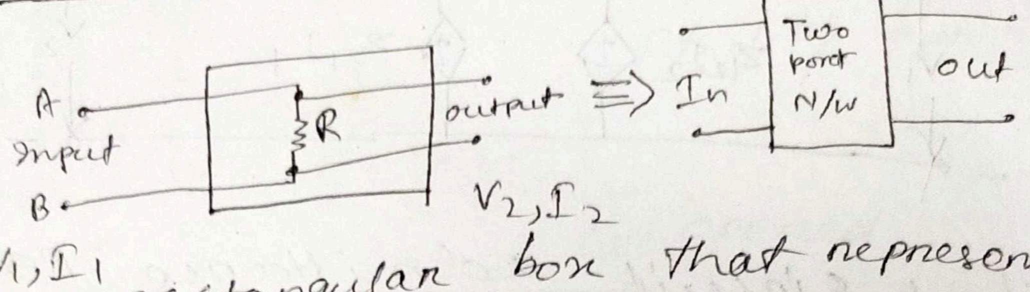
Circuit & N/W Th.
CA. Mishra ①

(2-port N/W)

Two port network :-



If the rectangular box that represents a network, consists of two pairs of terminals where one pair of terminals can be designated as input, the other pair being output, it is called a two port n/w or a four terminal n/w.

① Open circuit Impedance parameters :-

$$V_1 = f(I_1, I_2)$$
$$V_2 = f(I_1, I_2)$$
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

as $[V] = [Z][I]$

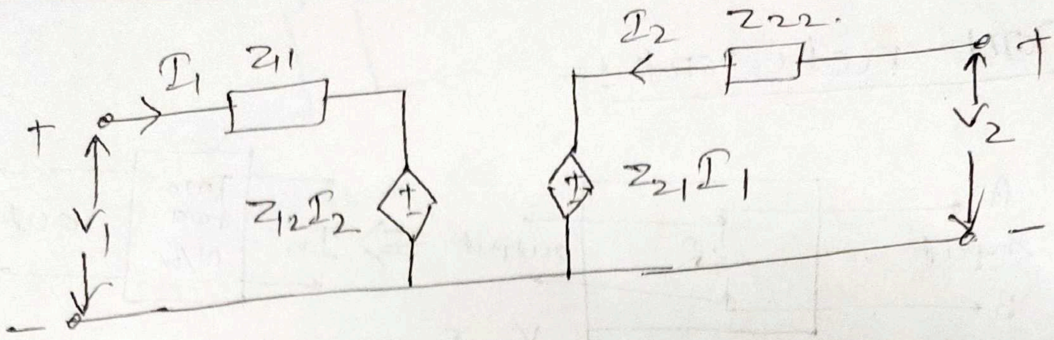
Z_{11}, Z_{22} are called i/p and o/p driving point impedances respectively.
Z_{12}, Z_{21} are called reverse and forward transfer impedances respectively.

ⓐ when port ② is open circuited,
i.e, $I_2 = 0$

$$Z_{11} = V_1/I_1 \Big|_{I_2=0} , \quad Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}.$$

ⓑ When port ① is open circuited,
i.e, $I_1 = 0$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} , \quad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}.$$



Short circuit admittance parameter

$I_1 = f(V_1, V_2)$

$I_2 = f(V_1, V_2)$

$I_1 = y_{11} V_1 + y_{12} V_2$

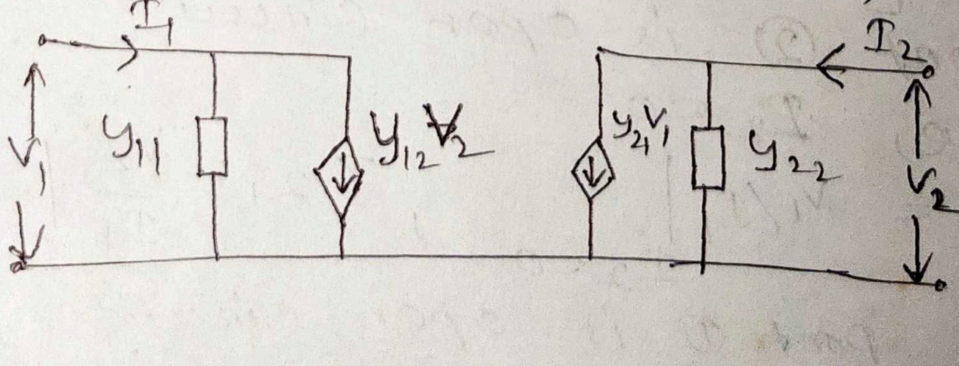
$I_2 = y_{21} V_1 + y_{22} V_2$

(a) port (2) is short ckted i.e, $V_2 = 0$

$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$, $y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$

(b) port (1) is short ckted i.e, $V_1 = 0$.

