ENGINEERING MATHEMATICS -I
FOR DIPOLMA STUDENTS

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PARTIAL FRACTION

Dt/29/10/24

Polynomial : $a_0 + a_1x + a_2x^2 \ldots a_nx^n$

$f(x) = ax^2 + bx + c \neq 0$

* **Rational fraction**

$$\frac{f(x)}{g(x)}, g(x \neq 0$$

ex:- $\frac{2x}{x+1}$, $\frac{x-1}{(x-2)(x-3)}$, $\frac{1}{x^2}$

1) **proper fraction** :-

If the degree of numerator is less than the degree of of denominator,

ex:- $\frac{1}{(x+2)(x-3)}$, $\frac{x}{x^2+1}$

2) **Improper fraction** :-

If the degree of numerator is greater then the degree of denominator.

ex:- $\frac{2x^2+1}{(x-1)}$, $\frac{x^2+2x}{(x+1)}$

* **partial fraction** :-

Expressing a rational fraction as the sum o two or more simple fraction is called resolvi a given fraction into partial fraction.

Case-1

(when the denominater contains non reapeted

Linear factors.)

Q. $\frac{x}{(x-1)(x-2)}$ split into P.F

Sol" $\frac{x}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$

$\Rightarrow \frac{x}{(x-1)(x-2)} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$

$\Rightarrow x = A(x-2) + B(x-1)$ ——— (1)

putting $(x-1) = 0$

$\Rightarrow x = 1$

$\Rightarrow 1 = A(1-2) + B(1-1)$

$\Rightarrow 1 = -A$, $\Rightarrow \boxed{A = -1}$

putting $(x-2) = 0$

$\Rightarrow x = 2$

$\Rightarrow 2 = A(2-2) + B(2-1)$

$\Rightarrow \boxed{2 = B}$

So put the value of A & B in the requirement P.F

$\therefore \frac{x}{(x-1)(x-2)} = \frac{-1}{(x-1)} + \frac{2}{(x-2)}$

Q:- $\frac{2x+1}{(x+1)(x-2)(x-5)}$

Soln $\frac{2x+1}{(x+1)(x-2)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$\Rightarrow \frac{2x+1}{(x+1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)}{(x+1)(x-2)(x-3)}$

$\Rightarrow 2x+1 = A(x-2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)$ ——(1)

Putting $(x+1) = 0$

$x = -1$

$\Rightarrow 2(-1)+1 = A(-1-2)(-1-3) + B(-1+1)(-1-3) + C(-1+1)(-1-2)$

$\Rightarrow -1 = 12A = \frac{-1}{12}$

Putting $(x-2) = 0$

$\Rightarrow x = 2$

$\Rightarrow 2 = A(2-2)(2-3) + B(2+1)(2-3) + C(2+1)(2-2)$

$\Rightarrow 5 = -3B \Rightarrow \boxed{B = -\frac{5}{3}}$

Putting $(x-3) = 0$

$x = 3$

$\Rightarrow 2 \cdot 3 + 1 = C(3+1)(3-2)$

$\Rightarrow 7 = 4C$

$\Rightarrow \boxed{C = \frac{7}{4}}$

Q: $\frac{x^2}{(x-1)(x-2)(x-3)}$

$\Rightarrow \frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$\Rightarrow \frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$\Rightarrow x^2 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

putting $(x-1) = 0$

$\Rightarrow x = 1$

putting $(x-2) = 0$

$x = 2$

$\Rightarrow 2 = A(2-2)(2-3) + B(2+1)(2-3) + C(2+1)(2-2)$

$\Rightarrow B = -\frac{2}{3}$

putting $(x-3) = 0$

$x = 3$

$\Rightarrow 2C = 9$

$C = 9/2$

Q. $\frac{x-2}{(x-4)(x-7)}$

$\Rightarrow \frac{(x-2)(x+4)}{(x-4)(x-7)} = \frac{A}{(x-4)} + \frac{B}{(x-7)}$

$\Rightarrow \frac{(x-2)(x+4)}{(x-4)(x-7)} = \frac{A(x-7)+B(x-4)}{(x-4)(x-7)}$

$\Rightarrow x = A(x-7) + B(x-4)$ ——— (1)

putting $(x-7) = 0$

$x = 7$,

Case-2

(when denominator contain repeated linear factor)

Q: $\frac{1}{(x-2)^2(x-1)}$

Sol^n $\frac{1}{(x-2)^2(x-1)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-1)}$

$\Rightarrow \frac{1}{(x-2)^2(x-1)} = \frac{A(x-2)(x-1) + B(x-1) + C(x-2)^2}{(x-2)^2(x-1)}$

