ENGINEERING MATHEMATICS -I
FOR DIPOLMA STUDENTS

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6-12-21

Co-ordinate Geometry
2D,

We read co-ordinate Geometry with the help of algebra.

8

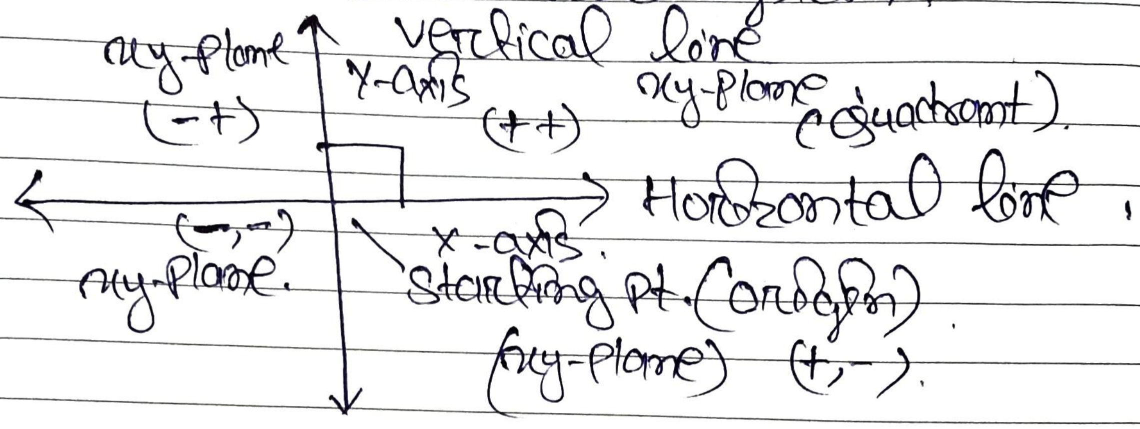
Co-ordinate Geometry: It is the branch of
9 mathematics in which all the concept of Geometry are analysed by using rules of
10 algebra

Co-ordinate Geometry

11
Plane co-ordinate Geometry (2D) → Space co-ordinate (3D).
12

1 2-dimensional Co-ordinate System.

2
3
4



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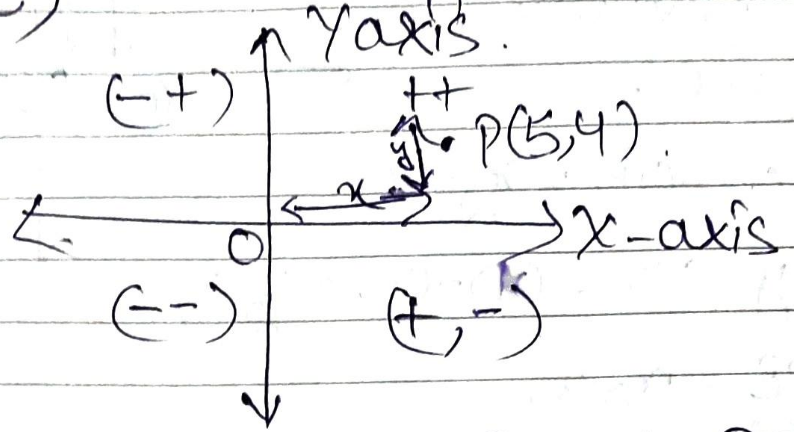
* Co-ordinate of a Point:
6 {a, b} → pair {Paint, Shirts}

(a, b) — ordered pair. ≠ (b, a) — Socks shoe.
1st component 2nd.

* Co-ordinates of a pt is an ordered pair which specifies the adress of a point in the xy-plane.

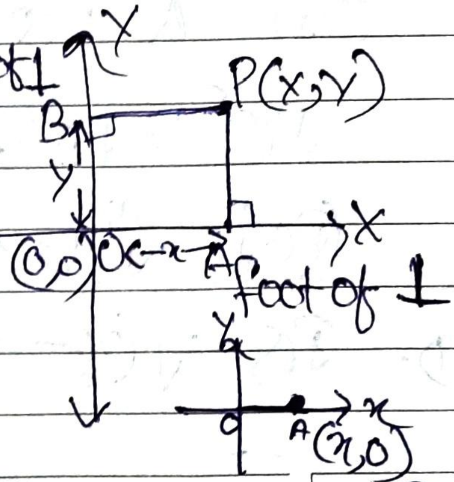
Cartesian co-ordinates.

In general the co-ordinates is denoted by P(x,y) where x is called x-co-ordinate or (abscissa), y is called y-co-ordinate or (ordinate)



Co-ordinate axes (both x-axis & Y-axis).

* If a ⊥ is drawn from any pt. P(x,y) to x axis then distance of origin from the foot of ⊥ is equal to abscissa of the pt.



* Any pt on the x-axis has y-co-ordinate = 0,
* Any pt on the y-axis has x-co-ordinate = 0
* The co-ordinate of origin is (0,0).

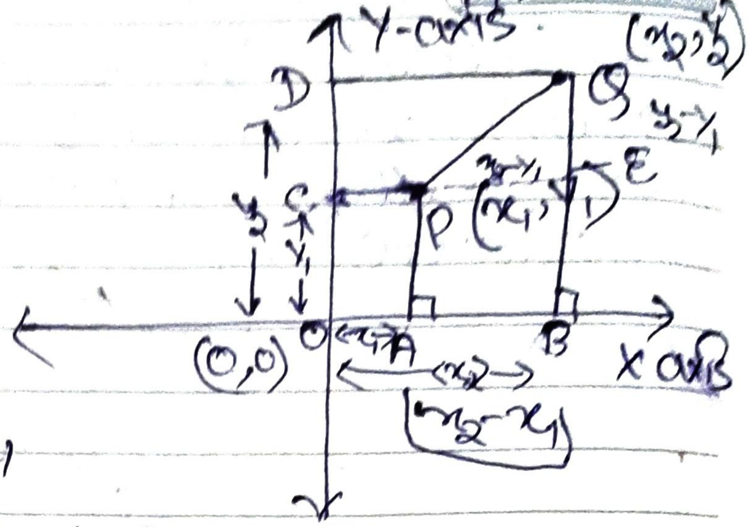
DISTANCE FORMULA

This formulae is used to find the distance betn 2 given points in the xy-plane.

from the figure.

$OA = x_1$, $OB = x_2$.

$OC = y_1$ $OD = y_2$.



from the geometry of the figure.

$AB = OB - OA = x_2 - x_1$

Similarly $CD = OD - OC = y_2 - y_1$

$AB = PE = x_2 - x_1$

$CD = QE = y_2 - y_1$

In Δ PQE Applying the pythagorous thm.