$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$, $y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$



Hybrid parameters
(h - parameter)

h - parameter representation is widely used in modelling of electronic components and ckts. Here both short ckt and open ckt terminal condition are utilized. Hence, this representation is known as hybrid parameter representation

$V_1 = f(I_1, V_2)$

$I_2 = f(I_1, V_2)$

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

When port ② is short ckted i.e, $V_2 = 0$.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

h_{11} = i/p impedance.
unit is 'Ω'.

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

(forward current gain) → unitless.

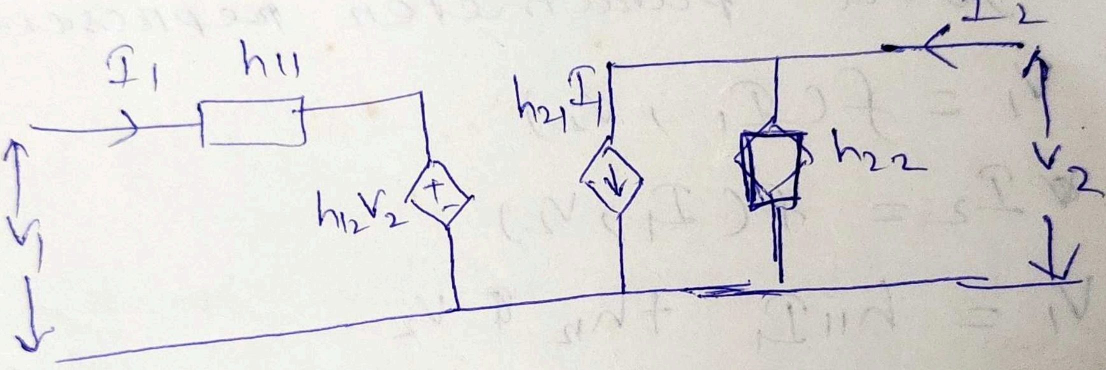
Port 1 is short ckted, i.e, $I_1 = 0$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

(reverse voltage gain) → unitless.

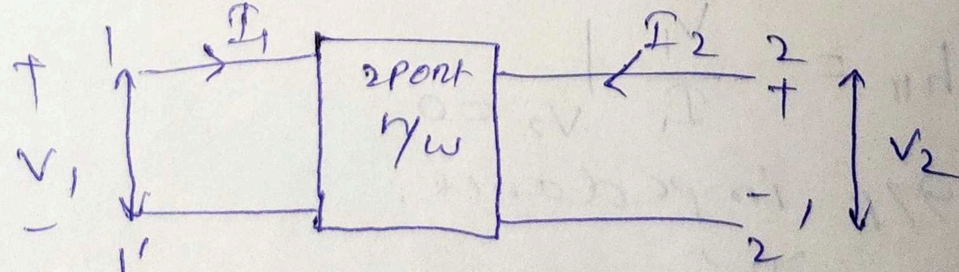
$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

(O/P admittance) → ℧



ABCD parameters

These are used in analysis of power transmission engineering. These are also called transmission parameters.



$$V_1 = f(V_2, I_2)$$

$$I_1 = f(V_2, I_2).$$

Port 2 is opened, $I_2 = 0$.