$\Rightarrow 1 = A(x-2)(x-1) + B(x-1) + C(x-2)^2$

$\Rightarrow 1 = A(x^2 - x - 2x + 2) + B(x-1) + C(x^2 + 4 - 4x)$

putting $(x-2) = 0$

$\Rightarrow x = 2$

$\Rightarrow 1 = A(2-2)(2-1) + B(2-1) + C(2-2)^2$

$\Rightarrow 1 = 0 + B + 0$

$\Rightarrow \boxed{B = 1}$

Putting $(x-1) = 0$

$\Rightarrow x = 1$

$\Rightarrow 1 = A(1-2)(1-1) + B(1-1) + C(1-2)^2$

$\Rightarrow 1 = 0 + 0 + C$

$\Rightarrow \boxed{C = 1}$

equate the co-efficient of higest degree

$\Rightarrow 0 = A + C$

$\Rightarrow A + 1 = 0$

$\Rightarrow \boxed{A = 1}$

put the value of A, B, C in the required

$\therefore \frac{1}{(x-2)^2(x-1)} = \frac{1}{(x-2)} + \frac{1}{(x-2)^2} + \frac{1}{(x-1)}$

* $\frac{x^2-4x}{(x-3)^2(x-1)(x-4)}$

Soln $\frac{x^2-4x}{(x-3)^2(x-1)(x-4)} = \frac{A}{(x-3)^2} + \frac{B}{(x-1)} + \frac{C}{(x-4)}$

$\Rightarrow \frac{x^2-4x}{(x-3)^2(x-1)(x-4)} = \frac{A(x-1)(x-4) + B(x-3)^2(x-4) + C(x-3)^2(x-1)}{(x-3)^2(x-1)(x-4)}$

$\Rightarrow x^2-4x = A(x-1)(x-4)+B(x-3)(x-4)+(x-3)^2(x-1)$

$\Rightarrow$

putting $(x-1)=0$

$x=1$

$\Rightarrow 1^2-(4.1)$

$*\ \dfrac{x^2-4x}{(x-3)^2(x-1)(x-4)}$

$\dfrac{x^2-4x}{(x-3)^2(x-1)(x-4)} = \dfrac{A}{(x-3)}+\dfrac{B}{(x-3)^2}+\dfrac{C}{(x-1)}+\dfrac{D}{(x-4)}$

$\Rightarrow x^2-4x = A(x-1)(x-4)+B(x-3)(x-1)(x-4)+C(x-3)^2(x-4)$
$+D(x-3)^2(x-1)$

putting $x-1=0$

$x=1$

$\Rightarrow 1-4 = C(1-3)^2(1-4)$

$-3 = C(-4)(-3)$

$1 = 4C$

$\Rightarrow \boxed{C=\frac{1}{4}}$

putting $x-3=0$

$x=3$

$-3 = B(3-1)(3-4)$

$\Rightarrow -3 = B(2)(-1)$

$\Rightarrow -3 = +2B$

$\Rightarrow \boxed{B=3/2}$

putting $x - 4 = 0$
$x = 4$

$0 = D(4-3)^2(4-1)$

$\Rightarrow 0 = D \cdot 3$

$\Rightarrow \boxed{0 = D}$

equate with the heigest co efficient of h degree of x.

$0 = A + C + D$

$0 = A + \frac{1}{4} + 0$

$\Rightarrow \boxed{A = \frac{1}{4}}$

putting the value of A, B, C in the required

$= \frac{x^2 - 4x}{(x-3)^2(x-1)(x-4)} = \frac{1}{4(x-3)} + \frac{3}{2(x-3)^2} + \frac{1}{4(x-1)}$

* $\frac{x}{(x-2)^2(x+3)}$

Soln $\frac{x}{(x-2)^2(x+3)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x+3)}$

$\Rightarrow \frac{x}{(x-2)^2(x+3)} = \frac{A(x-2)(x+3) + B(x+3) + C(x-2)^2}{(x-2)^2(x+3)}$

$\Rightarrow x = A(x-2)(x+3) + B(x+3) + C(x-2)^2$ ——— (1)

$\Rightarrow x = A(x^2 + 3x - 2x - 6) + B(x+3) + C(x^2 + 4 - 4x)$

$\Rightarrow x = Ax^2 + Ax - 6A + Bx + 3B + Cx^2 + 4C - 4Cx$

putting $(x-2) = 0$

$\Rightarrow x = 2$

$\Rightarrow 2 = A(2-2)(2+3) + B(2+3) + C(2-2)^2$

$\Rightarrow 2 = 0 + 5B + 0$

$\Rightarrow \boxed{B = \frac{2}{5}}$

putting $(x+3) = 0$

$\Rightarrow x = -3$

$\Rightarrow -3 = A(-3-2)(-3+3) + B(-3+3) + C(-3-2)^2$

$\Rightarrow 3 = 0 + 0 + 25C$

$\Rightarrow \boxed{C = \frac{-3}{25}}$

equate the co-efficient of x^2

$\Rightarrow 0 = A + C$

$\Rightarrow A - \frac{3}{25} = 0 \quad \Rightarrow A = \frac{3}{25}$

put the value of A, B, C in required P.F

$\therefore \frac{x}{(x-2)^2(x+3)} = \frac{\frac{3}{25}}{(x-2)} + \frac{\frac{2}{5}}{(x-2)^2} - \frac{\frac{3}{5}}{(x+3)}$