$PQ^2 = PE^2 + QE^2$

$(PQ)^2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\boxed{D = PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Distant $= \sqrt{(\text{Difference in abcessa})^2 + (\text{Difference in ordinate})^2}$

Q. find the distance betn the pts $(2,3)$, $(4,5)$

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(4-2)^2 + (5-3)^2}$

$\Rightarrow \sqrt{2^2 + 4^2}$

$D = \sqrt{20}$

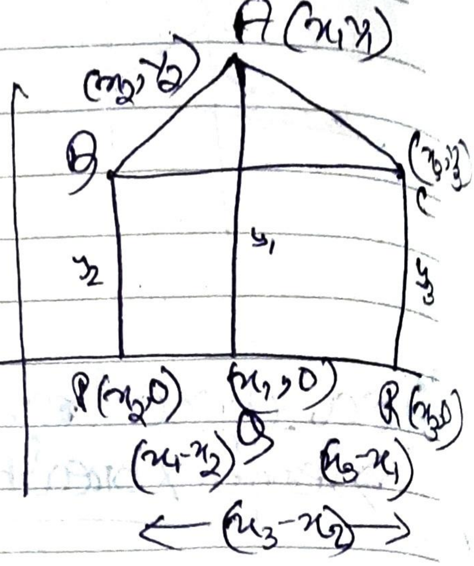
$2\sqrt{5}$.

$$\frac{1}{2}\left[x_3y_2 - x_3y_1 - x_1y_2 + x_1y_3 - x_2y_3 + x_2y_1\right]$$

$$\Rightarrow \frac{1}{2}\left[\left[x_1(y_3-y_1) + x_2(y_1-y_3) + x_3(y_2-y_1)\right]\right]$$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

~~OR~~



Δ = Area of trapezium ABPQ + area of trapezium AQRC − Area of trapezium BPRC,

$$\Rightarrow \frac{1}{2}(BP + AQ)(PQ) + \frac{1}{2}(AQ + CR)(QR) - \frac{1}{2}(BP + CR)(PR)$$

$$\Rightarrow \frac{1}{2}\left\{(y_1+y_2)(x_1-x_2) + (y_3+y_1)(x_3-x_1) - (y_3+y_2)(x_3-x_2)\right\}$$

$$\Rightarrow \frac{1}{2}\left\{x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\right\}$$

Since Δ turns out to be negative, we take the area as $|\Delta|$.

Collinearity of 3 points.

The 3 pt $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ will be collinear if & only if the expression for area of the triangle ABC is zero, which implies

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

* If we interchange the order of any 2 vertices of the $\triangle ABC$, we obtain a (-ve) of the area, However the area shall always be taken to the +ve.

DIVISION FORMULA

(Internal Division)

The co-ordinates of a point P which divides the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m:n$ are given by.

$$\bar{x} = \frac{mx_2 + nx_1}{m+n}, \quad \bar{y} = \frac{my_2 + ny_1}{m+n}.$$

* (Mid point formula):- The co-ordinates of the mid point of a line segment with end points $A(x_1, y_1)$ & $B(x_2, y_2)$ are given by

$$\boxed{\bar{x} = \frac{x_1 + x_2}{2}, \quad \bar{y} = \frac{y_1 + y_2}{2}.}$$

External division.

The co-ordinates of a point P which divides the line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $m:n$ are given by,

$$\bar{x} = \frac{mx_2 - nx_1}{m-n} \text{ and } \bar{y} = \frac{my_2 - ny_1}{m-n}.$$

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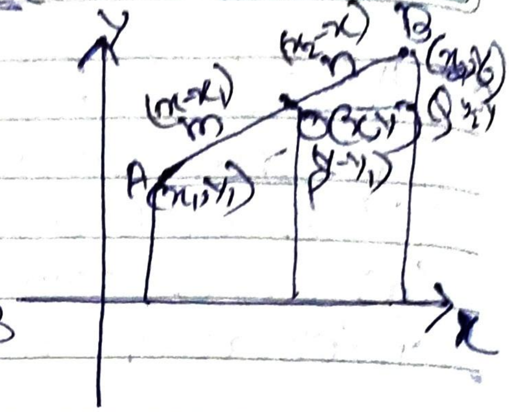
Proof of section formula for internal division.



$\Delta OAP \sim \Delta BOQ$

AAA - condition - similar.

$\angle OAP = \angle BOQ \Rightarrow$ same line AB

$\angle P = \angle Q = 90^\circ$ =

→ So third angle will automatically same.

$$\frac{OA}{BO} = \frac{AP}{OQ} = \frac{OP}{BQ}.$$

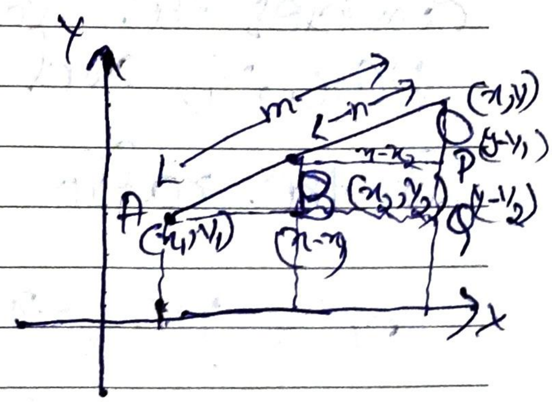
$$\Rightarrow \frac{m}{n} = \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

$$\frac{m}{n} = \frac{x-x_1}{x_2-x_1} \qquad\qquad \frac{m}{n} = \frac{y-y_1}{y_2-y_1}$$

$$m(x_2-x_1) = n(x-x_1) \qquad\qquad m(y_2-y_1) = n(y-y_1)$$

$$\boxed{x = \frac{mx_2+nx_1}{m+n}} \qquad\qquad \boxed{y = \frac{my_2+ny_1}{m+n}}$$

External division



$\Delta OAQ \sim \Delta OBP$

AAA = Condition of

$\angle OAQ = \angle OBP$

$\angle Q = \angle P = 90^\circ$

Third angle

$$\frac{OA}{OB} = \frac{AQ}{BP} = \frac{OQ}{OP}$$

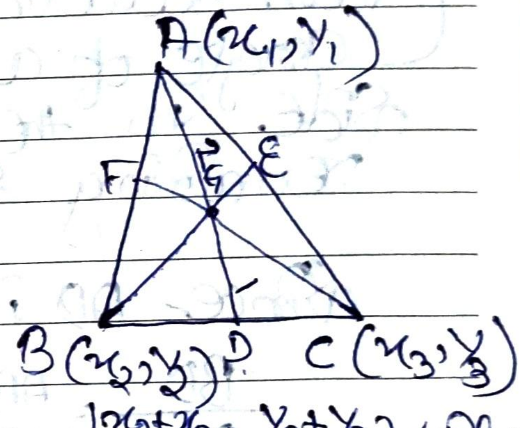
$$\frac{m}{n} = \frac{x-x_1}{x-x_2} = \frac{y-y_1}{y-y_2}$$

$$\frac{m}{n} = \frac{x-x_1}{x-x_2} \qquad \frac{m}{n} = \frac{y-y_1}{y-y_2}$$

$$m(x-x_2) = n(x-x_1) \qquad m(y-y_2) = n(y-y_1)$$

$$x = \frac{mx_2 - nx_1}{m-n} \qquad y = \frac{my_2 - ny_1}{m-n}$$

Centroid of a triangle



Let $G(\bar{x}, \bar{y})$ be a pt. dividing AD in the ratio 2 : 1

$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$ (mid pt)

Then $x = \dfrac{2\left(\frac{x_2+x_3}{2}\right) + 1 \cdot x_1}{2+1} = \dfrac{x_1+x_2+x_3}{3}$

$y = \dfrac{2\left(\frac{y_2+y_3}{2}\right) + 1 \cdot y_1}{2+1} = \dfrac{y_1+y_2+y_3}{3}$

Centroid is $\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

Incentre of a Triangle

The point at which the bisectors of the angles of a triangle intersect is called the incenter of the triangle.

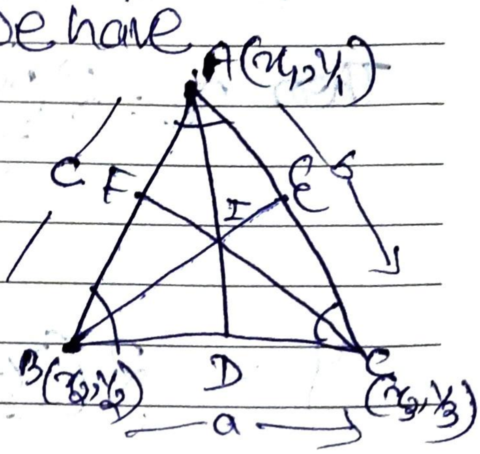
let ABC be a triangle with vertices of $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$

let $BC = a$, $AC = b$, & $AB = c$

let the bisector AD & BE of $\angle A$ & $\angle B$ respectively intersect ~~is called~~ at a pt, I.