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \longrightarrow \text{unitless}$$

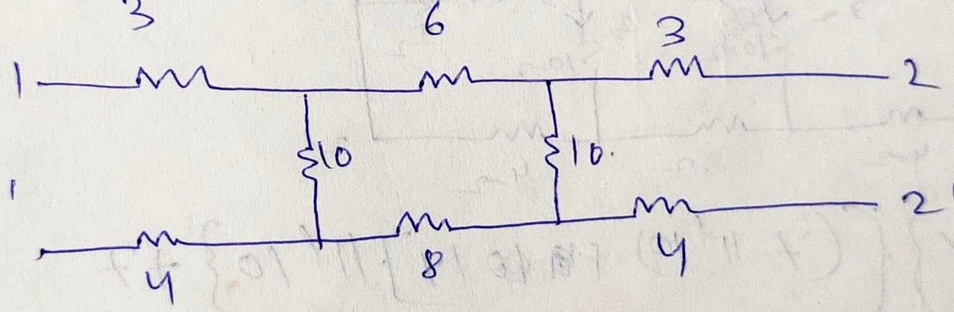
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \longrightarrow \text{Admittance } (\mho)$$

Port 2 is short ckted, $V_2 = 0$.

$$B = \left.\frac{V_1}{(-I_2)}\right|_{V_2=0} \longrightarrow \text{Impedance } (\Omega)$$

$$D = \left.\frac{I_1}{(-I_2)}\right|_{V_2=0} \longrightarrow \text{unitless.}$$

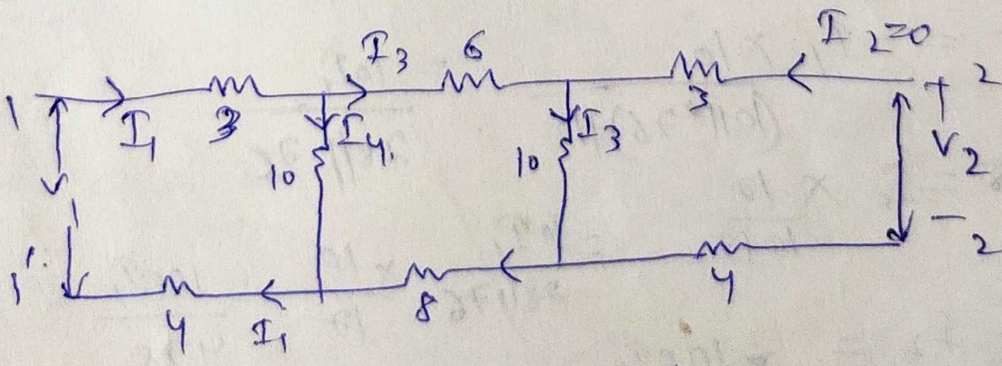
(Q)



Find A, B, C, D parameters.

Soln

(a) port (2) is opened, $I_2 = 0$.



$$V_1 = I_1 \times \left[3 + 4 + (10 \parallel 24)\right]$$

$$\Rightarrow \frac{V_1}{I_1} = 14.06.$$

$V_2 = I_3 \times 10$

$I_3 = \dfrac{I_1 \times 10}{10+24} = \dfrac{10 I_1}{34}$

$V_2 = \dfrac{10 I_1}{34} \times 10 = \dfrac{100}{34} I_1$

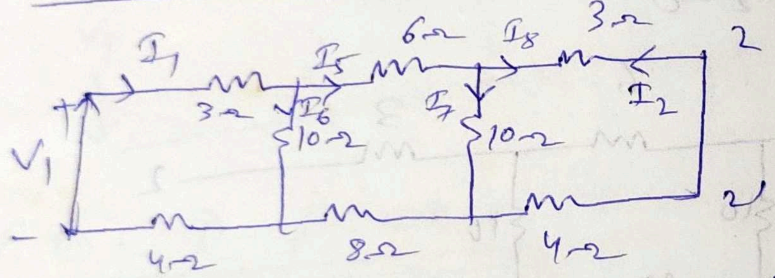
$\Rightarrow \dfrac{V_2}{I_1} = \dfrac{100}{34}$

$\Rightarrow \dfrac{I_1}{V_2} = \dfrac{34}{100} = 0.34\ \mho$

$\dfrac{V_1}{V_2} = A = \dfrac{14.04 I_1}{\frac{100}{34} I_1} = \dfrac{14.04}{100} \times 34$

$= 4.7804$

when $V_2 = 0$



$V_1 = I_1 \times \left\{ \left[(7 \parallel 10) + 14 \right] \parallel 10 \right\} + 7$

$\dfrac{V_1}{I_1} = 13.4435 I_1$

$I_2 = -I_8$

$I_5 = -I_1 \dfrac{\times 10}{(10 \parallel 76) + 10} = \dfrac{10 I_1}{28 \parallel 76}$

