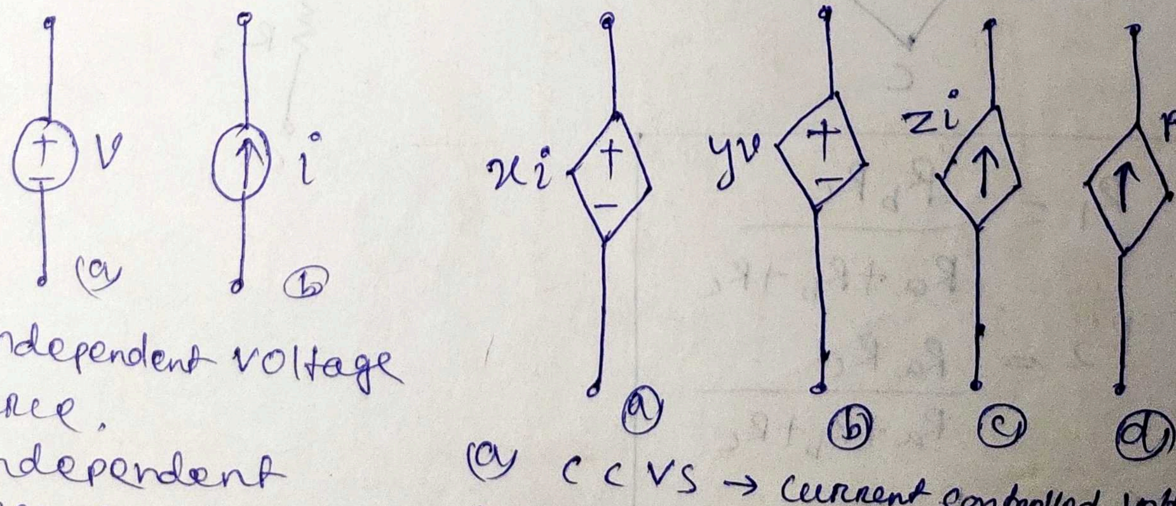
$v_s = i_s R$ or $i_s = \frac{v_s}{R}$



(Transformation of dependent sources)

Types of sources:

(1) Dependent sources
(2) Independent sources.



(a) Independent voltage source.
(b) Independent current source.

(a) CCVS → current controlled voltage source
(b) VCVS → voltage controlled voltage source
(c) CCCS → current controlled current source
(d) VCCS → voltage controlled current source