(from geometry, we know that the bisector of a angle of a triangle divide the opposite side in the ratio of lengths of remaining sides.)

Since AD is bisector of $\angle A$, we have



$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b} \quad \text{—— (I)}$$

$$\therefore \frac{DC}{BD} = \frac{b}{c} \quad \&$$

$$\frac{DC}{BD} + 1 = \frac{b}{c} + 1$$

$$\Rightarrow \frac{DC+BD}{BD} = \frac{b+c}{c} \Rightarrow \frac{BC}{BD} = \frac{b+c}{c}$$

$$\text{or} \quad BD = \frac{ac}{b+c} \quad \text{—— (II)}$$

Again BI is the bisector of $\angle B$ so it divides AD in the ratio AB : BD

$$\therefore \frac{AI}{ID} = \frac{AB}{BD} \Rightarrow \frac{c}{\frac{ac}{b+c}} \Rightarrow \frac{b+c}{a} \Rightarrow b+c : a \quad \text{(iii)}$$

Now, from (i) it follows that D divides BC in the ratio c : b

$\therefore$ The co-ordinates of D are

$$\left(\frac{cx_3+bx_2}{c+b}, \frac{cy_3+by_2}{b+c}\right)$$

~~Also from (i) it follows that D divides BC in the ratio c : b~~

Also from (iii) it follows that I divides AD in the ratio (b+c) : a

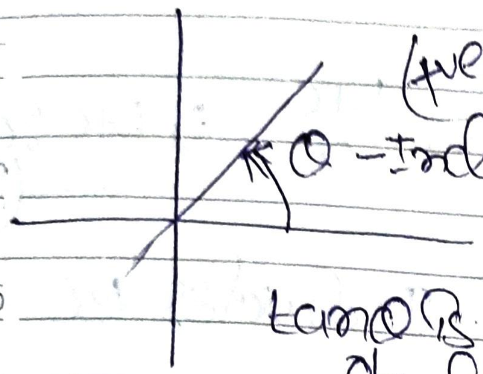
$\therefore$ The co-ordinates of I are (the incentre) are

$$\left(\frac{(b+c)\left(\frac{cx_3+bx_2}{c+b}\right)+ax_1}{a+b+c}, \frac{b+c\left(\frac{cy_3+by_2}{b+c}\right)+ay_1}{a+b+c}\right)$$

i.e $$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right).$$

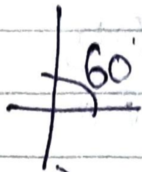
$\square$.

Slope of Line



(+ve direction of x-axis in anticlockwise direction)

θ - Inclination of line



$60°$

$\Rightarrow m = \tan 60°$ $\quad m = \tan 45°$

$\boxed{m = \sqrt{3}}$ $\quad \boxed{m = 1}$

$\tan\theta$ is called slope of line.

Sloped line $l = m$

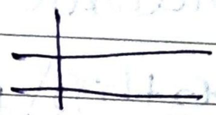
$$\boxed{m = \tan\theta}$$

* $0 < \theta < 90°$ slope is +ve.

* $\theta = 90°$ $= m = \tan 90° = \infty$ (not defined)
line is parallel to y-axis or line is y-axis itself.

(*) $\theta > 90°$ $= m$ (-ve) - slope.

* where line is parallel to x-axis then it's slope is 0.
or x-axis itself.

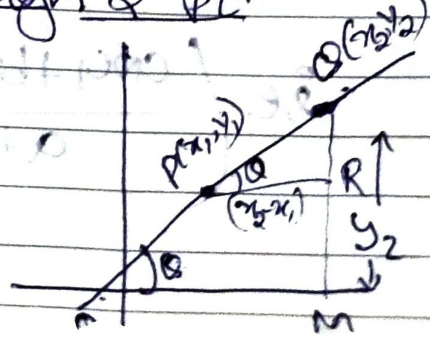


$\tan\theta = m = 0$.

Slope of a line passing through 2 pt.

Slop of line $PQ = \tan\theta$

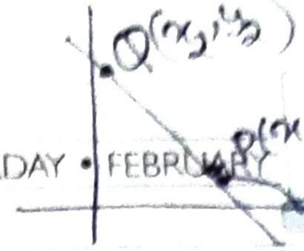
$$= \frac{QR}{PR} = \frac{y_2 - y_1}{x_2 - x_1}$$



$QM = y_2$, $RM = y_1$

$QR = y_2 - y_1$

$PR = (x_2 - x_1)$



If a line passing through 2 pts $P(x_1,y_1)$ & $Q(x_2,y_2)$

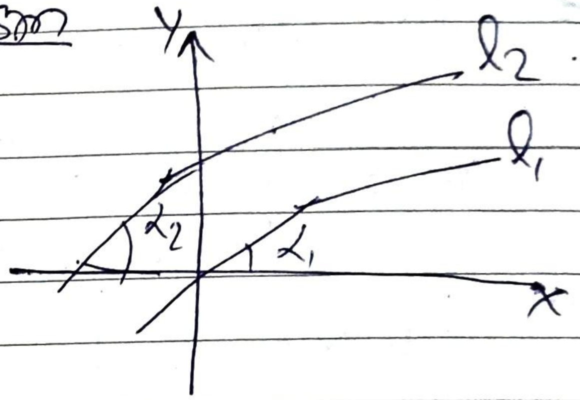
then slope is $\boxed{m = \dfrac{y_2-y_1}{x_2-x_1}}$

Q: $P(2,3)$ & $Q(4,5)$

$\Rightarrow \dfrac{5-3}{4-2} = \dfrac{2}{2} = 1 \Rightarrow$ ~~PQ is 45°~~

→ PQ is 45° with +ve direction of x-axis.

<u>Condition for Parallelism</u>



$m_1 =$ slope $l_1 = \tan\alpha_1$

$m_2 =$ slope of $l_2 = \tan\alpha_2$

As line $l_1 \parallel$ line l_2

$\Rightarrow \alpha_1 = \alpha_2$

$\tan\alpha_1 = \tan\alpha_2$ $\quad \boxed{m_1 = m_2}$

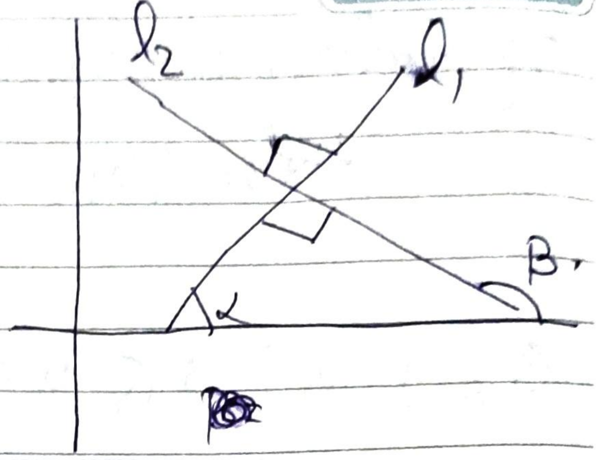
If 2 line with slope m_1 & m_2 are $\parallel$ then slope $\boxed{m_1 = m_2}$ are equal.

<u>Condition of perpendicularity</u>

If 2 lines with slope m_1 & m_2 are $\perp$ then $m_1 \times m_2 = -1$ $\quad \boxed{m_1 \times m_2 = -1}$

An exterior angle of a triangle is equal to the sum of it's opposite, non-adjacent interior angles.