$I_8 = I_5 \times \dfrac{10}{10+7} = \dfrac{10 I_1}{28 \parallel 76} \times \dfrac{10}{17} = \dfrac{100 I_1}{478}$

$-I_2 = \dfrac{100 I_1}{478}$ —— (2)

$\Rightarrow \dfrac{I_1}{(-I_2)} = D = \dfrac{478}{100} = 4.78$

$\dfrac{V_1}{(-I_2)} = B = \dfrac{13.44 \times 478}{100}$

$= 64.26\ \Omega$

Interrelationship between parameters of 2-port Network:

Y in terms of Z:-

$$[V] = [Z][I]$$

$$\Rightarrow [I] = [Z]^{-1}[V]$$

$$= [Y][V]$$

$$\therefore [Y] = [Z]^{-1}$$

$$\Rightarrow \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}^{-1}$$

Similarly,

$$[Z] = [Y]^{-1}$$

$$\Rightarrow \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}^{-1}$$

ABCD in terms of Z

$\Rightarrow$

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \text{--- (1)}$$
$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \text{--- (2)}$$

$$V_1 = AV_2 - BI_2 \quad \text{--- (3)}$$
$$I_1 = CV_2 - DI_2 \quad \text{--- (4)}$$

from eq^n (2),

$$z_{21} I_1 = V_2 - z_{22} I_2$$

$$\Rightarrow I_1 = \frac{1}{z_{21}} V_2 - \frac{z_{22}}{z_{21}} I_2 \quad \text{--- (5)}$$

Comparing eq^n (4) and (5),

$$C = \frac{1}{z_{21}}, \quad D = \frac{z_{22}}{z_{21}}$$

from eq^n (1),

$$V_1 = z_{11} I_1 + z_{12} I_2$$

putting the value of I_1 from eq^n (5), in eq^n (1),

$$V_1 = z_{11}\left[\frac{1}{z_{21}} V_2 - \frac{z_{22}}{z_{21}} I_2\right] + z_{12} I_2$$

$$= \frac{z_{11}}{z_{21}} V_2 - \frac{z_{11} z_{22}}{z_{21}} I_2 + z_{12} I_2$$

$$\Rightarrow \frac{z_{11}}{z_{21}} V_2 - I_2\left(-z_{12} + \frac{z_{11} z_{22}}{z_{21}}\right)$$

so, $A = \dfrac{z_{11}}{z_{21}}$, $B = -z_{12} + \dfrac{z_{11} z_{22}}{z_{21}}$

'h' in terms of 'Z'

$V_1 = z_{11} I_1 + z_{12} I_2$ — (1)
$V_2 = z_{21} I_1 + z_{22} I_2$ — (2)

$V_1 = h_{11} I_1 + h_{12} V_2$ — (3)
$I_2 = h_{21} I_1 + h_{22} V_2$ — (4)

$z_{22} I_2 = V_2 - z_{21} I_1$

$\Rightarrow I_2 = \frac{V_2}{z_{22}} - \frac{z_{21}}{z_{22}} I_1$ — (5)

Comparing eqⁿ (4) and (5),

$h_{22} = \frac{-z_{21}}{z_{22}}, \quad h_{21} = \frac{1}{z_{22}}$

$V_1 = z_{11} I_1 + z_{12} I_2$

$= z_{11} I_1 + z_{12}\left(\frac{V_2}{z_{22}} - \frac{z_{21}}{z_{22}} I_1\right)$

$= z_{11} I_1 + \frac{z_{12}}{z_{22}} V_2 - \frac{z_{12} z_{21}}{z_{22}} I_1$

$= I_1\left[z_{11} - \frac{z_{12} z_{21}}{z_{22}}\right] + \frac{z_{12}}{z_{22}} V_2.$

$\boxed{h_{11} = z_{11} - \frac{z_{12} z_{21}}{z_{22}}}, \quad \boxed{h_{12} = \frac{z_{12}}{z_{22}}}$

ABCD in terms of 'Y'