$= \frac{3}{25(x-2)} + \frac{2}{5(x-2)^2} - \frac{3}{25(x+3)}$

$$* \frac{x^2-2x+1}{(x-3)^2(x+5)}$$

Sol" $$\frac{x^2-2x+1}{(x-3)^2(x+5)} = \frac{A}{(x-3)} + \frac{B}{(x-3)^2} + \frac{C}{x+5}$$

$$\Rightarrow \frac{x^2-2x+1}{(x-3)^2(x+5)} = \frac{A(x-3)(x+5) + B(x+5) + C(x-3)^2}{(x-3)^2(x+5)}$$

$$\Rightarrow x^2-2x+1 = A(x-3)(x+5) + B(x+5) + C(x-3)^2 \quad \text{——— (1)}$$

$$\Rightarrow x^2-2x+1 = A(x^2+5x-3x-15) + B(x+5) + C(x^2+6-6x)$$

$$\Rightarrow x^2-2x+1 = Ax^2+Ax-15A+Bx+5B+Cx^2+9C-9Cx$$

putting $(x-3) = 0$

$$\Rightarrow x = 3$$

$$\Rightarrow 9-6+1 = A(3-3)(3+5) + B(3+5) + C(3-3)^2$$

$$\Rightarrow 4 = 0+8B+0$$

$$\Rightarrow \boxed{B = \frac{4}{8} = \frac{1}{2}}$$

putting $(x+5) = 0$

$$\Rightarrow x = -5$$

$$\Rightarrow -5 = A(-5-3)(-5+5) + B(-5+5) + C(-5-3)^2$$

$$\Rightarrow -5 = 0+0+$$

* case-3

(when denominator contains irreducible quadratic factor) (ax^2+bx+c)

Q $\dfrac{1}{(x-1)(x^2+1)}$

Sol^n $\dfrac{1}{(x-1)(x^2+1)} = \dfrac{A}{(x-1)} + \dfrac{Bx+C}{x^2+1}$

$\Rightarrow \dfrac{1}{(x-1)(x^2+1)} = \dfrac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$

$\Rightarrow 1 = A(x^2+1)+(Bx+C)(x-1)$ — (1)

$\Rightarrow 1 = Ax^2+A+Bx^2-Bx+Cx-C$

Putting $(x-1)=0$

$\Rightarrow x = 1$

$\Rightarrow 1 = A(1^2+1)+(B\cdot 1+C)(1-1)$

$\Rightarrow 1 = 2A+0, \Rightarrow \boxed{A = \frac{1}{2}}$

Equate the coefficient of x^2

$\Rightarrow 0 = A+B$

$\Rightarrow \frac{1}{2}+B=0, \Rightarrow \boxed{B = -\frac{1}{2}}$

equate the co-eff of (x)

$\Rightarrow 0 = -B+C$

$\Rightarrow -\left(-\frac{1}{2}\right)+C=0$

$\Rightarrow \boxed{C = -1/2}$

put the value of A, B, C in the required P.F

$$\therefore \frac{1}{(x-1)(x^2+1)} = \frac{\frac{1}{2}}{(x-1)} + \frac{\left(-\frac{1}{2}x - \frac{1}{2}\right)}{x^2+1}$$

$$= \frac{1}{2(x-1)} - \frac{(x+1)}{2(x^2+1)}$$

Q. $\frac{x}{(x-2)(x^2+2)}$

Soln $\frac{x}{(x-2)(x^2+2)} = \frac{A}{(x-2)} + \frac{Bx+C}{(x^2+2)}$

$$\Rightarrow \frac{x}{(x-2)(x^2+2)} = \frac{A(x^2+2)+(Bx+C)(x-2)}{(x-2)(x^2+2)}$$

$$\Rightarrow x = A(x^2+2) + (Bx+C)(x-2) \quad \text{---------- (1)}$$

$$\Rightarrow x = Ax^2 + 2A + Bx^2 - 2Bx + Cx - 2C$$

putting $x - 2 = 0$

$x = 2$

$$\Rightarrow 2 = A(2^2+2) + (B\cdot 2 + C)(2-2)$$

$$\Rightarrow 2 = 6A + 0 \quad \Rightarrow A = \frac{2}{6} = \frac{1}{3}$$

putting equate the coefficient of x^2

$\Rightarrow 0 = A + B$

$\Rightarrow \frac{1}{3} + B = 0, \quad \Rightarrow \boxed{B = \frac{-1}{3}}$

equate the co-eff of (x)

$\Rightarrow 1 = -2B + C$

$\Rightarrow -2\left(\frac{-1}{3}\right) + C = 0$

put the value of A, B, C in the required P.F

$$\therefore \frac{1}{(x-2)(x^2+2)} = \frac{\frac{1}{3}}{(x-1)} + \frac{\left(-\frac{1}{3}x + \frac{1}{3}\right)}{x^2+1}$$