Exterior Angle Property



$\beta = 90 + \alpha$

$\tan\beta = \tan(90+\alpha)$

$m_2 = -\cot\alpha$

$m_2 = -\frac{1}{\tan\alpha}$

$m_2 = -\frac{1}{m_1} \Rightarrow \boxed{m_1 \times m_2 = -1}$

Q:- line through $(-2,6)$ & $(4,8)$ $\perp$ to line through $(8,12)$ & $(x,24)$

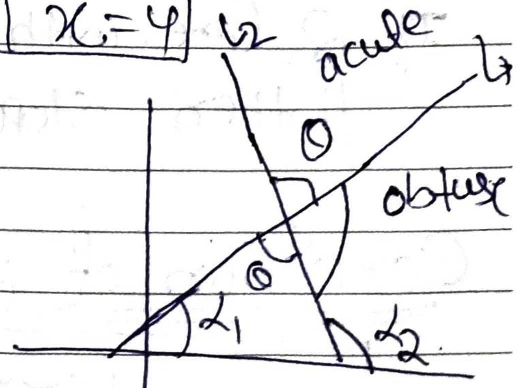
$m_1 = \frac{8-6}{4+2} = \frac{2}{6} = \frac{1}{3}$ $\quad m_2 = \frac{24-12}{x-8} = \frac{12}{x-8}$

$m_1 \times m_2 = -1, \quad \frac{1}{\cancel{3}} \times \frac{\cancel{12}^{4}}{x-8} = -1$ $\quad \cancel{4 = x-8}$ $\quad \cancel{x=12}$

$x-8 = -4$

$x = 8-4 \Rightarrow \boxed{x=4}$

Angle betn 2 lines



Slope of line $l_1 = m_1 = \tan\alpha_1$

Slope of line $l_2 = m_2 = \tan\alpha_2$

$\alpha_2 = \alpha_1 + \theta$

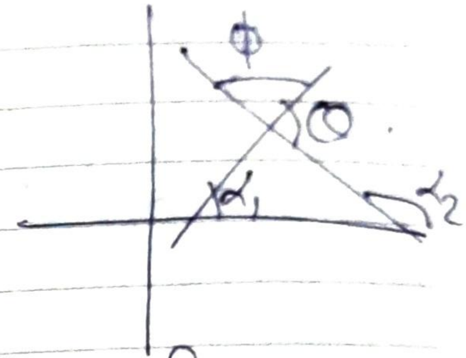
$\alpha_2 - \alpha_1 = \theta$

$\tan\theta = \tan(\alpha_2 - \alpha_1)$

$\tan\theta = \tan(\alpha_2 - \alpha_1)$

$= \dfrac{\tan\alpha_2 - \tan\alpha_1}{1 + \tan\alpha_2 \tan\alpha_1}$

$$\boxed{\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}}$$



$$\boxed{\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|}$$ Acute angle θ.

$$\boxed{\text{Obtuse Angle} = \phi = 180 - \theta}$$

Q/ Angle l_1 - $(0,0)$ & $(2,3)$, l_2 $(2,-2)$ & $(3,5)$.

Slope. $m_1 = \dfrac{3-0}{2-0} = \dfrac{3}{2}$ $\qquad \dfrac{5+2}{3-2} = \dfrac{7}{1} = 7$

$\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\dfrac{7 - \frac{3}{2}}{1 + 7\cdot\frac{3}{2}}\right| = \left|\dfrac{\frac{14-3}{\cancel{2}}}{\frac{2+21}{\cancel{2}}}\right| = \dfrac{11}{23}$

$\boxed{\theta = \tan^{-1}\left(\frac{11}{23}\right)}$ - acute.

obtuse angle $\phi = 180 - \theta$

$\Rightarrow \boxed{180 - \tan^{-1}\left(\frac{11}{23}\right)}$

NOTES

IMPORTANT

Record the list of things you might want to do.

Q: If Angle betⁿ line is $\theta = \pi/4$, $m_1 = 1/2$ find the slope of other line.

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$= \tan\left(\pi/4\right) = \left|\frac{m_2 - 1/2}{1 + \frac{1}{2} m_2}\right|$$

$$\frac{m_2 - 1/2}{1 + \frac{m_2}{2}} = \pm 1$$

(+ve)

$$\frac{\frac{2m_2 - 1}{2}}{\frac{2 + m_2}{2}} = 1 \Rightarrow 2m_2 - 1 = 2 + m_2$$

$$\Rightarrow \boxed{m_2 = 3}$$

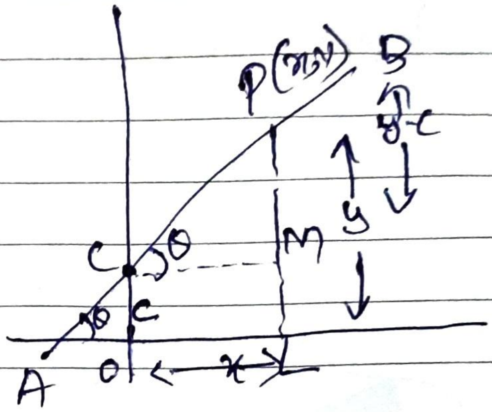
(-ve)

$$\frac{\frac{2m_2 - 1}{2}}{\frac{2 + m_2}{2}} = -1 \Rightarrow 2m_2 - 1 = -2 - m_2$$

$$\Rightarrow 3m_2 = -1, \boxed{m_2 = \frac{-1}{3}}$$

Equation of a line in slope-intercept form

Theorem-2 The equation of a line with slope m & making an intercept 'c' on y-axis is given by $y = mx + c$.



$$\tan\theta = \frac{PM}{CM}$$

$$m = \frac{y-c}{x}$$

$\Rightarrow mx = y - c$ or $\boxed{y = mx + c}$

Cor 1 :- The equation of line with slope m & passing through the origin is $y = mx$.

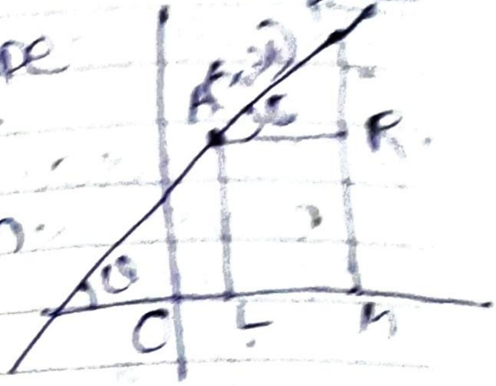
Proof $c = 0$, $y = mx$,

Cor-2 If $m = 0$, $c \neq 0$, then the eqn $y = mx + c$ reduces to $y = c$, which is the equation of line parallel to x-axis at a distance 'c' from it.

Cor-3 If $m = 0$, $c = 0$ then the equation becomes $y = 0$ which represents the x-axis,

(Equation of line point-slope form)

Theorem-2 The equation of line with slope 'm' & passing through a point (x_1, y_1) is given by $(y - y_1) = m(x - x_1)$



Let there be a line with slope m & passing ~~through~~ a point $A(x_1, y_1)$ is ~~given by~~.
Let $P(x, y)$ be any point on this line.