$I_1 = y_{11} V_1 + y_{12} V_2$
$I_2 = y_{21} V_1 + y_{22} V_2$

$V_1 = A V_2 - B I_2$
$I_1 = C V_2 - D I_2$

$$y_{21} V_1 = -y_{22} V_2 + I_2$$

$$\Rightarrow V_1 = \frac{-y_{22}}{y_{21}} V_2 + \frac{1}{y_{21}} I_2$$

$$V_1 = AV_2 - BI_2$$

$$\boxed{A = \frac{-y_{22}}{y_{21}} \quad , \quad B = \frac{-1}{y_{21}}}$$

$$I_1 = y_{11}\left[\frac{-y_{22}}{y_{21}} V_2 + \frac{1}{y_{21}} I_2\right] + y_{12} V_2$$

$$= y_2\left(y_{12} - y_{11}\frac{y_{22}}{y_{21}}\right) + \frac{y_{11}}{y_{21}} I_2$$

$$= V_2\left(y_{12} - y_{11}\frac{y_{22}}{y_{21}}\right) + \frac{y_{11}}{y_{21}} I_2$$

$$I_1 = CV_2 - DI_2$$

$$C = y_{12} - \frac{y_{11} y_{22}}{y_{21}}$$

$$D = -\frac{y_{11}}{y_{21}}$$

h in terms of y!

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$y_{11} V_1 = -y_{12} V_2 + I_1$$

$$\Rightarrow V_1 = \frac{-y_{12}}{y_{11}} V_2 + \frac{1}{y_{11}} I_1$$

$$\Rightarrow \boxed{h_{11} = \frac{1}{y_{11}}} , \boxed{h_{12} = \frac{-y_{12}}{y_{11}}}$$

$$I_2 = y_{21}\left[-\frac{y_{12}}{y_{11}}V_2 + \frac{1}{y_{11}}I_1\right] + y_{22}V_2$$

$$= -\frac{y_{21}\,y_{12}}{y_{11}}V_2 + \frac{y_{21}}{y_{11}}I_1 + y_{22}V_2$$

$$= V_2\left[y_{22} - \frac{y_{21}\,y_{12}}{y_{11}}\right] + \frac{y_{21}}{y_{11}}I_1$$

$$\boxed{h_{22} = \left[y_{22} - \frac{y_{21}\,y_{12}}{y_{11}}\right]}$$

$$\boxed{h_{21} = \frac{y_{21}}{y_{11}}}$$

Q) $A = 2,\ B = -1,\ C = 3,\ D = -2$

Find Z - parameters.

Sol^n:

$V_1 = 2V_2 + I_2$

$I_1 = 3V_2 + 2I_2$

$V_1 = z_{11}I_1 + z_{12}I_2$

$V_2 = z_{21}I_1 + z_{22}I_2$

$I_1 = 3V_2 + 2I_2 \Rightarrow 3V_2 = I_1 - 2I_2$

$\Rightarrow V_2 = \frac{I_1}{3} - \frac{2}{3}I_2.$

$$\boxed{z_{21} = \frac{1}{3},\ z_{22} = -2/3\ \Omega}$$

$V_1 = 2V_2 + I_2$

$= 2\left[\frac{I_1}{3} - \frac{2}{3}I_2\right] + I_2 = \frac{2}{3}I_1 - \frac{4}{3}I_2 + I_2$

$V_1 = \frac{2}{3}I_1 - \frac{1}{3}I_2$

$$\boxed{z_{11} = \frac{2}{3},\ z_{12} = -1/3\ \Omega}$$

(Q) $h_{11} = 1$, $h_{12} = -2$, $h_{21} = -3$, $h_{22} = 2$

Find z- parameters.

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$V_1 = I_1 - 2V_2$

$I_2 = -3I_1 + 2V_2$

$V_1 = z_{11} I_1 + z_{12} I_2$

$V_2 = z_{21} I_1 + z_{22} I_2$

$I_2 = -3I_1 + 2V_2$

$\Rightarrow 2V_2 = 3I_1 + I_2$

$\Rightarrow V_2 = \frac{3}{2} I_1 + \frac{I_2}{2}$

$\boxed{z_{21} = 3/2\,\Omega,\ z_{22},\ 1/2\,\Omega}$

~~$V_1 = z_{11} I_1 + z_{12}$~~

$V_1 = I_1 - 2V_2$

$= I_1 - 2\left(\frac{3}{2} I_1 + \frac{I_2}{2}\right)$

$= I_1 - 3I_1 - I_2$

$V_1 = -2I_1 - I_2$

$\therefore z_{11} = -2\,\Omega$, $z_{12} = -1\,\Omega$.