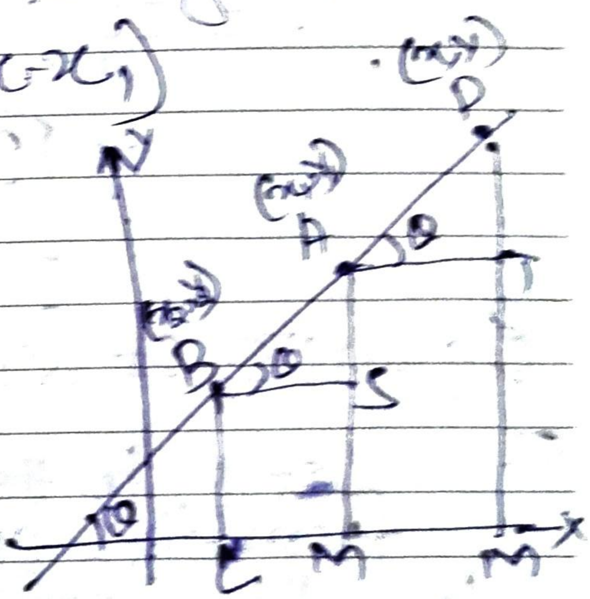
Now slope of a line through (x_1, y_1) & (x, y) is $\frac{y - y_1}{x - x_1}$

$$m = \frac{y - y_1}{x - x_1} \Rightarrow \boxed{y - y_1 = m(x - x_1)}$$

Equation of line in (2 pt form)

(3)- The equation of a line through two points (x_1, y_1) & (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$



Clearly the slope of AP = the slope of AB.

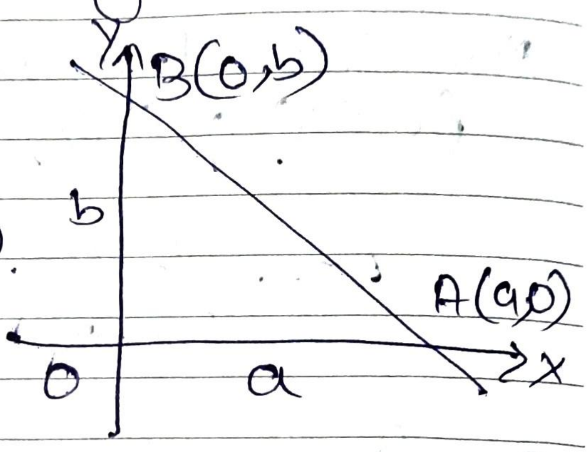
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow (y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Equation of line in Intercept form.

4- The equation of a straight line which makes intercepts of length 'a' & 'b' on x-axis & y-axis respectively is

$$\frac{x}{a} + \frac{y}{b} = 1$$



Proof: The co-ordinates of A, B are $A(a,0)$ & $B(0,b)$.

∴ The equation of the line joining A & B is

$$(y-0) = \frac{b-0}{0-a}(x-a)$$

$$y = \frac{-b}{a}(x-a) \Rightarrow \frac{y}{b} = \frac{-x}{a} + 1$$

$$\Rightarrow \boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

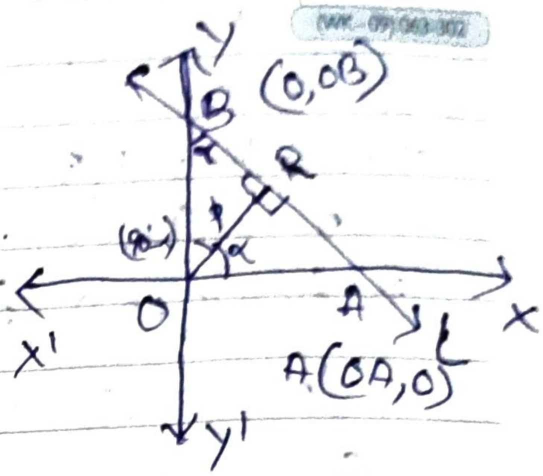
Equation of line in normal (perpendicular) form:

5- Let p be the length of perpendicular from the origin to a given line & α be the angle made by this perpendicular with the positive direction of x-axis. Then the equation of the line is given by

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$

$\cos\alpha = \frac{B}{h}$

$\sin\alpha = \frac{P}{h}$



Proof: In ΔORA

$\cos\alpha = \frac{OR}{OA}$

$\Rightarrow \cos\alpha = \frac{P}{OA}$

$\boxed{OA = \frac{P}{\cos\alpha}}$

In BOR,

$\sin\alpha = \frac{OL}{OB}$

$\Rightarrow \sin\alpha = \frac{P}{OB}$

$OB = \frac{P}{\sin\alpha}$.

$\frac{x}{a} + \frac{y}{b} = 1$

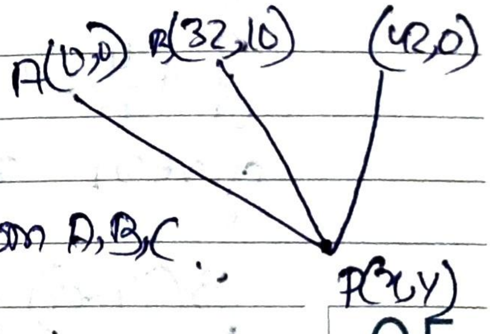
$\frac{x}{P/\cos\alpha} + \frac{y}{P/\sin\alpha} \Rightarrow \frac{x\cos\alpha}{P} + \frac{y\sin\alpha}{P} = 1$

$\Rightarrow \boxed{x\cos\alpha + y\sin\alpha = P}$

Q: 12:

let A(0,0), B(32,0) & C(42,0) be 3 pt's so,

given P(x,y) is equidistance from A, B, C.



PA = PB = PC

Now, PA = PB.

$\Rightarrow x^2 + y^2 = (32-x)^2 + (10-y)^2$

$\Rightarrow \cancel{x^2} + \cancel{y^2} = (32)^2 + \cancel{x^2} - 2(32)x + (10)^2 + \cancel{y^2} - 2\,10.y$.

$\Rightarrow 1024 + 100 - 64x - 20y = 0$

$64x + 20y = 1124$

$\Rightarrow 4(16x + 5y) = 1124$

$16x + 5y \longrightarrow 281 \longrightarrow (1)$.

$PA = PC$

$PA^2 = PC^2$

$\Rightarrow x^2+y^2 = (42-x)^2+y^2$

$x^2-(42-x)^2=0$

$\Rightarrow (x-42+x)\,(x-42-x)=0.$

$\boxed{x=21}$

$(x=21, y=11)$

13: $\boxed{(a-b)^2-(a+b)^2=4ab}$

Q: Find the eqn of straight line which passes through the pt (3,4) & the sum of the intercepts on the axes is 14.

$\frac{x}{a}+\frac{y}{b}=1$, Given line passes through the pt3.

$\frac{3}{a}+\frac{4}{b}=1$ or $3b+4a=ab$. —①

given ~~x+y~~ $a+b=14$. —②

$\boxed{b=14-a}$

$3(14-a)+4a=a(14-a)$.

so $a=6$ or $b=8$, & $a=7, b=7$,

$\frac{x}{6}+\frac{y}{8}=1$, & $\frac{x}{7}+\frac{y}{7}=1$

or $4x+3y=24$ & $x+y=7$ are the eqn of required line.

Q: Find eqn of the line which passes through P(1,2) & ‖ to the line $x+2y+3=0$.

Soln $m=-\frac{1}{2}$, , $y-y_1=m(x-x_1)$

$y-2=-\frac{1}{2}(x-1)$

Equation of a line in General form

1- The equation $ax+by+c=0$ always represents a Straight line provided a & b are not simultaneously zero.

Proof $ax+by+c=0$ where $a \neq 0$ or $b \neq 0$.

~~i.e~~ $\cancel{y=\left(\frac{-c}{b}\right)}$

case I:- when $a=0$, $b \neq 0$,

$$by+c=0 \Rightarrow y=\left(\frac{-c}{b}\right)$$

which represent line parallel to x-axis.

Case-2: when $a \neq 0$ & $b=0$.

In this case the eq^n reduces.

$$ax+c=0 \Rightarrow x=\left(\frac{-c}{a}\right)$$

which represent line parallel to y-axis.

case-III $a \neq 0$, $b \neq 0$,

In this case, the eq^n may be re-written as:

$$y=\left(\frac{-a}{b}\right)x+\left(\frac{-c}{b}\right)$$

This clearly represents a line in slop intercept form with slop $\left(-\frac{a}{b}\right)$ & y intercept $\left(-\frac{c}{b}\right)$.

Angle bet^n 2 ~~line~~ Intersecting lines:

① - Suppose that the line $y=m_1x+c_1$ & $y=m_2x+c_2$ intersect at a pt, Then the angle bet^n them is given by:

$$\boxed{\theta=\tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|}$$

Condition of parallelism of 2 lines:

The lines $y = m_1x_1 + c_1$ & $y = m_2x + c_2$ are parallel if and only if $m_1 = m_2$.

$\Rightarrow$ Given lines are parallel.

$\Leftrightarrow$ the angle betn them is zero.

$$\tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = 0, \quad \frac{m_1 - m_2}{1 + m_1m_2} = 0 ;$$

$\Leftrightarrow m_1 - m_2 = 0$ i.e $\boxed{m_1 = m_2}$.

Condition of perpendicularity of two lines:

The lines $y = m_1x + c_1$ & $y = m_2x + c_2$ are perpendicular if & only if $m_1m_2 = -1$

The given lines are perpendicular.

$\theta = 90°$, where θ is angle betn the lines.

$$\Rightarrow \cot 90° = \frac{1 + m_1m_2}{m_1 - m_2}$$

$$\Rightarrow 1 + m_1m_2 = (m_1 - m_2)\cot 90° = 0$$

$$\Rightarrow \boxed{m_1m_2 = -1}$$

4- The angle betⁿ the intersecting lines

$a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$

is $\tan^{-1}\left|\dfrac{a_2b_1-a_1b_2}{a_1a_2+b_1b_2}\right|$

Proof

Let θ be the angle betⁿ the given lines and m_1 & m_2 be their slopes. Then.

$a_1x+b_1y+c=0$ & $a_2x+b_2y+c=0$

$y=\left(\dfrac{-a_1}{b_1}\right)x+\left(\dfrac{-c}{b_1}\right)$ $\quad y=\left(\dfrac{-a_2}{b_2}\right)x+\left(\dfrac{-c}{b_2}\right)$

$\therefore m_1=\left(\dfrac{-a_1}{b_1}\right)$ & $m_2=\left(\dfrac{-a_2}{b_2}\right)$

$$\tan\theta=\left|\dfrac{-\dfrac{a_1}{b_1}+\dfrac{a_2}{b_2}}{1+\left(\dfrac{-a_1}{b_1}\right)\left(\dfrac{-a_2}{b_2}\right)}\right|=\left|\dfrac{\dfrac{a_2}{b_2}-\dfrac{a_1}{b_1}}{1+\dfrac{a_1a_2}{b_1b_2}}\right|$$

$$\Rightarrow \left|\dfrac{\dfrac{a_2b_1-a_1b_2}{\cancel{b_1b_2}}}{\dfrac{b_1b_2+a_1a_2}{\cancel{b_1b_2}}}\right|=\left|\dfrac{a_2b_1-a_1b_2}{a_1a_2+b_1b_2}\right|$$

$$\theta=\tan^{-1}\left|\dfrac{a_2b_1-a_1b_2}{a_1a_2+b_1b_2}\right|$$

$$\text{Slope} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } y}$$

Transformation of general Equation to different Standard forms.

8
Slope Intercept form ($y=mx+c$)

9
We have $Ax+By+C=0$

10
$By = -Ax - C$

11
$\Rightarrow y = \left(-\frac{A}{B}\right)x + \left(\frac{-C}{B}\right)$ which is of the form $y=mx+c$.

12
$m = \left(\frac{-A}{B}\right)$, $c = \left(\frac{-C}{B}\right)$.

Intercept on x-axis $= \frac{-\text{Constant term}}{\text{coefficient of } x}$

Intercept on y-axis $\frac{-C}{B} = \frac{-\text{Constant term}}{\text{coefficient of } y}$

1
Intercept form ($\frac{x}{a}+\frac{y}{b}=1$)

2
$Ax+By+C=0$

$Ax+By=-C$

3
or $\frac{Ax}{-C}+\frac{By}{-C}=1$ or $\frac{x}{\left(\frac{-C}{A}\right)}+\frac{y}{\left(\frac{-C}{B}\right)}=1$

which is of the form $\frac{x}{a}+\frac{y}{b}=1$

Normal form ($x\cos\alpha + y\sin\alpha = p$)

$Ax+By+C=0$ —(1)

$x\cos\alpha + y\sin\alpha - p = 0$ —(2)

If eqn (1) & (2) represent same straight line,

$\Rightarrow$ Therefore $\frac{A}{\cos\alpha} = \frac{B}{\sin\alpha} = \frac{C}{-p}$

or $\cos\alpha = \frac{-Ap}{C}$ & $\sin\alpha = \frac{-Bp}{C}$.

$\cos^2\alpha + \sin^2\alpha = \frac{A^2p^2}{C^2} + \frac{B^2p^2}{C^2} \Rightarrow 1 = \frac{p^2}{C^2}(A^2+B^2)$

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To transform the general eq^n to normal form we perform the following steps:
(i) Shift the constant term on the RHS & make it positive
(ii) Divide both sides by $\sqrt{(\text{coefficient of } x)^2 + (\text{co-efficient of } y)^2}$

$$P = \pm \frac{C}{\sqrt{A^2+B^2}}$$ (But P denote the length of ⊥ from the origin to the line and is always +ve).

Putting in eq^n we get.

$$-\frac{A}{\sqrt{A^2+B^2}}x - \frac{B}{\sqrt{A^2+B^2}}y = \frac{C}{\sqrt{A^2+B^2}}$$

which is the required form of the line
$Ax+By+C=0$

Q: Transform the eq^n of line $\sqrt{3}x+y-8=0$
(i) Slope intercept form & find its slope & y-intercept
(ii) Intercept form & find the inclination of perpendicular segment from the origin.
(iii) normal form & find the inclination of the perpendicular segment from the origin on the line with the axis & its length.

Sol (i) we have $\sqrt{3}x+y-8=0$ or $y=-\sqrt{3}x+8$
Slope $=-\sqrt{3}$ & y-intercept $=8$

(ii) we have $\sqrt{3}x+y-8=0$,

or $$\frac{x}{\left(\frac{8}{\sqrt{3}}\right)}+\frac{y}{8}=1,$$ x-Intercept $\left(\frac{8}{\sqrt{3}}\right)$
y-Intercept $=(8)$.

(iii) $\sqrt{3}x+y=8$

$$\Rightarrow \frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2+1^2}}x+\frac{1}{\sqrt{(\sqrt{3})^2+1^2}}y=\frac{8}{\sqrt{(\sqrt{3})^2+1^2}}$$

Equation of a line parallel given line.

Let m be the slope of the line $ax+by+c=0$

Then slope $m = -\frac{a}{b}$ $\left(\text{using } m = -\frac{\text{coefficient of } x}{\text{coefficient of } y} \right)$

Let c_1 be the y-intercept of required line

Therefore, the eq^n of the required line is

$y = mx + c_1$ (using slope-Intercept form)

or $y = -\frac{a}{b}x + c_1$

or $ax + by - bc_1 = 0$

or $ax + by + \lambda = 0$, (where $\lambda = -bc_1 = \text{constant}$)

Therefore the eq^n of a line parallel to a given line $ax+by+c=0$, is $ax+by+\lambda=0$ where λ is a constant.

(To write a line parallel to a given line, we keep the expression containg x & y same and simply replace the given constant by a new constant λ, The value of λ can be determined by some given condition)

Equation of a line perpendicular to a given line

Let m_1 be the slope of the given line & m_2 be the slope of a line perpendicular to the given line

Then $m_1 = -\frac{a}{b}$ & $m_1 m_2 = -1$ (using $\perp$ condition)

Therefore $m_2 = -\frac{1}{m_1} = \frac{b}{a}$

Let c_2 be the y-intercept of the required line. Then it's equation is.

$$y = m_2 x + c_2.$$

$$y = \frac{b}{a}x + c_2 \quad , bx - ay + ac_2 = 0$$

or $bx - ay + \lambda = 0$, where $\lambda = ac_2 = \text{Constant}$

Therefore, the eq^n of a line perpendicular to a given line $ax + by + c_1 = 0$, is $bx - ay + \lambda = 0$ where λ is a constant.

To write a line perpendicular to a given line

(i) Interchange x & y

(ii) If the co-efficients of x and y in the given eq^n are of the same sign, make them of opposite signs & if the coefficients are of opposite signs, make them of the same sign.

(iii) Replace the given constant by a new constant λ, which is determined by a given condition.

Corollary-1 The line $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_1=0$

are parallel if & only if $\frac{a_1}{a_2}=\frac{b_1}{b_2}$

Proof: slopes must be same

$$-\frac{a_1}{b_1}=-\frac{a_2}{b_2} \quad [\because m_1=m_2]$$

$$\Rightarrow \frac{a_1}{b_1}=\frac{a_2}{b_2} \Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}.$$

Corollary-2 The lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are perpendicular if and only if $\boxed{a_1a_2+b_1b_2=0}$.

Proof $a_1x+b_1y+c_1=0$

$$y=\left(-\frac{a_1}{b_1}\right)x+\left(-\frac{c_1}{b_1}\right)$$

and $a_2x+b_2y+c_2=0$

$$\Rightarrow y=\left(-\frac{a_2}{b_2}\right)x+\left(-\frac{c_2}{b_2}\right)$$

Given lines are perpendicular.

$$\left(-\frac{a_1}{b_1}\right)\left(-\frac{a_2}{b_2}\right)=-1 \quad [m_1m_2=-1]$$

$$\Rightarrow \boxed{a_1a_2+b_1b_2=0}$$

Point of Intersection of two lines.

The co-ordinates of the point of intersection of the 2 intersecting lines

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

Proof let the lines $a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$ intersect at point $P(x_1, y_1)$

Then (x_1, y_1) must satisfy each one of the given eqn.

$$a_1x_1 + b_1y_1 + c_1 = 0 \text{ --- (i)}$$
$$a_2x_1 + b_2y_1 + c_2 = 0 \text{ --- (ii)}$$

Solving (i) & (ii) by cross multiplication.

$$\frac{x_1}{b_1c_2 - b_2c_1} = \frac{y_1}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\therefore x_1 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ \& } y_1 = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

Hence the co-ordinates of the point of intersection are.

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

For finding the co-ordinates of the point of intersection of 2 lines whose equations are general form, we solve the given eqn simultaneously & the value of x & y so obtained determine the co-ordinates of the points.

Condition of concurrency of three lines.

If they pass through a common point.

$$a_1x+b_1y+c_1=0 \quad \text{—(1)}$$
$$a_2x+b_2y+c_2=0 \quad \text{—(2)}$$
$$a_3x+b_3y+c_3=0 \quad \text{—(3)}$$

to be concurrent is that:

$$\boxed{a_1(b_2c_3-b_3c_2)+b_1(c_2a_3-c_3a_2)+c_1(a_2b_3-a_3b_2)=0}$$

or

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Proof. Solving (ii) & (iii) we get

$$\frac{x}{b_2c_3-b_3c_2}=\frac{y}{c_2a_3-c_3a_2}=\frac{1}{a_2b_3-a_3b_2}.$$

$$\therefore x=\frac{b_2c_3-b_3c_2}{a_2b_3-a_3b_2} \ \& \ y=\frac{c_2a_3-c_3a_2}{a_2b_3-a_3b_2}$$

So, the point of intersection (ii) & (iii) is.

$$\left(\frac{b_2c_3-b_3c_2}{a_2b_3-a_3b_2}, \frac{c_2a_3-c_3a_2}{a_2b_3-a_3b_2}\right)$$

The 3 given eqn of lines will be concurrent if this point lies on (i) i.e. if

$$a_1\left(\frac{b_2c_3-b_3c_2}{a_2b_3-a_3b_2}\right)+b_1\left(\frac{c_2a_3-c_3a_2}{a_2b_3-a_3b_2}\right)+c_1=0$$

$$\Rightarrow a_1(b_2c_3-b_3c_2)+b_1(c_2a_3-c_3a_2)+c_1(a_2b_3-a_3b_2)=0$$

* Distance betⁿ 2 ∥ line $= \left| \dfrac{c_1 - c_2}{\sqrt{a^2+b^2}} \right|$

Conditions for 2 lines to be coincident, parallel perpendicular or intersect :

Two lines $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$ are

(i) ~~Coincident~~ coincident, if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$.

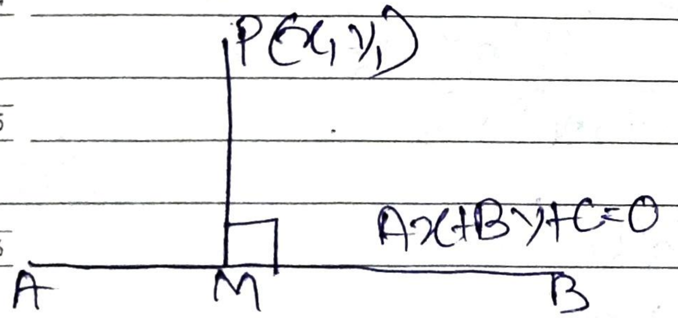
(ii) parallel if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$.

(iii) Perpendicular, if $a_1a_2+b_1b_2=0$

(iv) Intersecting, if they are neither coincident nor parallel.

Perpendicular distance

Length of perpendicular from a point $P(x_1,y_1)$ to a line $Ax+By+C=0$



PM is the length of ⊥ from the point $P(x_1,y_1)$ to the line AB which has equation $Ax+By+C=0$.

$$|PM| = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}},$$

* The length of perpendicular from ~~a pt~~ the origin to the line $Ax+By+C=0$ is $\dfrac{C}{\sqrt{A^2+B^2}}$

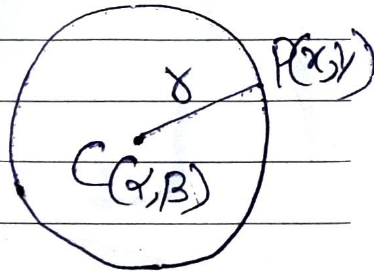
CIRCLE

A circle is the locus of a point which moves on a plane in such a way that it's distance from a fixed point is always constant. The fixed point is called the centere of the circle & the constant distance is called the radius of the circle.

(Locus: If a pt's moves according to some given geometrical conditions).

(1): Equation of a circle (centre & radius form).

Let C(α, β) be the centre of the circle & radius of the circle be 'r'. Let P(x, y) be any point on the circumference of the circle.



$CP = r$.

By distance formular.

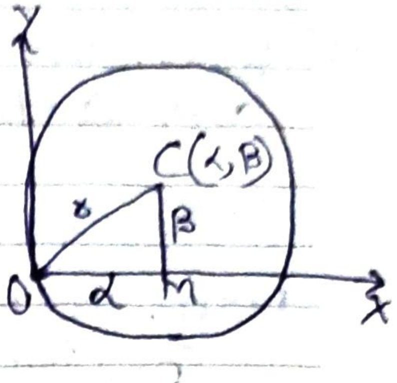
$$\sqrt{(x-\alpha)^2+(y-\beta)^2} = r$$

$$(x-\alpha)^2+(y-\beta)^2 = r^2 \quad \text{—(1)}.$$

Some particular cases:

the standard eqn of circle with centre at C(α, β) & radius r, is $(x-\alpha)^2+(y-\beta)^2 = r^2$

(i) When the circle passes through the origin.



In right angle triangle ΔOCM

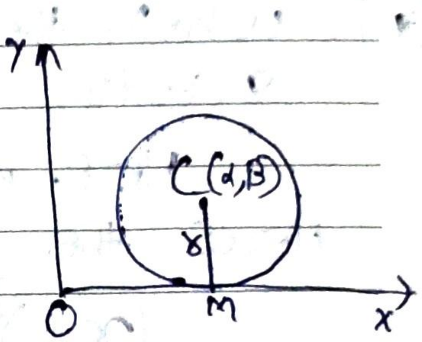
$OC^2 = OM^2 + CM^2$ i.e $r^2 = \alpha^2 + \beta^2$

eqn (i) becomes.

$$(x-\alpha)^2 + (y-\beta)^2 = \alpha^2 + \beta^2$$

or $$x^2 + y^2 - 2\alpha x - 2\beta y = 0$$

(ii) When the circle touches x-axis.



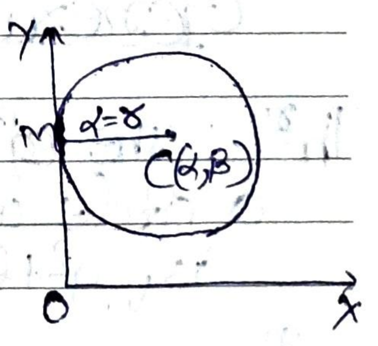
Here $r = \beta$

Hence the eqn of circle becomes.

$$(x-\alpha)^2 + (y-\beta)^2 = \beta^2$$

or $$x^2 + y^2 - 2\alpha x - 2\beta y + \alpha^2 = 0$$

(iii) When the circle touches y-axis.



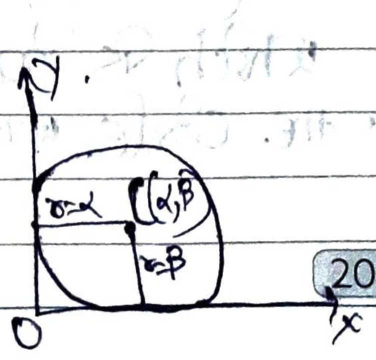
Here $r = \alpha$,

Hence the eqn of the circle becomes,

$$(x-\alpha)^2 + (y-\beta)^2 = \alpha^2$$

or $$x^2 + y^2 - 2\alpha x - 2\beta y + \beta^2 = 0$$

(iv) When the circles touches both the axes. : Here $\alpha = \beta = r$



the eqn of the circle.

$$(x-r)^2 + (y-r)^2 = r^2$$

$$\Rightarrow x^2 + y^2 - 2rx - 2ry + r^2 = 0$$

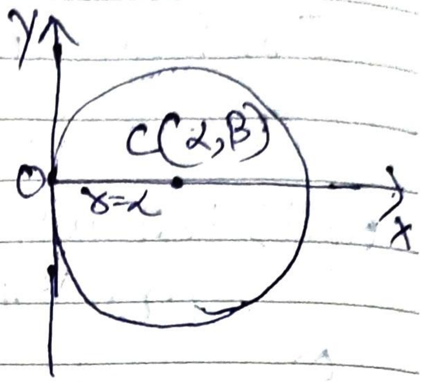
(v) When the circle passes through the origin and centre lies on x-axis :-

Here $r = \alpha$ & $\beta = 0$

Hence the eq^n of the circle becomes.

$$(x-\alpha)^2+(y-0)^2=r^2$$

$$x^2+y^2-2ry=0$$



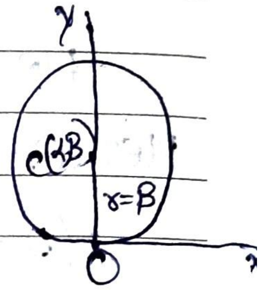
(vi) When the circle passes through the origin & centre lies on y-axis.

Here $\alpha=0$ & $\beta=r$

Hence, the eq^n of the circle becomes.

$$(x-0)^2+(y-r)^2=r^2$$

or $$x^2+y^2-2ry=0$$



General form

Th^m : The eq^n $x^2+y^2+2gx+2fy+c=0$ always represents a circle whose centre is at $(-g,-f)$ & radius is $\sqrt{g^2+f^2-c}$.

Sol^n $(x-(-g))^2+(y-(-f))^2=\left(\sqrt{g^2+f^2-c}\right)^2$

which is in the standard form (i.e. $(x-\alpha)^2+(y-\beta)^2=r^2$) of the circle with centre at (α,β) & radius 'r'

Note: To find the centre & radius of the circle, which is in the form $ax^2+ay^2+2gx+2fy+c=0$ where $a \neq 0$.

Divide both sides of the eq^n by co-efficient of x^2 or y^2 (i.e. a) to get

$$x^2+y^2+\frac{2g}{a}x+\frac{2f}{a}y+\frac{c}{a}=0$$

which is the general form of the circle.

Hence, the co-ordinates of the centre are

$$\left(-\frac{1}{2}\text{coeff. of } x, -\frac{1}{2}\text{coeff. of } y\right)$$

$$=\left(-\frac{1}{2}\frac{2g}{a}, -\frac{1}{2}\frac{2f}{a}\right) = \left(\frac{-g}{a}, \frac{-f}{a}\right)$$

and radius $\sqrt{\left(-\frac{1}{2}\text{coeff. of } x\right)^2+\left(-\frac{1}{2}\text{coeff. of } y\right)^2 - \text{constant term}}$

$$=\sqrt{\frac{g^2}{a^2}+\frac{f^2}{a^2}-\frac{c}{a}}$$

Diameter form: (Equation of circle with given end pt of a diameter).

Let $A(x_1, y_1)$ & $B(x_2, y_2)$ be 2 pt's of diameter. Let $P(x, y)$ be any pt's on the circle.

Join AP & BP $\therefore \angle APB = 90°$

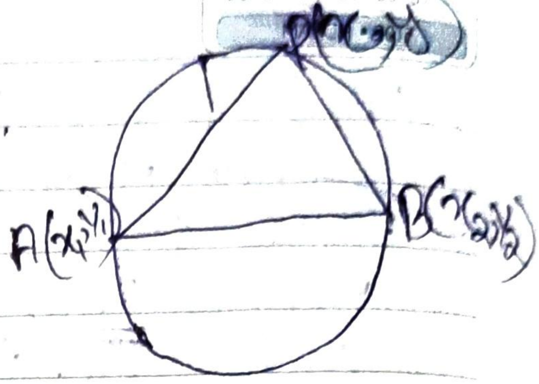
($\because$ An angle on a semi-circle is right angle)

Now, slope of AP $= \frac{y-y_1}{x-x_1}$ & slope of BP $= \frac{y-y_1}{x-x_2}$.

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Since $AP \perp BP$

By condition of perpendicularity, the product of their slopes = -1.



8

9 $$\left(\frac{y-y_1}{x-x_1}\right)\left(\frac{y-y_2}{x-x_2}\right) = -1$$

10 $$\Rightarrow (y-y_1)(y-y_1) = -(x-x_1)(x-x_2)$$

11 $$\Rightarrow (x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

12

1

2

3

4

5

6